Title

Effective mechanical potential of cell–cell interaction explains basic structures of three-

dimensional morphogenesis

Short title

Effective potential of cell–cell interaction on morphogenesis

Authors

Hiroshi Koyama[1,2,*], Hisashi Okumura[2,3,4], Atsushi M. Ito[5], Tetsuhisa Otani[2,6], Kazuyuki

Nakamura[7,8], Kagayaki Kato[2,9,10], and Toshihiko Fujimori[1,2]

[1]Division of Embryology, National Institute for Basic Biology, 5-1 Higashiyama,

Myodaiji, Okazaki, Aichi 444-8787, Japan

[2]SOKENDAI (The Graduate University for Advanced Studies), Hayama, Kanagawa 240-

0193, Japan

[3]Biomolecular Dynamics Simulation Group, Exploratory Research Center on Life and

Living Systems (ExCELLS), National Institutes of Natural Sciences, 5-1 Higashiyama,

Myodaiji, Okazaki, Aichi 444-8787, Japan

[4]Institute for Molecular Science, National Institutes of Natural Sciences, 5-1 Higashiyama,

Myodaiji, Okazaki, Aichi 444-8787, Japan

[5]Department of Helical Plasma Research, National Institute for Fusion Science, National

Institutes of Natural Sciences, 322-6 Oroshi-cho, Toki, Gifu 509-5292, Japan

[6]Division of Cell Structure, National Institute for Physiological Sciences, 5-1

Higashiyama, Myodaiji, Okazaki, Aichi 444-8787, Japan

[7]School of Interdisciplinary Mathematical Sciences, Meiji University, 4-21-1 Nakano,

Nakano-ku, Tokyo 164-8525, Japan

[8]JST, PRESTO, 4-1-8 Honcho, Kawaguchi, Saitama 332-0012, Japan

[9]Bioimage Informatics Group, Exploratory Research Center on Life and Living Systems

(ExCELLS), National Institutes of Natural Sciences, 38 Nishigonaka, Myodaiji, Okazaki,

Aichi 444-8585, Japan

[10]Laboratory of biological diversity, National Institute for Basic Biology, 38 Nishigonaka,

Myodaiji, Okazaki, Aichi 444-8585, Japan

*Correspondence: Hiroshi Koyama

Division of Embryology, National Institute for Basic Biology, 5-1

Higashiyama, Myodaiji, Okazaki, Aichi 444-8787, Japan

Phone: +81 564 59 5862,

hkoyama@nibb.ac.jp.

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Abstract

Mechanical properties of cell–cell interactions have been suggested to be critical for the

emergence of diverse three-dimensional morphologies of multicellular organisms.

Mechanical potential energy of cell–cell interactions has been theoretically assumed,

however, whether such potential can be detectable in living systems remains poorly

understood. In this study, we developed a novel framework for inferring mechanical

forces of cell–cell interactions. First, by analogy to coarse-grained models in molecular

and colloidal sciences, cells were approximately assumed to be spherical particles, where

microscopic features of cells such as polarities and shapes were not explicitly

incorporated and the mean forces (i.e. effective forces) of cell–cell interactions were

considered. Then, the forces were statistically inferred from live imaging data, and

subsequently, we successfully detected potentials of cell–cell interactions. Finally,

computational simulations based on these potentials were performed to test whether these

potentials can reproduce the original morphologies. Our results from various systems,

including Madin-Darby canine kidney (MDCK) cells, *C.elegans* early embryos, and

mouse blastocysts, suggest that the method can accurately infer the effective potentials

and capture the diverse three-dimensional morphologies. Importantly, energy barriers

were predicted to exist at the distant regions of the interactions, and this mechanical

property of cell–cell interactions was essential for formation of cavities, tubes, cups, and

two-dimensional sheets. Collectively, these structures constitute basic structures observed

during morphogenesis and organogenesis. We propose that effective potentials of cell–

cell interactions are parameters that can be measured from living organisms, and represent

a fundamental principle underlying the emergence of diverse three-dimensional

morphogenesis.

