Pollinators in food webs: Mutualistic interactions increase diversity, stability, and function in multiplex networks

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https://github.com/kayla-hale/Multiplex-Dynamics/

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ABSTRACT

Ecosystems are composed of complex networks of many species interacting in different

ways. While ecologists have long studied food webs of feeding interactions, recent studies

increasingly focus on mutualistic networks including plants that exchange food for

reproductive services provided by animals such as pollinators. Here, we synthesize both

types of consumer-resource interactions to better understand the controversial effects of

mutualism on ecosystems at the species, guild, and whole-community levels. We find that

consumer-resource mechanisms underlying plant-pollinator mutualisms can increase the

persistence, productivity, abundance, and temporal stability of both mutualists and non-

mutualists in food webs. These effects strongly increase with floral reward productivity

and are qualitatively robust to variation in the prevalence of mutualism and pollinators

feeding upon resources in addition to rewards. This work advances the ability of

mechanistic network theory to synthesize different types of interactions and illustrates how

mutualism can enhance the diversity, stability, and function of complex ecosystems.

INTRODUCTION

As elegantly illustrated by Darwin's "tangled bank," [1] ecosystems are complex,

composed of many different types of interactions among many different species. However,

theory has classically predicted that complexity in terms of the number and strength of

interspecific interactions destabilizes ecological systems [2]. Mutualistic interactions like

those between plants and their pollinators are thought to be particularly destabilizing [3,4].

Robert May famously emphasized this point by calling mutualism an "orgy of mutual

benefaction" (pg. 95) [5] whose instability due to positive feedback loops helps explain why

mutualism is infrequent and unimportant in natural systems [3]. Yet, mutualisms appear to

be not only frequent but key to maintaining much of the biodiversity that drives ecosystems

[6,7], especially agricultural ecosystems essential to human wellbeing [8,9]. Here, we address

such disparities between theory and observation by developing and applying consumer-

resource theory of feeding and reproductive mechanisms that integrates food webs and

mutualistic networks into "multiplex" networks containing different types of interactions.

We use our multiplex model to study how mutualism affects the diversity, stability, and

function of complex ecosystems.

The integration pursued here benefits from long but largely separate traditions of

research on feeding and mutualistic interactions [8,10]. For example, "mutualistic" feeding

interactions between species with positive effects on each other (+,+), e.g. pollinators

foraging on the nectar of flowering plants, are often excluded from food web data (e.g., ref

[11]), while "antagonistic" feeding interactions (+, −), e.g. herbivory and predation, are

typically excluded from mutualistic networks [12]. Additionally, food web research has

focused more on aquatic systems [13] where feeding interactions are strongly structured by

body mass or gape size [13,14], while mutualism research has focused more on terrestrial

systems where feeding interactions may be more strongly structured by other organismal

traits like chemical defense and shape of mouth parts [13–15]. Within aquatic ecosystems, the

allometric trophic network theory [16,17] of food webs has leveraged body-size considerations

to successfully simulate the seasonal dynamics of many interacting species [18,19] and predict

the quantitative effects of experimental species manipulations [20–22]. However, the failure of

these predictions in the presence of facilitation (+, 0), e.g. habitat provisioning for mussels

by barnacles, highlights the need for food-web theory to better address interactions with

positive effects beyond nutrition [20]. Mutualistic network theory has focused on animal-

mediated pollination, an interaction involving trophic and reproductive effects [12,23],

motivated in part by large agricultural and evolutionary significance [9,24,25]. Merging these

distinct traditions requires a more unified approach that addresses several problems.

One significant problem is that classifying interactions based on positive and/or

negative effects ignores logical inconsistencies such as when "antagonistic" herbivory or

predation respectively increase plant fitness [26] or prey abundance [27] and when "mutualistic"

pollinators parasitize plants by robbing their floral rewards without transferring pollen [24].

We resolve such conflicts by modelling mechanisms by which organisms interact and

allowing effects to emerge from the interactions rather than asserting such effects *a priori*.

We do this by developing consumer-resource theory that has long been applied to food web

theory [16,28] and more recently applied to mutualistic interactions [29,30] with success

predicting pollinators' foraging preferences in the field [31]. Both feeding and mutualistic

interactions typically involve food consumption while mutualistic interactions also often

involve reproductive services provided by animals such as pollinators and seed dispersers

[27,29,30]. Positive, negative, and neutral interspecific effects dynamically emerge from the

benefits and costs of participating in these interactions, which often both involve the same

organisms and jointly determine much of the stability and function of ecosystems [8,10,32–36].

For example, feeding interactions such as parasitism and predation on pollinators,

herbivory on animal-pollinated plants, and feeding by pollinators on animals and plant

vegetation in addition to floral rewards such as nectar and pollen profoundly affect

pollination dynamics, crop yields [37], and long-term sustainability of agroecosystems [38].

Another outstanding problem with understanding the joint effects of feeding and

mutualistic interactions concerns contradictory conclusions of previous theoretical work.

Classic "effects-based" theory has long held that mutualistic interactions are generally

destabilizing (Table 1) especially at high complexity [3,4]. However, more recent theory

finds that mutualisms *stabilize* ecological systems under conditions such as high levels of

complexity of mutualism relative to antagonism in "merged" plant-pollinator and plant-

herbivore networks [35,39], low levels of complexity in hierarchical networks of all types of

interactions [40], or intermediate levels of mutualism when mutualistic links are randomly

assigned and animals allocate effort to feeding and mutualistic interactions separately [41–43].

Contradictions among these findings may arise from different definitions of stability (i.e.

local stability or persistence, Table 1) and network architectures [4,41] that often misrepresent

empirically-observed structure [40]. A broader problem is that narrowly focusing on stability

develops inefficient theory [44] that ignores how mutualisms alter the diversity, population

dynamics, and overall functioning of complex ecosystems.

To more broadly understand the ecology of mutualistic interactions, here, we follow

repeated calls to synthesize different types of interactions within networks [8,10,32–36,45] by

developing and applying mechanistic consumer-resource theory to "multiplex" ecological

networks (Table 1) [34,36]. Our model based on this theory integrates the structure and

dynamics of feeding and reproductive mechanisms from which effects of interspecific

interactions emerge including predation (+, –), mutualism (+, +), and resource and apparent

competition (–, –) [46,47]. We simulate network *dynamics* by extending Brose *et al.*'s

allometric trophic network theory [16] to incorporate Valdovinos *et al.*'s theory of the

exchange of food for reproductive services between plants and their pollinators (Fig. 1)

[30,31]. We simulate network *structures* by integrating Williams and Martinez' "niche model"

of food webs [48] with Thébault and Fontaine's model of mutualistic networks [49] (Fig. 2).

Using this multiplex model, we investigate how the presence, prevalence, and intensity of

mutualism affect multiple dimensions of ecological stability [50] by assessing diversity,

persistence, dynamics, and function at the species, guild, and ecosystem levels.

RESULTS

The multiplex model.

We synthesize the *structure* and *dynamics* of feeding and reproductive interactions

by integrating food webs and mutualistic networks and extending Allometric Trophic

Network (ATN) theory [16-18,20,51] to include the consumption of floral rewards produced by

plants and reproductive services produced by pollinators (Fig. 1, Methods). In addition to

ATN theory's logistically growing "plants without pollinators," "plants with pollinators" in

our multiplex model must consume reproductive services produced by their pollinators to

vegetatively grow. This involves partitioning the biomass of plants with pollinators into

coupled pools of vegetation and floral rewards [30] that plants produce at an energetic cost

[33,52,53]. The vegetative growth rate of plants with pollinators is a saturating function of

reproductive services consumed by the plants [53] determined by the quantity (consumption

rate) and quality (fidelity) of pollinator visits [30] limited by community-wide carrying

capacity [18,51]. The foraging rates and metabolic maintenance costs of all animals including

pollinators scale allometrically with body size [16].