Introduction

The mechanical properties of interactions between objects are among the most

fundamental parameters of various physical, chemical, and biological phenomena at a

wide range of spatial scales in the molecular, colloidal, cellular, and astrophysical

sciences. Interactions among particulate matter such as ions, molecules, and colloids are

primarily mediated by electromagnetic forces, and the properties of these interactions,

including attractive and repulsive forces, substantially affect the dynamics and stability

of systems [1,2]. In multi-cellular living systems, various three-dimensional morphologies

are observed. The emergence of these diverse morphologies is thought to be primarily

dependent on the mechanical properties of the constituent cells. In particular, mechanical

properties of cell–cell interactions are involved in morphogenetic events such as epithelial

cell movement and cell sorting [3–7]. However, the mechanical basis of morphogenesis,

which gives rise to a variety of structures, remains to be elucidated, and it is not yet known

whether any unifying principle can explain the morphological diversity of organs and

tissues.

The mechanical properties of cell–cell interactions are regulated by various

factors. In epithelial cells, whose shapes are typically columnar polygons, cell–cell

adhesion energy is controlled by cadherin family proteins (Fig. S1A-i and B) [3,4,8], whereas

actomyosin proteins counteract adhesion through the indirect effect of cell surface tension

(Fig. S1A-i and B) [9,10]. Moreover, cellular polarities such as apico-basal polarity are

linked to biased localization of cadherin proteins, leading to directional differences in

properties of cell–cell interactions. By contrast, in mesenchymal cells, which are

unstructured shapes, in addition to cadherins, cell–cell interactions are regulated by

indirect interactions through extracellular matrix (i.e. substrate) (Fig.S1A-i) [11–13].

Therefore, cell–cell interactions entail many microscopic parameters determined by

various proteins and cellular processes, resulting in very complex physics with many

degrees of freedom (Fig.S1A-i). To elucidate the principles underlying morphological

diversity, it is essential to investigate how the integrated effects of many microscopic

parameters can be described as simple meso or macroscopic parameters (Fig. S1A-ii).

In non-living materials such as ions, molecules, and colloids, interactions among

particulate matter are determined by electron clouds (i.e. quantum state), which have

complex shapes (Fig. S1A-iii) [14]. On the other hand, mechanical potential energies of

particle–particle interactions have been successfully described as a function of distances

of the interactions, e.g. the Lenard–Jones potential which is usually applied to rare gas

atoms (Fig. S1A-iv). Therefore, it can be interpreted that the microscopic components

conferred from electron clouds can be integrated into the potentials of particle–particle

interactions as mesoscopic parameters. Typically, such distance–potential curve contains

repulsive forces around regions of short distances, which are provided from excluded

volume effect of the particles, and attractive forces around regions of middle and far

distances (Fig. 1B, and Fig. appx 1). Intriguingly, in the case of non-spherical or polarized

molecules including water and amino acids, potentials of their interactions have been

sometimes defined as a function of distances where the molecules are assumed as

isotropic particles. These coarse-grained models have been succeeded to explain overall

behaviors of the systems with a trade-off of microscopic accuracy [14].

In atomic and molecular sciences, how various microscopic components are

integrated into potentials of particle–particle interactions have been theoretically

established well in relatively simple systems [2,14], which we call here a bottom-up

approach. On the other hand, there are top-down approaches to obtain the potentials.

Potential energies of whole systems according to atomic or molecular configurations are

calculated from simulations on the basis of quantum chemistry which is more accurate

than classical mechanics. Then, by using the calculated potential energies and the

configurations as reference data, potentials of particle–particle interactions have been

inferred through trial-and-errors searches or machine learning [1,2,15]. The simplest example

is a usage of radial distribution functions as reference data, because the functions mean

probability of particle positions along particle–particle distances, which can be linked to

potential energy (Fig. appx 1 and SI, Appendix section 12-1) [2]. Because these potentials

are the outcomes from various microscopic components, they are "effective" potentials

of particle–particle interactions. Using these potentials obtained from the bottom-up or

top-down approaches as an integrative parameter, many molecular dynamics simulations

have been successfully performed to understand the dynamics and shapes of various

systems (Fig. S1) [16].

The potential of cell–cell interaction is also conceptually well recognized, and

has been used in theoretical studies (Fig. 1B and S1A-ii) [11,17–25]. The potential energy can

be defined regardless of whether two cells directly contact each other. Although adhesive

forces between two isolated cells have been measured (i.e. forces required to dissociate

one cell from the other) [6,26], they have neither been measured nor inferred in more

complex multi-cellular systems, which differ from isolated cells. For instance, cell shapes

in multi-cellular systems are entirely different from those of isolated cells: they can be far

from spherical shapes (e.g. flattened shape). Apico-basal polarity cannot be defined in

isolated cells but can be in cell sheets whose three-dimensional directions are determined

by extracellular matrix (ECM) or substrate (Fig. 1 and S1). If effective potentials of cell–

cell interactions could be defined as a parameter integrating these various factors, the

effective potentials would be useful to explain shapes and dynamics of multi-cellular

systems. However, it is challenging to identify all these factors and measure the values of

their related-parameters, which is a bottleneck to obtain effective potentials through

bottom-up approaches.

In this study, by analogy to non-living materials (Fig. 1), we tried a top-down

approach: we developed a method for statistically inferring the effective potential of cell–

cell interactions using nuclear tracking data from live imaging as reference data, which

are relatively easy to obtain. Thus, shapes and polarities of cells were not explicitly

considered at this moment, whose effects we tried to decode after acquiring inference

results (Fig. 1I). Our method was validated by using various artificially-generated data

with or without external factors such as egg shells, cavities, and traction forces exerted

between cells and substrates. Then, we tested whether effective potentials of cell–cell

interactions are detected in living systems. We applied this method to relatively small

multi-cellular systems (4–350 cells) including mouse pre-implantation embryos bearing

cavities and early embryos of the nematode *Caenorhabditis elegans* bearing egg shells.

We clearly detected effective potentials as a function of cell–cell distances in these

systems. Further theoretical analyses suggested that effective potential represents a

unifying principle capable of explaining various structures including cell aggregates,

cavities, tubes, cups (round and hollow shapes), and two-dimensional sheets (Fig. 1H).

Because these structures are formed prior to the morphogenesis of complex tissue

structures, we consider these as basic structural units during three-dimensional

morphogenesis. We also describe the differences in these potentials between living and

non-living materials, as well as among biological species.

Theory and principle for inferring effective potential

Overview of strategy for inferring effective potentials/forces of cell–cell interaction

To measure or infer the effective potentials or forces of cell–cell interactions *in*

vivo, we developed a particle-based cell model in which attractive or repulsive forces are

exerted between each pair of cells (Fig. 1A, B and D). This coarse-grained model does

not explicitly incorporate microscopic features of cells such as cell shape, polarities, size,

heterogeneity, cell type or origin (e.g., epithelial, mesenchymal, or blastomere cells), etc.

at this moment (Fig. 1C, D and S1A). Simultaneously, we obtained time series of three-

dimensional cell positions from confocal microscopic images (Fig. 1E) and statistically

inferred the forces of cell–cell interactions by systematically searching for force values

that could reproduce the cell movements observed *in vivo* (Fig. 1F). Since the inferred

forces are the outcome from the integrative effects of the microscopic features of cells,

we call the forces "effective forces". To validate the method, we utilized artificial cell

movement data generated by performing simulations under given potentials of cell–cell

interactions, and confirmed that the inferred forces were consistent with the given-

potentials. The details of each part of the analysis are explained in the following sections.

Particle-based cell models

In our model, particles interact with each other and attractive or repulsive forces

(F_i) are assumed (Fig. 1D), where i is an identifier for particle–particle interactions. In

three-dimensional cellular systems with no attachment to substrates, we did not assume

persistent random walks which are originated from cellular traction forces on substrates

[27,28].