We integrated realistic food-web and mutualistic-network structures into multiplex

networks by generating 102 niche-model food webs [48] of $S_f = 50$ species including exactly

20 plant species (Fig. 2a, Methods) and 238 plant-pollinator networks [49] (Fig. 2b) of

varying species diversity ($S_p = A + P = 9, 12, ..., 57$) and empirically-observed pollinator-

to-plant ratio ($A/P = 2$) and ranges of connectance and nestedness (S1). We pair each food

web with each plant-pollinator network ($N = 102 * 238 = 24,276$ pairs) and add the

pollinators of each plant within the plant-pollinator network to its paired food web by

linking the pollinators to a randomly selected plant species in the food web (Fig. 2c). Our

Rewards Only (RO) treatment links only floral rewards to the pollinators (Fig. 2d). Our

Rewards Plus (RP) treatment links floral rewards, vegetative biomass, and prey (Fig. 2e) to

the pollinators. Pollinators are preyed upon by predators of herbivores in RO networks or

predators of herbivores and low trophic-level omnivores in RP networks (Fig. 2d-e). As

such, RO and RP treatments generate two different topological classes of multiplex

networks for which we generate two groups of topologically-comparable food webs (RO

FW and RP FW), described below.

Sensitivity and uncertainty analyses (S2) revealed a pivotal role of floral rewards in

determining ecological effects of mutualism (Tables S1-S3, Fig. S5). We illustrate this role

by presenting results from networks with High ($\beta = 1.0$), Low ($\beta = 0.2$), and no rewards

productivity (Eqn. 4). High and Low productivities apply to both RO and RP multiplex

networks (High RO, Low RO, High RP, Low RP) containing both feeding and reproductive

interactions. Treatments with no rewards productivity convert all feeding on rewards to

feeding on the same plants' vegetative biomass and eliminate reproductive interactions

resulting in networks with only traditional Food Web (FW) interactions where comparable

plants with pollinators in multiplex networks instead grow as plants without pollinators and

their former pollinators graze only on their plant partners' vegetation instead of floral

rewards. This conversion transforms the added pollinators in multiplex networks into

"added animals": herbivores in RO FW networks or herbivores and omnivores in RP FW

networks (Fig. 2d, 2e). These two Food Web treatments (RO FW, RP FW) control for

network structure, including the varying numbers and trophic levels of species and links in

RO and RP networks, to help elucidate the effects of mutualism in multiplex networks.

By integrating food webs of fixed size with plant-pollinator networks of varying

size, the initial diversity (S = 50 + added animals = 56, 58, ..., 88) of our 24,276 networks

corresponds to prevalence of herbivory in our FW treatments or prevalence of mutualism in

our multiplex treatments via the fraction of plants that are animal-pollinated and the

fraction and number of species and links directly involved in mutualistic interactions (Fig.

5a). This allows us to analyze our results in terms of increasing initial diversity (S) for all

treatments and increasing *prevalence* of mutualism for multiplex treatments. The *intensity*

of mutualism in our treatments varies with rewards productivity (Table 1) from high (High

RO, High RP) to low (Low RO, Low RP) to none (RO FW, RP FW). We simulate the

24,276 networks subjected to these six treatments by initializing each species and pool of

rewards with a biomass of 10 and recording results after 5000 timesteps when species have

persisted or gone extinct and system dynamics are at, or close to, steady-state (Table 1, Fig.

3). We use these results to compare multiple measures of stability and function among

treatments averaged over all initial diversity classes (Fig. 4) or within initial diversity

classes (Figs. 5-6) at the species, guild, and ecosystem levels (Table 1). Below, we

describe the effects of the presence, prevalence, and intensity of mutualism first on

biodiversity, then on ecosystem function, and finally on temporal stability.

Diversity.

Multiplex networks with High rewards productivity (High RO, High RP) had higher

average diversity (Fig. 4a) and persistence (Fig. 4b) than their counterparts with less (Low

RO, Low RP) and no rewards productivity (RO FW, RP FW). Diversity and persistence

were also slightly elevated in the Low RP compared to the corresponding RP FW

treatment. Persistence decreased with increasing initial diversity and prevalence of

mutualism in all treatments except High RO (Fig. 5c). However, these decreases were not

strong enough to prevent overall increased final diversity (Fig. 5b) with increased initial

diversity and mutualism (Fig. 5a) in all treatments. These results indicate that effects of

mutualisms on biodiversity depend both on the prevalence and intensity of mutualism

expressed as rewards productivity.

Since all 20 plant species nearly always persisted in all treatments (Fig. S1),

differences in animal persistence among treatments underlie the overall patterns in

diversity. Most notably, relatively high and increasing persistence of omnivores with

mutualism (Fig. 5e) and consistently high persistence of added pollinators (Fig. 5f-g)

doubled animal diversity in High RO and High RP treatments over that in FW, Low RO,

and Low RP treatments (Fig. 5b). The few carnivores (~3 initial species) made smaller

contributions to elevated diversity in High rewards treatments via substantial increases in

persistence with increasing mutualism (Fig. 5d). In contrast, FW and Low rewards

treatments had much lower persistence of carnivores (Fig.5d), omnivores (Fig. 5e), and

added animals (Fig. 5f-g) that, except for carnivores, decreased with increasing mutualism.

In the High RO treatment, the few herbivores (~5 initial species) achieved higher

persistence than in the corresponding FW (Fig. 5h). However, in the other multiplex

treatments (Low RO, Low RP, High RP), herbivore persistence was lower and declined

dramatically with increasing mutualism.

Function.

Similar to final diversity, the total biomass (Fig. 4c), productivity (Fig. 4d), and

consumption in all multiplex treatments were comparatively higher than in FW treatments

with some exceptions in the Low RO treatment. Specifically, biomass was up to twice as

high while productivity and consumption were up to an order of magnitude higher in

multiplex compared to FW treatments. The Low RO treatment also had higher biomass,

but lower diversity, productivity, and consumption than the corresponding RO FW

treatment (Fig. 4). Overall, this indicates that both the presence and intensity of pollination

mutualisms increase key measures of ecosystem function.

In all treatments, total biomass (Fig. 6b), productivity (Fig. 6c), and consumption

(Fig. 6d) increased with initial diversity and prevalence of mutualism, with the strongest

increases occurring in the High rewards treatments. Plant biomass substantially decreased

below carrying capacity (Fig. 6b) with increasing initial diversity in all but Low RO

treatments, but these decreases were strongly compensated for by increases in the biomass

of animals and floral rewards. As might be expected, biomass and productivity of

pollinators, plants with pollinators, and rewards increased with increasing mutualism in

multiplex treatments. Concurrently, plant productivity increased with initial diversity and

mutualism in all but Low RO treatments. Total consumption (Fig. 6d) in all treatments

very closely matched total production (Fig. 6c) and was distributed similarly to that of

animal species' biomass (Fig. 6b).