The equation of particle motions is defined below. In cellular-level phenomena,

viscous drag force or frictional force provided by the surrounding medium or tissue is

dominant, as expected from the low Reynolds number; consequently, the inertial force is

negligible in general [5,11,13,29,30]. The forces in such a system can be assumed to be

correlated with the velocities of the objects [5,13,24,25,31]. Thus, the velocity (V_p) of a particle

is calculated by the net force (F_p) exerted on the particle as follows:

$V_p = F_p / \gamma$ (Eq. 1),

where p is an identifier for particles and γ is the coefficient of viscous drag and frictional

forces. F_p is the summation of F_i: $F_p = \sum_{i=1}^{I} \delta(p) F_i$, where I is the total number of

interactions, and in the case that the ith interaction is formed with the pth particle, $\delta(p)$ is

1, otherwise $\delta(p)$ is 0 (Fig. 1D). We assumed the simplest situation, i.e., γ is constant

(=1.0). Thus, by giving the values of F_i from, for instance, a distance–potential curve, a

simulation can run. The influence of these assumptions on simulation results is discussed

in SI (Section 4-4, 4-6-2, and 7).

Data acquisition of time series of cell positions

To define cell positions, we focused on nuclei, because they are most easily

imaged by microscopy (Fig. 1E). In fact, nuclear detection followed by temporal tracking

has been performed in a wide range of organisms from *C. elegans* to mammals, and these

data are accumulating [32–36]. We utilized publicly available nuclear tracking data of a

developing embryo of *C. elegans* [32]; nuclear tracking data of developing mouse embryos

and MDCK cultured cells were obtained in this study (SI, Section 2, 8, and 11 with text

data). Procedures for tracking cell division are described in SI (Section 3) and Figure S2B.

Development of method for inferring effective potentials/forces of cell-cell interaction

In the case of ions, molecules, etc., radial distribution functions are often

measured and used to infer the effective potentials of their interactions (Fig. appx 1 and

SI, Appendix section 12-1) [1,2]. Although this method is simple and applicable to

thermodynamically equilibrium systems, it is not suitable for non-equilibrium systems,

including morphogenetic events. To infer the effective potentials of non-equilibrium

systems, we fitted the particle-based cell model to the time series of nuclear positions in

the spirit of data assimilation, where the fitting parameters are forces between all pairs of

cell–cell interactions (the total number is *I*, as previously defined) for each time frame.

Data assimilation is a technique to solve an inverse problem for a simulation-based model

[37,38]. One of well-known model fittings is based on least squares in combination with a

linear model (function-based model), resulting in inference of parameter values of the

linear function. In data assimilation, models to be fitted are simulation-based models

instead of function-based models, whereas least squares are also often used to infer

parameter values which are usually numerically solved (Fig. appx 3 and SI, Appendix

section 12-3). Through repeated cycles of simulations of the model, we systematically

searched for the values of the effective forces between all pairs of cell–cell interactions

that minimized the differences (i.e. corresponding to least squares) between the particle

positions in the simulations and the *in vivo* nuclear positions (Fig. 1, and S2C). The

differences (G_{xyz}) to be minimized are defined as follows (SI, Section 4-1 and 4-2).

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$$G_{xyz} = \sum_{t=1}^{T} \sum_{p=1}^{P(t)} \frac{\left\{ x_p(t) - x_p^{\text{ref}}(t) \right\}^2 + \left\{ y_p(t) - y_p^{\text{ref}}(t) \right\}^2 + \left\{ z_p(t) - z_p^{\text{ref}}(t) \right\}^2}{\Delta t}$$

235   (Eq. 2)

Here, *p* is an identifier for particles, *t* is an identifier for time frames, and *Δt* is

the time interval between the time frames. The *x*, *y*, and *z* coordinates of the *p*th particle

obtained from microscopic images are x_p^{ref}, y_p^{ref}, and z_p^{ref}; ref means reference (Fig. S2C).