The higher biomass of multiplex compared to FW treatments was primarily due to

increases in animal biomass (Fig. 6b), while differences in productivity were due to

decreases in vegetative productivity coupled with strong increases in productivity of

rewards and smaller increases in animals (Fig. 6c). These differences emerge primarily due

to the interactive dynamics of rewards in which growth potential, unlike all other stocks of

biomass, depends not on its own abundance but on the abundance of another component,

i.e., the vegetative biomass of plants with pollinators (Methods, Eqn. 6). This allows

rewards to be highly productive even when very rare whereas other network components

would be unproductive or could even go extinct. In FW treatments, the overall weak

increases in ecosystem biomass and strong increases in total productivity with increasing

initial diversity emerge from increasing herbivore biomass that reduces plant vegetation

below carrying capacity (Fig. 6b), freeing plants from competition. This increases plant

productivity (Fig. 6c) and animal biomass enough to lead to small net increases in total

biomass with initial diversity (Fig. 6b). Multiplex treatments experience similar decreases

of plant biomass and corresponding increases in vegetative plant productivity (Fig. 6b-c),

but rewards productivity dramatically increases as does animal biomass. These increases

are mostly due to increases in pollinator abundance that stimulate rewards productivity by

depleting rewards below their self-limitation threshold. Then, animal biomass and

productivity are further elevated by increases in the biomass of omnivores and carnivores

that feed on the increasingly abundant pollinators. These patterns in production,

consumption, and increased animal biomass are greatly enhanced in High rewards

treatments.

Stability.

We evaluated the temporal stability of our networks by analyzing coefficients of

variation (CV = standard deviation / mean) of biomass during our simulations' final 1000

timesteps for each species, for the sum of each species within each guild (Fig. 6f), and for

the sum of each species within the ecosystem. We calculated a species-level CV for each

guild by summing each species' CV within the guild and dividing by the number of species

in the guild (Fig. 6e) and a species-level CV for each ecosystem by doing the same thing

for all species within the network (Fig. 4e). We calculated a guild-level CV for each guild

as the CV for the total biomass in each guild (Fig. 6f), and a guild-level CV for each

ecosystem by summing all guilds' CVs and dividing by the number of guilds (standardized

across treatments, Fig. 4f). Finally, we calculated an ecosystem-level CV as the CV of the

total biomass of the ecosystem.

At the ecosystem level, all treatments were exceedingly stable (CV < 0.001). In

contrast, species on average were much more variable (Fig. 4e), especially in Low rewards

treatments where plants with pollinators and their rewards contributed large amounts of

variability (Fig. 6e). Large variability in plant and reward biomass in multiplex treatments

was caused by the very low biomass of a few plant with pollinators species whose biomass

decreased throughout the end of the simulations (e.g. Fig. 3). This low and decreasing

biomass yields large CVs at the species level but contributes very little to guild-level

variation (Fig. 6e-f) due to the tiny fraction of their guild's biomass comprised by these

very rare species. In FW treatments, where extinctions occur relatively early (Fig. S2),

species-level and guild-level variation are comparable (Fig. 4e-f). In contrast, average

guild-level variation (Fig. 4f) is only a fraction of the species-level variation (Fig. 4e) in

multiplex treatments. Variation at both the species and guild levels decrease with initial

diversity and mutualism in all treatments except for species-level variation in High RP

networks where large decreases in animal variability only partly compensate for larger

increases in rewards variability (Fig. 6e-f). High RP networks are by far the most stable at

the guild level however, both on average (Fig. 4f) and with increasing mutualism (Fig. 6f).

Overall, mutualism broadly stabilizes the dynamics of multiplex networks by reducing

variability of animal populations compared to those in FWs.

Overall effects of mutualism.

Excepting Low RO networks, multiplex treatments had higher average diversity,

persistence, biomass, productivity, and consumption than their FW counterparts (Fig. 4).

Multiplex treatments were also more temporally stable than FW treatments at the guild

level and, for animal populations, at the species and guild levels. Interestingly, though Low

RO networks displayed lower average diversity, persistence, productivity, and overall

species-level stability than FW treatments, these effects were ameliorated in Low RP

networks, in which pollinators had additional food available to them in the form of plant

and animal resources. This suggests that the positive effects of mutualism are reasonably

restricted to systems that provide sufficient food for mutualistic animals to survive (also see

S2) and that the low persistence of the many added pollinators in Low RO networks (Fig.

5g) throttles the positive effects of mutualisms from more generally increasing ecosystem

diversity, function, and stability beyond that of FW treatments. These observations indicate

that intensity of pollination mutualisms in terms of rewards productivity is critical to

positive effects of mutualism on ecosystems.

Mutualistic feedbacks.

We studied the degree to which mutualistic feedbacks affect ecosystems beyond

broadly providing food for animals by developing "feedback controls." At steady-state of

the multiplex simulations, plants' production of rewards interacts with their vegetative

production and their pollinators' consumption. These interactions emerge from the

dynamic feedbacks between plants and pollinators whereby plants produce rewards, which

pollinators consume while providing reproductive services, which increase vegetative

growth rate, which affects vegetative biomass, which affects rewards productivity, etc. (see

Fig. 1). Our feedback controls are non-mutualistic systems initialized with rewards and

forced to produce rewards at rates seen in a steady-state mutualistic system, but without the

mutualistic feedbacks (blue arrows in Fig. 1 removed). This allows us to test whether the

additional biomass produced by plants with pollinators is the sole cause of diversity,

stability, and function in our multiplex networks or whether sustained plant-pollinator

feedbacks are required for these effects (Methods, S3).

The overall ecosystem diversity, persistence, biomass, and productivity in our

feedback controls equilibrate to similar values as in the multiplex simulations (Fig. S7).

However, ecosystem *composition* in controls differed from that of multiplex treatments.

Eliminating dynamic feedbacks increased vegetative biomass of plants with pollinators,

decreased biomass of plants without pollinators, and decreased persistence and biomass of

omnivores and herbivores (Fig. S8). These guild-level differences were tiny in the High RP

treatment but much larger in the RO and Low RP treatments. This pattern suggests that the

combination of sufficient rewards productivity and increased trophic connectedness of

mutualists in High RP networks dampen mutualistic feedback effects.

Overall, our results suggest that the added productivity of mutualistic rewards drive

our observations of ecosystem stability and function in the multiplex treatments (see S2).

However, our results also suggest that the *dynamics* of mutualistic feedbacks alter the

composition of ecosystems by increasing biomass and persistence of consumers,

particularly omnivores, and decreasing biomass of plants with pollinators.

DISCUSSION

Our investigation of the effects of mutualism on the diversity, stability, and

ecosystem function of realistically complex ecological networks used six treatments that

varied the intensity (no, low, and high productivity of floral rewards) and prevalence

(number and fraction of species directly involved in mutualisms) of mutualistic interactions

between plants and their pollinators that either ate only floral rewards (rewards only, RO)

or other sources of biomass in addition to floral rewards (rewards plus, RP). We found that

adding more intense mutualisms strongly increases diversity, persistence, productivity,

consumption, biomass, network complexity (number of species and interactions), and the

temporal stability of biomass over that in networks with weaker or no mutualisms (Fig. 4).

With a few exceptions, these effects increased with the prevalence of both strong and weak

mutualisms (Fig. 5-6). Perhaps most strikingly, overall persistence increased with the

prevalence of strong mutualisms involving RO pollinators (Fig. 5c, High RO). Temporal

stability more generally increased in all six treatments with diversity and prevalence of

mutualism at the levels of species (Fig. 6e) and guilds (Fig. 6f) except for the slightly

negative relationship at the species-level in networks with strong mutualisms involving RP

pollinators (Fig. 6e, High RP). With one exception (Low RO productivity), networks with

the most prevalent mutualistic interactions had higher abundance (Fig. 6b), productivity

(Fig. 6c), and guild-level stability (Fig. 6d) than in corresponding networks with no

mutualism.

These increases of stability and function due to increases in mutualism are broadly

consistent with empirical observations of the effects of increased pollinator diversity in

blueberry, watermelon, and other agroecosystems [54–57]. Though consistent with empirical

observations, our results are unexpected in light of classic theory that mutualism decreases

stability to small perturbations around assumed equilibria [2–4], which we did not asses.

Instead, we more comprehensively addressed both stability in terms of diversity,

persistence, and biomass variability and also function in terms of productivity,

consumption, and biomass abundance at species, guild, and ecosystem levels of

organization (Table 1) [50]. Mutualism tends to increase stability and ecosystem function

according to *all* of our measures in treatments with stronger mutualistic interactions and by

several metrics in treatments with weaker mutualisms (Fig. 4).

Examining our results in more detail indicates that these strong positive effects of

mutualism on ecosystems emerge because mutualistic rewards at Low and even more so at

High productivity levels stabilize ecological networks by increasing persistence of

omnivores (Fig. 5e), omnivorous pollinators (Fig. 5f), and carnivores (Fig. 5d) above that

of comparable species in food webs of only feeding interactions (FW treatments). In

contrast, pollinators that only consume rewards (RO treatments) exceed the persistence of

comparable herbivores in FW treatments at High rewards productivity only. Our feedback

controls show that these positive effects of mutualism are not fully reproduced by

traditional food web dynamics when supplementing food webs with rewards productivity

similar to that in multiplex networks. The removal of dynamic feedbacks between

pollinators and animal-pollinated plants alters species composition and abundance by

reducing persistence and abundance of omnivores but increasing abundance of plants

formerly dynamically partnered with pollinators (Fig. S7-8). Thus, the combination of

dynamic feedbacks and mutualistic interaction strength, in terms of rewards productivity,

lead to the increased stability and function we observed in our multiplex ecological

networks.

Our theory embraces Levin's assertion that "The key to prediction and

understanding lies in the elucidation of mechanisms underlying observed patterns" (p.1943)

[58] by using trophic and reproductive mechanisms underlying the observed structure and

dynamics of multiplex networks to understand and predict how mutualism affects complex

ecosystems. Previous theory obscures key dynamics of such mechanisms by assuming

their effects [23,47,59]. For example, the few studies of ecological networks involving both

feeding and mutualistic interactions assert species have constant per-capita direct effects on

each other's fitness or abundance [36,45]. Such effects can rarely be reproducibly measured

due to, e.g., context-dependence and temporal variability [59,60]. We instead focused on

mechanisms such as more reproducibly observed rates of production and consumption of

food in terms of biomass [18,61] and reproductive services in terms of pollinator visits [36]. We

also focused on more realistic (i.e., empirically-corroborated) network architectures – as

have others with results similar to ours. For example, García-Callejas et al. [40] found that

mutualism increases or leaves unaffected persistence of ecological networks containing

different types of interactions structured according to realistic species abundance

distributions among different trophic levels. Several studies of near-equilibrium stability

similarly find that destabilizing effects of mutualism may be overstated in models of fewer

species [32] and interaction types [41] compared to more realistic ecological networks.

However, unlike studies that require the "peculiar" constraint that animals' interaction

efforts are allocated separately between mutualistic and non-mutualistic interactions [41–43],

we find that mutualism is stabilizing according to empirically-relevant metrics (Table 1)

when unpartitioned effort is allocated to both types of interactions.

Our multiplex treatments may bound the diet breadth of pollinators in the field by

providing pollinators unrealistically few resources in RO networks and unrealistically many

resources in RP networks. This suggests results intermediate between RO and RP networks

may best predict empirical observations. For example, in both RO and RP treatments, High

rewards productivity increases the persistence, biomass, and productivity on average of all

consumer guilds over that in Low rewards networks (Figs. 5d-h, 6b-c) excepting the

decrease in herbivore persistence in RP networks (Fig. 5h). This suggests that, in natural

systems, we may expect the weakest increases in persistence, biomass, and productivity due

to increased mutualism to occur among herbivores. The different effects of our treatments

on herbivores may be due to RP pollinators' additional herbivorous and carnivorous

feeding links compared to RO pollinators. These additional links increase competition and

predation pressure on herbivores by pollinators, omnivores, and carnivores. More broadly,

our results suggest that, as the prevalence of pollination mutualisms increase, the diversity

and biomass of upper trophic-level consumers will increase while our predictions for the

effects on herbivores are less certain.

Key limitations of our work concern how the network architecture and bioenergetic

parameters in our models match those seen in nature. While the multi-trophic and

mutualistic components of our multiplex network structures correspond to empirical

patterns, we lack observations of their interconnection into multiplex networks with which

to test against our simulated networks [8,45], though recent multiplex networks for rocky

intertidal systems that include facilitation suggest progress in this direction [36]. Regarding

dynamics, the metabolic rates of animals should be better related to the intrinsic

productivities of plants. Most such estimates come from aquatic ecosystems that often

differ from those in aboveground terrestrial systems [13,14]. The cryptic yet critically

important rates determining reward availability are also only rarely estimated. For

example, Baude *et al.* [62] suggest that nectar productivity of meadows in the UK can be up

to ~5-10% of net primary productivity and Adgabe *et al.* [63] estimate nearly 1000 kg/ha of

floral sugar is produced in a 5-month flowering season by a single tree species. Pollen

production may significantly increase such estimates and accounting for seasonality of

reward production and pollinator activity could greatly decrease estimates of resource

demand needed to sustain pollinators. This highlights the need for improved theory of

seasonal effects on both food webs [18] and mutualistic networks [64]. Bioenergetic costs of

reward production compared to vegetation production should also be considered.

Accounting for such anabolic efficiencies greatly increased the ability of allometric trophic

network theory [16,18,20] to predict realistic stocks and flows of carbon and energy in complex

food webs [18,65]. Compared to the relatively complex compounds that comprise vegetative

biomass, efficiencies of synthesizing simple sugars that provide nearly all the usable energy

in nectar may be much higher. Such efficiencies are suggested by estimates for animals

that indicate, for example, the energetic efficiency of producing milk is almost six times

than that of beef [66]. Attending to these less studied aspects of the structure and function of

multiplex networks may advance research on networks including plants and pollinators as

well as those involving coral, mycorrhizal fungi, and frugivores that disperse seeds, all of

which involve the exchange of autotrophic food for increased growth of primary producers.

More broadly, our theoretical development shows how non-trophic effects can more

generally include effects that directly depend on interaction *dynamics*, such as quality and

quantity of pollinator visits, in addition to depending on the abundance of interactors as in

previous work [10,40,41,67].

We have advanced theory on multiplex networks in order to explore the effects of

mutualism on ecological systems. Our focus on reproductive interactions follows previous

extensions of allometric trophic network theory involving plant nutrients [20], detritus [18],

ontogenetic niche shifts [68], environmental stochasticity [69], fishing [51,68], economics [51] and

other non-trophic effects [67] integrated into food webs comprised of feeding interactions

distinguished by their relative body sizes [16] such as diseases [11], parasites [11,70], parasitoids [11],

and predators [13]. Such integration of multiple interaction types into multiplex networks is

increasingly recognized as an effective means of accommodating different mechanisms

responsible for ecosystem structure and function [8,10,36,67]. Our application to mutualistic

mechanisms finds a striking ability of mutualism to enhance ecosystems including their

diversity, stability, and function when mutualistic rewards suffice to sustain mutualistic

partners. Further work incorporating mutualism into multiplex network theory (e.g.

mutualisms between zooxanthellae and coral polyps or plants and mycorrhizal fungi) may

illuminate whether mutualisms more generally enhance other systems. Such research

would help compensate for ecologists' emphasis on competition by elucidating the much

less studied roles of mutualistic and other facilitative interactions [24] in biodiversity

maintenance [6], ecosystem function [7], and evolution [25].