The coordinates during the repeated cycles of simulations are x_p, y_p, and z_p. The effective

forces were inferred for each cell–cell pair for each time frame (Fig. S2B and C). Note

that, because γ was assumed to be constant in Equation 1, we can only infer relative but

not absolute values of effective forces. To determine whether our method can correctly

infer effective forces, we first applied it to artificial data generated under given potentials

of particle–particle interactions, and examined whether the inferred effective forces were

consistent with the given potentials (Fig. S3A). We tried the Lenard–Jones (LJ) potential

as a test case of the given potentials, which is originated from atom–atom interactions but

is one of the most well-known potentials used in various fields including biology [39]. By

performing particle simulations under the LJ potentials, we obtained time series of the

particle positions (Fig. S3A, SI, Section 4-3-4-1, and Movie S1A). Then, we applied the

inference method to the time series, yielding effective forces for each pair of particles for

each time frame (Fig. S3B). The inferred effective forces were plotted against particle–

particle distance (Fig. S2D, S3C, and SI, Section 5). By averaging the plots for each

shallow bin of distances, we obtained distance–force (DF) curves (Fig. S3C). We found

that the DF curves were not consistent with the curve from the LJ potentials, suggesting

that Equation 2 was not optimal. Therefore, we tried to incorporate various additional

constraints (i.e. cost functions during the minimization procedures) into Equation 2. We

found that, when a cost function was set so that force values approach zero at long distant

regions of cell–cell interactions (SI, Section 4-3, Equation S6), inferred DF curves

became consistent with that from the LJ potentials (Fig. S3). Detailed principles, rational,

and procedures are described in SI (Section 4). Additionally, if this cost function was not

incorporated, this minimization problem exhibited indefiniteness (i.e., a unique solution

could not be determined) and over-fitting also occurred (Fig. appx 3). This cost function

is considered as the prior in the Bayesian inference, and, in general, a unique solution

could be ensured by introducing such a prior [37].

Using Equation S6 containing the above cost function (SI, Section 4-3), we also

tried other potentials generated by freehand drawing, called "FH" potentials in this study,

as an alternative to the LJ potentials (Fig. S3A and Movie S1B). The FH potentials were

chosen because the simulation outcomes under the FH potentials differ from those under

the LJ potentials as shown later (Fig. S11). The inferred DF curves were almost consistent

with the given potentials (Fig. S3C, E, and SI, Section 4-3-4). Conversely, we also tried

using time series of random–walk simulations as negative controls (Fig. S3A and Movie

S1C). The plots of the inferred effective forces against distances were widely distributed,

and the DF curves generated from the plots by averaging were zigzag and disorganized

(Fig. S3C and E). These results indicate that our method can correctly infer DF curves.

Results

Systematic validation of inference method using artificial data

We first systematically validated our inference method using artificial data which

include various external factors. Specifically, we assessed whether spatial constraints

such as egg shells or cavities can affect inference results of effective forces of cell–cell

interactions. Similar to the previous section, "Theory and principle for inferring effective

potential", we generated artificial data based on simulations under LJ potential, to which

we applied our inference method.

In the absence of spatial constraints, inferred distance–force (DF) curves were

well consistent with the DF curves derived from the LJ potential under various settings

of simulations: the settings are related to time interval of sampling, cell proliferation, and

fluctuation of forces between cell–cell interactions (Fig. 2A-B, S4-I, -II, -III, SI, Section

4-4, 7-4, and Movie S8A-E). Next, we introduced spatial constraints corresponding to

eggshells (SI, section 7-4-2 and Movie S8F-I). Even under narrow cylindrical constraints,

inferred DF curves were well consistent with the DF curves from the LJ potential (Fig.

2C; length of cylinder = 25μm, S4-V, and SI, Section 4-4). By contrast, under constraints

with smaller volume where each particle was highly compressed, the profiles of inferred

DF curves were shifted leftward in the distance–force graph (Fig. 2C; length of cylinder

= 15μm, and S4-V). This shift can be reasonable, if radial distribution functions under

compressed conditions are considered (Fig. appx 1 and SI, Appendix section 12-1). In

addition, in molecular science, pressure or density of particles affects profiles of DF

curves [14]. However, in cell biology, we think that the above compressive conditions are

not physiologically relevant, because cells are essentially composed of non-compressive

liquid and their volumes are not changed by external pressure. These trends were similarly

detected under spherical spatial constraints (Fig. S4-IV). These results suggest that

inferred DF curves are not significantly affected by spatial constraints corresponding to

egg shells.