METHODS

Network architecture.

We created multiplex networks (Fig. 2) by generating food webs using the "niche

model" (Fig. 2a) parameterized with 50 species ($S_f = 50$) and 10% directed connectance (C_f

$= L_f / S_f^2 = 0.1$ where L_f is the number of feeding links) [48]. The niche model stochastically

assigns each species i three traits: (1) a niche value (n_i) drawn randomly from a uniform

distribution between 0 and 1, (2) a feeding range (r_i) where $r_i = xn_i$ and x is drawn from

randomly from a beta distribution with expected value $2C_f$, and (3) a feeding centre (c_i)

drawn randomly from a uniform distribution between $r_i/2$ and $min(n_i, 1 - r_i/2)$. Species i

feeds on j if n_j falls within i's feeding interval $[c_i - r_i, c_i + r_i]$. We selected niche-model

food webs with $0.0976 < C_f < 0.1024$ that were comprised of 50 species ($S_f = 50$), of which

exactly 20 were plants and five were herbivores that only feed on plants (i.e. have trophic

level [TL] = 2), yielding 102 food webs. We also generated plant-pollinator networks

using a stochastic model [49] with 3 to 19 plant-with-pollinator species (P) and exactly twice

as many animal-pollinator species ($A = 2 * P$) to maintain pollinators' average resource

availability in networks of increasing diversity. This yielded approximately 14 networks

within each of 17 diversity classes ranging from 9 to 57 species ($S_p = P + A = 9, 12, …, 57$)

for a total of $14 * 17 = 238$ plant-pollinator networks that covered the empirically observed

range of nestedness (Fig. 2b, see S1 for more details). We constrained the number of

pollination links (L_p) to ensure that pollination connectance ($C_p = L_p / PA$) broadly

decreased as S_p increased in an empirically realistic manner (Fig. S4a). We integrated each

of the 238 plant-pollinator networks with one of the 102 food webs yielding $N = 238*102 =$

24,276 networks of increasing species diversity ($S = S_f + A = 56, 58, …, 88$). We did this

by randomly choosing P of the 20 plant species already in the food web and assigning the A

pollinators to those P plant species as determined by the plant-pollinator network (Fig. 2c).

This leaves $20 - P$ plant species without pollinators.

We linked pollinators to consumers in the food web in Rewards Only (RO)

treatments by setting each pollinator's n_i to $+/-$ 5% of the n_i of a randomly selected (i.e.,

non-omnivorous) strict herbivore (TL = 2) from the food web (Fig. 2d). Pollinators' r_i and

c_i were set to zero. This causes pollinators to be preyed upon by predators similar to

predators of herbivores and to consume only floral rewards as determined by the plant-

pollinator network. Because the connectance (C_p) of our simulated plant-pollinator

networks decreases with increasing diversity (S_p) and pollinators have no other resources in

RO networks, connectance ($C = L/S^2$, where L is the total number of links) decreases on

average from 0.091 to 0.06 as initial species diversity (S) increases from 56 to 88 in the RO

multiplex and corresponding Food Web (FW) treatments (Fig. S3). The Rewards Plus (RP)

treatments link herbivorous and omnivorous pollinators to food webs (Fig. 2e) by setting

each pollinator's n_i, r_i and c_i to $+/-5\%$ of the corresponding n_i, r_i and c_i of a randomly

selected herbivore or omnivore that eats plants ($2 \le TL \le 2.3$). This procedure maintains a

constant average connectance ($C = 0.102$) with increasing S (Fig. S3) among RP networks.

Feeding on both vegetation and floral rewards of the same plant species allows two links

between plants and pollinators in RP networks. The corresponding FW treatment has

slightly less C than the RP multiplex network because the FW eliminates the link to

rewards and maintains only the herbivory link (Fig. 2d, Fig. S3). We ignore this issue to

simplify comparisons between all treatments.

Overall, as initial species diversity (S) increases, plants with pollinators in the

multiplex networks increase from 3 to 19 of the 20 total plant species and mutualistic

interactions increase from directly involving 16% to 65% of species in the networks.

Correspondingly, initial herbivory in Food Web (FW) treatments increases from directly

involving approximately half to three quarters of the species in the networks. We thus

analyze how networks vary with *increasing initial diversity*, which corresponds to

increasing prevalence of mutualism in multiplex treatments or *increasing herbivory* in FW

treatments.

Network dynamics.

To model multiplex dynamics, we extended Allometric Trophic Network (ATN)

theory [16,18,20,51] by integrating a consumer-resource approach to pollination mutualisms in

which pollinators feed on floral rewards (R) and plants consume reproductive services

produced by plants [30,31]. Plants benefit from pollinators depending upon on the quantity

and quality of pollinators' visits in terms of the rate at which pollinators consume plants'

rewards and the fidelity of pollinators' visits to conspecific plants [30,31]. Pollinators in RP

treatments also feed on species' biomass according to ATN theory.

More specifically, ATN theory models the change in biomass B_i over time t for

consumer i as

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$$\frac{dB_i}{dt} = \sum_{j \in resources} C_{ij}(B_j) - x_i B_i - \sum_{j \in consumers} C_{ji}(B_i)/e_{ji} \qquad (1)$$

where x_i is the allometrically-scaled mass-specific metabolic rate of species i and e_{ji} is the

assimilation efficiency of species j eating i. C_{ij} is the rate of species i assimilating B_j, the

biomass of species j:

$$C_{ij}(B_j) = x_i y_{ij} B_i F_{ij}(B_j) \qquad\qquad (2)$$

where y_{ij} is the maximum consumption rate per metabolic rate. $F_{ij}(B_j)$ is the functional

response of i eating j:

$$F_{ij}(B_j) = \frac{\omega_{ij} B_j^h}{B_{0ij}^h + \sum_{k \in resources} \omega_{ik} B_k^h} \qquad\qquad (3)$$

where ω_{ij} is i's relative preference for j, h is the Hill coefficient [71], and B_{0ij} is the "half-

saturation" density of resource j at which i's consumption rate is half y_{ij} [18]. The form of the

preference term, ω_{ij}, determines if a trophic generalist (i) is treated either as a "strong

generalist" ($\omega_{ij} = 1$) or "weak generalist" ($\omega_{ij} = 1/(\text{\# of species in } i\text{'s diet})$) [72]. Here, we

present results only for weak generalists that search for each of their resources equally even

if one or more of their resources are extinct. Eqn. 3 is a Type II functional response when h

$= 1$ and a Type III response when $h = 2$. We use $h = 1.5$ for a weak Type III response [71].

We use ATN theory's logistic growth model [18] to simulate biomass dynamics of

plants without pollinators as:

$$\frac{dB_i}{dt} = \left(1 - \frac{1}{K}\sum_{j \in plants} B_j\right) r_i B_i - \sum_{j \in consumers} C_{ji}(B_i)/e_{ji} \qquad\qquad (4)$$

where r_i is the maximum mass-specific growth rate of plant i, and K is the carrying capacity

of the plant community. For plant with pollinators i (Fig. 1), we model its vegetative

biomass dynamics as:

$\frac{dB_i}{dt} = \left(1 - \frac{1}{K}\sum_{j \in plants} B_j\right) r_i B_i P(R_i) - \sum_{j \in consumers} C_{ji}(B_i)/e_{ji} - \kappa_i(\beta_i B_i - s_i R_i)$ (5)

and the dynamics of its floral rewards biomass as:

$$\frac{dR_i}{dt} = \beta_i B_i - s_i R_i - \sum_{j \in pollinators} C_{ji}(R_i)/e_{ji}$$ (6)

where β_i is the production rate of floral rewards, s_i is the self-limitation rate of floral reward

production, and κ_i is the cost of producing rewards in terms of total vegetative growth.

$P(R_i)$ is the functional response describing how benefit to i accrues due to reproductive

services provided by i's pollinators:

$$P(R_i) = f\bigg(\sum_{j \in pollinators} \overbrace{C_{ji}(R_i)}^{quantity} \overbrace{\frac{C_{ji}(R_i)}{\sum_{k \in resources} C_{jk}(B_k \ or \ R_k)}}^{quality}\bigg)$$ (7)

with the overall bracket labeled *reproductive services*.

which is a function of the quantity and quality of pollination visits by pollinator j. Quantity

is j's consumption rate on i's floral rewards. Quality is j's consumption of i's rewards as

compared to j's consumption of all the resources it consumes. Quality is therefore j's

relative consumption rate of i's floral rewards, a measure of j's fidelity that ensures more

specialist pollinators typically provide higher quality services than generalist pollinators by,

for example, depositing higher concentrations of conspecific pollen [30]. The form of the

functional response describing benefit accrual due to pollination (*f*) reflects the assertion

that reproductive services saturate [53] at 1 according to: *reproductive services* / (0.05 +

reproductive services). As $P(R_i)$ approaches 1, the realized growth rate of plant with

pollinators i's vegetative component approaches r_i, its maximum growth rate.

Pollinators follow the dynamics typical of ATN consumers (Eqn. 1) with the

exception that they access rewards biomass R_i instead of B_i in RO treatments (Eqn. 1-1) or

in addition to the biomass of other resource species (Eqn. 1-2) in RP treatments:

$$\frac{dB_i}{dt} = \sum_{j \in resources} C_{ij}(R_j) - x_i B_i - \sum_{j \in consumers} C_{ji}(B_i)/e_{ji}$$ (1-1)

$$\frac{dB_i}{dt} = \sum_{j \in resources} C_{ij}(R_j \text{ and } B_j) - x_i B_i - \sum_{j \in consumers} C_{ji}(B_i)/e_{ji}$$ (1-2)

Parameterization.

Vital rates for consumers follow previously described allometric scaling for

invertebrates [51]. Specifically, we set plant species' "body mass" to a reference value (m_i =

1) [16] and calculated consumers' body mass as $m_i = Z_i^{swTL_i - 1}$, where $swTL_i$ is i's short-

weighted trophic level [73] and Z_i is i's average consumer-resource body size ratio sampled

from a lognormal distribution with mean = 10 and standard deviation = 100. Then, for i

eating j, i's mass-specific metabolic rate (x_i) is $0.314 m_i^{-0.25}$, its maximum consumption rate

per metabolic rate (y_{ij}) is 10, and its assimilation efficiency (e_{ij}) is 0.85 if j is an animal or

0.66 if j is plant vegetation. We set the maximum mass-specific growth rate (r_i) of plant i

to be 0.8 for plants without pollinators or 1.0 for plants with pollinators, so that when

sufficient reproductive services are provisioned by pollinators, the mass-specific growth

rate of plants with pollinators is comparable or can even exceed that of the plants without

pollinators.

The remaining parameters are not allometrically constrained. We assigned a "half-

saturation" density for consumers of species' biomass or rewards of B_0 = 60 or 30,

respectively. This reflects the decreased "handling time" for rewards compared to typically

more defended vegetation. We also assigned a Hill coefficient of h = 1.5, a community-

wide carrying capacity for plant vegetative biomass of $K = 480$, and an assimilation

efficiency of $e_{ij} = 1.0$ for pollinator species i consuming the floral rewards of j. For plants

with pollinators, we used a rewards production rate of $\beta_i = 0.2$ or 1.0 (Low or High rewards

productivity treatments, respectively), a self-limitation rate of $s_i = 0.4$, and a vegetative cost

of rewards production of $\kappa_i = 0.1$. In FW treatments, rewards are zeroed out ($R_i = \beta_i = 0$)

and all plants are parameterized so that they behave as plants without pollinators while

pollinators are parameterized as "added animals" (herbivores or omnivores) that consume

vegetation with the associated lower assimilation efficiency ($e_{ij} = 0.66$) but have otherwise

unchanged vital rates. See Table S4 for a summary of model parameters and values.

Simulations.

587   We simulated each of our $N = 24,276$ networks subjected to each six treatments

(High RO, Low RO, RO FW, High RP, Low RP, and RP FW) for a total of 145,656

simulations. We used MATLAB's [74] differential equation solvers (ode15s for the multiplex

treatments and ode45 for FWs) to simulate these networks for 5000 timesteps (Fig. 3). By

2000 timesteps, the simulations were approximately at dynamical steady-state, which we

assessed through small changes in persistence with increased simulation length. More

specifically, persistence decreased by 5% on average between 2,000 and 500,000 timesteps

in a sample of 90 networks from each treatment (Fig. S2). We initialized all biomasses (B_i

and R_i) to 10 and used an extinction threshold of $B_i < 10^{-6}$. Statistical analyses were

performed in JMP 14 [75]. Our results are qualitatively robust to simulation length (Fig. S2).

Sensitivity of our results to parameter variation are reported in the Supplementary

Information (Tables S1-S3) and qualitative effects of each parameter are summarized in

Table S2.

Outputs.

We quantified ecosystem stability and function using species persistence, biomass, productivity, consumption, and variability at or near the end of the simulations, when the dynamics were approximately at steady state (Table 1). We calculated these metrics for the whole ecosystem (Fig. 4) and for seven guilds of species (Figs. 5-6). Two guilds are self-evidently described as species of plants without pollinators and plants with pollinators. Herbivores, omnivores, and carnivores refer only to species present in the niche-model food webs prior to integrating animals from plant-pollinator networks in Fig. 2c. Herbivores eat only vegetative biomass. Omnivores eat vegetation and animals. Carnivores eat only animals. The meanings of the two remaining guilds (collectively referred to as the "added animals") depend on the treatment that adds them to the food web. Added herbivores/pollinators refer to herbivores added by the FW treatments, pollinators added by the RO or RP multiplex treatments that consume only rewards, and pollinators added by the RP multiplex treatment that consume rewards and vegetation. Added omnivores/pollinators refer to omnivores added by the RP FW treatment and analogous pollinators added by the RP multiplex treatment that consume rewards, other animals, and potentially vegetation. When relevant (e.g. in Fig. 6), we considered the rewards biomass of all plants with pollinators as an eighth guild.

We calculated all outputs at the end of the simulations (timestep 5000) except for biomass variability, which we calculated over the last 1000 timesteps. Final diversity and persistence are the *number* and the *fraction*, respectively, of the initial species whose biomass stayed above the extinction threshold throughout the simulation. Biomass abundance, productivity, and consumption are calculated as summed totals for the whole

ecosystem and/or each guild of species. Plant productivity is the rate of biomass increase

due to growth minus loss due to rewards production. Rewards productivity is the rate

rewards production minus self-limitation. Animal productivity is the rate of biomass

increase due to assimilation minus losses due to metabolic maintenance. Consumption is

the rate of biomass assimilated by consumers divided by assimilation efficiency. Species-

level variability for the whole ecosystem (Fig. 4e) is the averaged coefficients of variation

of biomasses (CV = standard deviation / mean) of all surviving species in the ecosystem.