Next, we introduced spatial constraints corresponding to cavities. We assumed

that particles located on the surface of a cavity cannot penetrate into the cavity (Fig. 2D,

S4-VI, SI, Section 7-4-2, and Movie S8J-K). Inferred DF curves were almost consistent

with the DF curves from the LJ potential (Fig. 2D, S4-VI, and SI, Section 4-4). However,

we detected additional repulsive forces at distant regions, which we call distant energy

barrier (DEB). Repulsive forces around the corresponding distances were also detected

in DF curves inferred from simulation data where the forces between cell–cell interactions

were assumed to be absent but the cavities to be present (Fig. S4-VI-E, and SI, Section 4-

4), suggesting that DEB was derived from the cavities. Interestingly, profiles similar to

DEB are well-known in molecular and colloidal sciences: by considering hydration of

particles by solvents as an external factor, effective potentials are modified so as to

contain energy barriers around distant regions [1,2]. We conclude that the effects of cavities

can be incorporated into effective potential of cell–cell interactions. In principle, the

primary effect of a cavity would be to provide potential of positions for each cell but not

potential of cell–cell interactions. Thus, a cavity indirectly affects the effective potential

of cell–cell interactions. To avoid confusion in usage of the terms, we call inferred

potential of cell–cell interactions "effective potential for each cell modeled as cell–cell

interactions".

Validation of inference method by using non-adhesive cells

Before applying our inference method to three-dimensional multi-cellular

systems, we performed an additional negative control experiment using two-

dimensionally cultured cells that exhibit negligible cell–cell interactions but exert traction

forces between the cells and substrate, leading to random–walk-like movements. We

knocked out cadherin function in MDCK cells (E-cadherin and cadherin-6 in the case of

MDCK cells) by disrupting the gene encoding α-catenin, an essential regulator of the two

cadherins (Fig. S5 and SI, Section 8-3) [40]. We found that, in contrast to wild-type MDCK

cells which had organized DF curves (Fig. S5B), DF curves obtained from the α-catenin

mutant cells were disorganized (Fig. S5A), similar to those obtained from random–walk

simulations. This result supports the idea that our method is accurate enough to infer

intercellular forces in multi-cellular systems.

Inference of effective forces in C. elegans early embryo

We next investigated whether effective potential could be detected as a function

of cell–cell distance in three-dimensional systems. The nematode *C. elegans* has well-

defined embryogenesis: cell movements in early embryos are almost identical among

different individuals, and the time series of nuclear positions have been reported

previously [32]. In addition, the spatiotemporal patterns of cell differentiation are absolutely

the same: cell lineage and cell fate are invariant. During early *C. elegans* embryogenesis,

a fertilized egg (i.e., one cell) repeatedly undergoes cell division and cell differentiation,

ultimately forming an ovoid embryo containing ~350 cells (Fig. S6A and Movie S3A).

The cells become smaller through cell division [41], and the total volume of the embryo

remains constant because the embryo is packed into a stiff egg shell. Thus, the volume of

each cell is reduced by a factor of 350 relative to the original egg, meaning that cell

diameter is reduced by a factor of 7. Because particle diameter is approximately reflected

on a DF curve as the distance with force = 0 (Fig. 1B and S2D) corresponding to the

minimum of the potential, we expected that inferred DF curves should be gradually

shifted to the left side of the distance–force graph throughout embryogenesis.

352   Figure 3A is a snapshot that shows nuclear positions and inferred effective forces

in the embryo; the temporal change is also visualized (Fig. S6B and Movie S3B), where

dynamic changes in the effective forces were observed. Importantly, almost identical

values of the effective forces were obtained from different initial values during the

minimization process of Equation S6 described previously (Fig. 3B, S10A, and SI,

Section 4-4), suggesting that our inference problem may have a unique solution. To

investigate whether DF curves change during the embryogenesis, we divided the whole

embryogenesis into segments containing different time frames, and plotted the inferred

forces against the distances for each segment (Fig. 3C and S6C, time frame = 16–55, 36–

75, 76–115, 116–155, and 156–195). The plots were widely distributed in all the segments,

but the DF curves generated by averaging in each shallow bin of the distances exhibited

organized patterns. This was in striking contrast to the results of the random–walk

simulations and the α-catenin mutant cells. All of the DF curves had very similar profiles:

repulsive and attractive forces were detected around the core of the cells and longer

interaction distances, respectively (Fig. 3D). The repulsive forces are likely to be derived

from the volume effect of the cells, and the attractive forces might be derived from cell-

cell adhesion. These patterns are typical in various non-living particle–particle

interactions such as atoms and molecules, whose potentials are given by the LJ potential,

etc. (Fig. S3A). We also calculated the distance–potential (DP) curves by integrating the

force values; potential energy is obtained from the integral of forces along distances (Fig.

1B, S2D, and SI, Section 5). Importantly, the distances in the minimum of the effective

potentials were gradually shifted toward the left side (Fig. 3D), as we had expected (Fig.

3E), indicating that the inferred effective potentials were consistent with the reduction in

cell volume during embryogenesis (Fig. 3E). Taken together, the patterns of the DP curves

seem to be physically reasonable. As speculated from the previous analyses using

artificial simulation data (Fig. 2C), we think that the eggshell does not significantly affect

the profiles of the DP curves. In addition, the data points were widely distributed from

the DF curves as shown by the heat maps, which may originate in biological properties,

including variety of cell sizes, the intrinsic heterogeneity of cells and/or experimental or

methodological errors during detection of cell centers or the inferring process, as

discussed in SI (Section 4-6).

Effect of inferred distance–force curves on C.elegans embryonic morphologies

The inferred DF curves for *C. elegans* embryogenesis seem to be reasonable.

However, one concern may be that the averaged DF curves are oversimplification, given

the broad distribution of the effective force, as discussed later. This prompted us to further

investigate whether the simple DF curves can explain embryonic morphologies. Because

the dynamics of all cells during embryogenesis are very complicated, largely due to the

repeated cell division and cell differentiation, we focused on a few typical features of the

morphologies: aggregated cells and the ovoid embryo shape. The embryonic cells can

keep a cell aggregate even when the eggshell is removed, and subsequent culture leads to

a cell aggregate with an ovoid or distorted shape, except for very early stage of

development (< ~20 cells) [42,43].

We performed simulations based on the particle-based cell model by considering

the DF curves, and investigated whether aggregated states with ovoid shapes were stable.

In multi-cellular systems, morphologies of systems are determined by both DF curves and

initial configurations, because energetic local minimum states (i.e. metastable state) as

well as global minimum states are meaningful (SI, Appendix 12-2 and Fig. appx 2). To

find stable states under the DF curves, we performed simulations starting from various

initial configurations of the particles, and the systems were relaxed. When we used the

DF curves obtained from the *C. elegans* embryos, we observed a tendency for the particles

to aggregate with an ovoid or distorted shape (Fig. 6A and S11). As a control simulation,

we utilized the LJ potential, and found that it did not generate an ovoid or spherical shape,

but rather a shape similar to its initial configuration (Fig. S11). Moreover, the FH

potentials generated a spherical but not ovoid shape, and the outcomes were not

significantly affected by the initial configurations (Fig. S11). These results suggest that

the inferred effective DF curves are capable of recapitulating the basic morphology of the

C. elegans embryos.

Inference of effective force in mouse pre-implantation embryos

To further investigate whether effective forces could be obtained in three-

dimensional systems, we focused on mouse pre-implantation embryos, including morulae

at the 8-cell and compaction stages, as well as blastocysts bearing cavities (Fig. 4A,

illustration). These embryos are surrounded by a spherical egg shell–like structure called

the zona pellucida, but even if that structure is removed, the embryos maintain their

spherical shapes and development normally [44]. In 8-cell stage embryos before compaction,

cell–cell adhesion is weak, and individual cells can be easily discerned (Fig. 4A, bright

field). In the compaction-stage embryos composed of ~16-32 cells, cell–cell adhesion

becomes stronger due to elevated expression of cadherin proteins on cell membrane, and

the cells are strongly assembled [45]. The surface of the embryo becomes smooth, and the

embryonic shape becomes more spherical (Fig. 4A, bright field). In blastocyst-stage

embryos composed of >64 cells, an inner cavity is formed, and the embryos expand while

maintaining their spherical shape. Trophectoderm (TE) cells form a single–cell-layered

structure at the outermost surface of the embryo, whereas the cells of the inner cell mass