Species-level variability for each guild (Fig. 6e) is the averaged CVs of all surviving

species within that guild. Guild-level variability for each guild (Fig. 6f) is the CV of the

summed biomass of all species in that guild. Guild-level variability of the whole ecosystem

(Fig. 4f) is the averaged CVs for five guilds (all plants, herbivores, all added animals,

omnivores, and carnivores), which standardizes the grouping of species into guilds across

treatments. Ecosystem-level variability (not shown) is the CV of the summed biomass of all

species in the ecosystem.

Feedback control.

To disentangle effects of mutualistic feedbacks from effects of floral rewards, we

ran multiplex simulations with mutualism "turned off" ("feedback control"), in which all

feedbacks (blue arrows in Fig. 1) between vegetation, rewards, and pollinators are severed.

This control transforms plants with pollinators into two *independent* biomass pools: a plant-

without-pollinators (vegetation) pool and a rewards pool, both with constant production

rates. In this way, rewards production is forced to match that of the multiplex model in the

absence of mutualistic feedbacks even though these feedbacks also generated the

production rate through dynamics over the course of the multiplex simulations. All feeding

interactions (gray arrows) remain the same.

Specifically, we modified the dynamics of each former plant with pollinators i so

that its vegetative biomass follows the dynamics and parametrization of plants without

pollinators (Eqn. 4, $r_i = 0.8$) and its rewards biomass follows:

$$\frac{dR_i}{dt} = (\overline{\beta_i B_i - s_i R_i}) - \sum_{j \in pollinators} C_{ji}(R_i)/e_{ji} \qquad (6\text{-}1)$$

with fixed production rate $(\overline{\beta_i B_i - s_i R_i})$ equal to i's average net rewards production during

the last 1000 timesteps of the multiplex simulations. In this manner, vegetation is not

dependent upon pollinator consumption of rewards nor on rewards production, and rewards

production is fixed and not dependent upon vegetation. All other species followed the

same dynamic equations and parameterization as in the multiplex simulations.

We applied these feedback controls to the four multiplex treatments (RO Low, RO

High, RP Low, RP High) and initialized all species at biomass $B_i = 10$ and rewards nodes at

$R_i = \overline{R_\iota}$, the average rewards biomass for each plant with pollinator species i during the last

1000 timesteps of the multiplex simulations. Simulations were run for 5000 timesteps to

approximate steady state. We compared the results of these simulations with those of the

original multiplex simulations by measuring absolute differences in persistence and total

biomass at timestep 5000, where the *effect of feedback = multiplex – control*. To assess

differences in these ecosystem metrics due to guilds, we calculated absolute differences in

the fraction of persisting species composed by each guild:

$$\frac{multipex\ final\ guild\ diversity}{multiplex\ final\ diversity} - \frac{control\ final\ guild\ diversity}{control\ final\ diversity} \qquad (8)$$

and the fraction of ecosystem biomass composed by each guild:

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$$\frac{multipex\ guild\ biomass}{multiplex\ total\ biomass} - \frac{control\ guild\ biomass}{control\ total\ biomass}. \tag{9}$$

If these *effects of feedbacks* evaluate to positive numbers, feedbacks in multiplex

simulations have a positive effect, i.e. they increase persistence or biomass of the

ecosystem or guild. If *effects of feedbacks* are negative, feedbacks decrease persistence or

biomass. If, instead, *effects of feedbacks* are approximately zero, stability and function in

our multiplex treatments can be attributed to the overall rates of plant (vegetative and

rewards) productivity that emerge during those simulations.

Data availability.

Network structures and simulation outputs are available in the online repository at

https://github.com/kayla-hale/Multiplex-Dynamics/.

Code availability.

Simulation and analysis code are available in the online repository at

https://github.com/kayla-hale/Multiplex-Dynamics/.

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Table 1. Definitions of terms.

Ecosystem "Stability" and "Function"	We use a range of complementary metrics to assess ecosystem "stability" and "function" in a broad sense [50]. We apply the metrics both to the whole **ecosystem** (all species in the network) and to individual **guilds** in the network at **steady-state dynamics** of our simulations.
Biomass Variability (CVs)	**Coefficient of Variation** (CV = standard deviation / mean) of species' or guilds' biomass evaluated over the last 1000 timesteps of the simulations when their dynamics are approximately at **steady state**. Increased **temporal stability** corresponds to decreased biomass variability.
Consumption Rate	Total rates of biomass extracted by consumers.
Diversity	Number of species within a network. Networks of **initial diversity** S are subjected to the dynamical simulations, during which species may go extinct or persist. The resulting number of species is the **final diversity**, or simply **diversity**.
Guilds	Guilds are groups of "species" with similar consumer-resource interactions: **plants without pollinators**; **plants with pollinators**; **floral rewards** of plants with pollinators (when relevant to analyze their biomass and flows separately from vegetation, Fig. 1); **herbivores**: species that only eat plant vegetation in the original niche-model food web (Fig. 2a); **omnivores**: species that eat vegetation and animals in the original niche-model food web (Fig. 2a); **carnivores**: species that eat only animals; **added (+) herbivores/pollinators**: herbivores or herbivorous pollinators added by the RO or RP treatments that consume rewards (multiplex treatments) and/or vegetation (FW treatments; Fig. 2, yellow-green nodes); **added (+) omnivores/pollinators**: omnivores or omnivorous pollinators added by the RP treatment that consume rewards (multiplex treatment), animals, and/or vegetation (Fig. 2e, orange nodes). "**Added animals**" include both +herbivores/pollinators and +omnivores/pollinators.
Local Stability	Categorizes the behavior of a system around an equilibrium point as "locally stable" if the system tends to return to that equilibrium after a very small perturbation [4].
Persistence	Fraction of species that survive to the end of simulations (= initial diversity / final diversity).
Productivity	Total rates of biomass increase due to plant growth and food assimilated by animals minus loss due to animals' metabolic maintenance costs and plants' costs of producing rewards.
Steady-State Dynamics	Formally, dynamics in which all species have constant abundance ($dB_i/dt = dR_i/dt = 0$ for all i). At the end of 5000 timesteps, our systems *approximate* steady-state dynamics (Fig. 3) as quantified by very small variability in total ecosystem biomass over the last 1000 timesteps of the simulations (CVs < 0.0001).

Multiplex Networks	Ecological networks that include more than one type of species interaction. Here, we focus on multiplex networks that combine food webs (including carnivory and herbivory, Fig. 2a) and pollination networks (Fig. 2b).
Rewards Only (RO) Treatment	Network construction treatment in which pollinators can access floral rewards of plants w/ pollinators as their only resource (Fig. 2d). These networks are subjected to multiplex dynamics, which include pollination in addition to traditional food web dynamics.
Rewards Plus (RP) Treatment	Network construction treatment in which pollinators can access floral rewards plus plant vegetation and/or animal biomass resources (Fig. 2e). These networks are subjected to multiplex dynamics, which include pollination in addition to traditional food web dynamics.
Pollination Link *or* Mutualistic Interaction	A pollination link or mutualistic interaction between pollinator i and plant w/ pollinator j describes both the consumption of j's floral rewards by i and the reproductive services provisioning to the vegetative growth rate of j by i (Fig. 1). In the FW treatments, pollination links are switched to links in which i consumes the vegetative biomass of j, i.e. to herbivory links.
Food Web (FW) Treatments	Ecological networks with the same structure as multiplex networks, but with all **pollination links** switched to herbivory (also corresponding to zero rewards productivity). In the Rewards Only Food Web (RO FW) construction, animal i is strictly an herbivore (Fig. 2d), while in the Rewards Plus Food Web (RP FW) construction, animal i could be an omnivore or herbivore (Fig. 2e). These networks are subjected to traditional food web dynamics.
Rewards Productivity (β)	Parameter in our multiplex model specifying the rate of rewards biomass produced by plants w/ pollinators per unit of their vegetative biomass. "Low" ($\beta = 0.2$) and "High" ($\beta = 1.0$) are arbitrary values chosen to illustrate two behaviors of the multiplex model compared to traditional food web dynamics. "None" corresponds to Food Web treatments. We interpret rewards productivity as a proxy for the **intensity** of pollination interactions. See Fig. S5 for persistence results across a range of β values.
Feedback Control	To test whether transient mutualistic feedbacks isolated from rewards availability lead to the differences between our multiplex and Food Web treatments, we initialized simulations forced with rewards availability from multiplex simulations but with feedbacks (blue arrows in Fig. 1) turned "off." We then observed potential changes in steady-state ecosystem stability and function.

Figure Legends

Figure 1. Interaction mechanisms in the multiplex model. Interspecific and

intraspecific mechanisms of feeding and reproduction combine to describe pollination

mutualisms and traditional trophic interactions. Biomass of plants with pollinators is

partitioned into two pools, vegetation (blue node) and floral rewards (purple node), coupled

by intraspecific dynamic feedbacks (dotted arrows). Rewards production is proportional to

vegetative biomass but subject to self-limitation such that its productivity per unit biomass

decreases with increasing rewards abundance. Producing rewards incurs costs (reduced

vegetative productivity), which create tradeoffs between producing rewards to attract

pollinators and benefiting from the *quantity* (number of visits measured as feeding rate on

rewards) and *quality* (conspecific feeding / total feeding) of pollinators' reproductive

services (blue arrow) that are required for vegetative production. At saturation,

reproductive services allow plants with pollinators to potentially achieve a 25% higher per-

biomass growth rate than that of plants without pollinators whose intrinsic growth rate is

independent of consumers' behavior. All plants are also subjected to competition from the

plant community in the form of a community-level carrying capacity, which reduces per-

biomass vegetative growth rate as the plant community approaches its carrying capacity.

Gray arrows show herbivores feeding on vegetation and pollinators feeding on rewards.

Figure 2. Steps for constructing food-web and multiplex-network treatments. Nodes

are vertically arranged by trophic level with plant species at the bottom and carnivores at

the top. All (grey, red, orange) links represent feeding by the consumer above the resource

except the bi-directional pollination links (blue, simplified from Fig. 1) that represent

pollinators consuming plants' floral rewards (e.g., nectar) and plants consuming

pollinators' reproductive services. **Steps: a)** Generate food web with diversity S_f and

connectance C_f [48]. **b)** Generate plant-pollinator network with diversity S_p and connectance

C_p [49]. **c)** Integrate the food web and plant-pollinator network by adding pollinators and

their links from b) to the food web in a) by randomly assigning plant species in a) to

become the animal-pollinated plants in b). **d) Rewards Only (RO) treatments:** following

a), b), and c), construct the RO multiplex treatment by stochastically linking predators of

herbivores in c) to pollinators and then, for the RO FW treatment, transform pollinators into

herbivores and plants with pollinators into plants without pollinators. **e) Rewards Plus**

(RP) treatments: following a), b), and c), construct the RP multiplex treatment by

stochastically linking predators and diets of herbivores and herbivorous omnivores in c) to

pollinators and then, for the RP FW treatment, transform omnivorous and herbivorous

pollinators and plants with pollinators into omnivores, herbivores, and plants without

pollinators, respectively. Resulting diversity (S) and connectance (C) is shown under each

network treatment (d–e). See Methods.

Figure 3. Timeseries of a 72-species ecological network subjected to six treatments.

This example uses a 50-species niche-model food web integrated with a 33-species plant-

pollinator network according to the Rewards Only (RO, top row) and Rewards Plus (RP,

bottom row) network treatments subjected to traditional Food Web (FW) dynamics (left) or

multiplex dynamics with Low (center) or High (right) rewards productivity. Note that 11

plant species in the food web are chosen to represent the 11 plant species in the pollination

network. Simulations last 5000 timesteps and are presented on log-log axes. Each line

(colored by guild following Fig. 2) is the trajectory of a species' biomass over time.

Species that fall below a biomass of 10^{-4} continue to extinction (10^{-6}). Resulting overall

persistence is labeled in the upper right corner of each panel. Plants with pollinators are

considered extinct when their vegetative biomass (light blue) drops below 10^{-6}; their

rewards biomass (purple) is strongly coupled and declines accordingly. The vast majority

of species' biomasses achieve steady state by 2000 timesteps with nearly all animal

extinctions occurring before then, while several low-abundance plants with pollinators

continue slow declines well past 2000 timesteps.

Figure 4. Overall effects of mutualism on stability and function in complex

ecosystems. Column headings label the treatments described in Fig. 3. Gray bars and

associated μ's are means over all levels of initial diversity for all networks or "ecosystems"

within each treatment at the end of $N = 24,276$ simulations. Shown are the total a) diversity,

b) persistence, c) biomass, d) productivity and consumption rates, and the mean CVs of

biomass of all e) species within each ecosystem and f) guilds within each ecosystem

averaged over all the ecosystems within each treatment. Black error bars show 95%

confidence intervals. Mutualism is absent in Food Web (FW) treatments and present in

corresponding multiplex treatments. Mutualisms are less intense in Low than in High

rewards productivity treatments. Total rates of productivity and consumption were

approximately equal (i.e. at steady-state, all production is being consumed) so they are

shown in one row (d).

Figure 5. Effects of increasing mutualism on diversity and persistence in complex

ecosystems. Colors represent guilds of species described in Fig. 2 and Table 1. Initial

diversity (*S*) on the x-axis and in a) describes the initial number of species in food webs

plus added herbivores and omnivores in the Food Web (FW) treatments or pollinators in

the multiplex treatments. Increasing *S* corresponds to both to an increasing *number* of

added (+) animals and to an increasing *fraction* of the 20 plants without pollinators that are

assigned to be plants with pollinators, and thus to *increasing prevalence of mutualism* in

multiplex treatments. Bars show mean values for networks of a given initial S in

increments of 2 species. Stacked bar graphs show the mean number of species in each

guild (colors) that contribute to a) initial or b) final ecosystem diversity. Persistence, the

fraction of the initial diversity that persists to the end of the simulations, is shown for c) the

entire ecosystem (i.e. network of species) and d-h) for each guild of animals. c) Error bars

show standard deviations. d-h) μ is the mean guild persistence over all $N = 24{,}276$

simulations in each treatment. Plants nearly always persist in our treatments (Fig. S1), so

their persistence is not shown.

Figure 6. Effects of increasing mutualism on diversity, abundance, function, and

temporal stability in complex ecosystems. Formatting follows Fig. 5 with the addition of

floral rewards shown in purple. a) Final diversity from Fig. 5 is shown again for reference.

Stacked bar graphs show the contribution of each guild (colors) to total ecosystem b)

abundance, c) productivity, d) consumption, and average variability over time of e) species

and f) guilds, with colors representing the average variability of the guild in f) or species

within the guild in e). b) The community-wide carrying capacity for plant vegetative

biomass (K) is marked on the y-axis; total plant vegetative biomass (summed vegetation of

plants with and without pollinators) does not exceed this value.

Figure 1.

Figure 2.

Figure 3.

Figure 4.

Figure 5.

Figure 6.

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