(ICM) were tightly assembled, forming a cluster (Fig. 4A, illustration). Using confocal

microscopy, we performed live imaging of fluorescently labeled nuclei, obtained time

series of the nuclear positions (Fig. 4A, Movie S4A-C, and SI, Section 8-1), and then

applied the inference method.

Figure 4B shows the snapshots of the inferred effective forces of cell–cell

interactions during the three embryonic stages that we examined (Movie S4D–G). The

uniqueness of the solutions was confirmed (Fig. S10). We calculated the DF and DP

curves in a manner similar to *C. elegans* study described above, and obtained curves for

all three stages (Fig. 4C and D). The DF curves derived from the 8-cell and compaction

stages had typical profiles, with repulsive and attractive forces detected at short and long

interaction distances, respectively (Fig. 4D and S6D). On the other hand, the DF curves

derived from the blastocyst stage contained DEB (Fig. 4D, arrows), similar to the case of

artificial simulation data considering a cavity (Fig. 2D and S4-VI). Thus, we think that

the DEBs in the blastocysts are derived from the cavities. Such DEBs have not been

discovered in multi-cellular systems. Together, these results indicate that the profile of

DF curves changes during the development. Moreover, the profiles of the DF curves

suggest the consistency with previous experimental knowledge as discussed in Figure S7.

Effect of inferred distance–force curves on mouse embryonic morphologies

We next asked whether the temporally evolved DF curves derived from the

mouse embryos were sufficient to explain the key morphological features of the embryos.

Using the DF curves, we searched for stable states by an approach similar to the one we

used for *C. elegans* embryos. In contrast to the case of *C. elegans* embryos, in which an

ovoid shape was generated, the DF curve derived from the compaction stage embryo

yielded a spherical shape where particles were aggregated (Fig. 6B and S11). In the case

of the 8-cell stage embryo, simulation results were similar to those obtained for the

compaction stage, except that the spherical shape seemed to be slightly distorted (Fig.

S11). In the case of the blastocyst stage embryo, when aggregated particles were given as

initial configurations, the particles gradually scattered and formed a spherical structure

harboring a cavity at the center (Fig. 6B). We then examined whether DEB contribute to

the formation of a cavity. We artificially eliminated DEB from the inferred DF curves,

and performed a simulation. We found that no cavity was generated (Fig. 6B). These

results suggest that all of the DF curves from the various embryonic stages are sufficient

to explain their different morphologies. We also showed DF curves that were obtained

from interactions between adjacent cells (Fig. S8), by which we may evaluate direct cell–

cell interactions excluding indirect effect of cavities. Later we will discuss the differences

in the DF curves between the two cell populations TE and ICM (Fig. S7). We will also

discuss cellular polarities, such as apico-basal and planar cell polarities that were not

explicitly considered in this study (Fig. S9 and SI, Appendix section 12-1).

Inference of effective distance–force curves and their effect on morphology in MDCK cyst

467   The above results led us to hypothesize that DEB of the DF curves plays essential

roles in the morphogenesis of structures harboring cavities. To examine this hypothesis,

we focused on another system with a cavity. MDCK cells, derived from dog kidney

epithelium, can form cysts containing a cavity when cultured in suspension (Fig. 5A,

illustration) or in gels composed of extracellular matrix [46]. To exclude mechanical

interactions between the cysts and the external environment, we chose suspension

conditions in which a cyst can be assumed to be a mechanically isolated system for which

external forces are negligible. We obtained time series of the nuclear positions of the cysts

by a procedure similar to the one used for the mouse embryos (Fig. 5A, Movie S5A, and

SI, Section 8-2). We then inferred the effective forces and calculated the DF curves (Fig.

5B–D and S6E). The uniqueness of the solutions was confirmed (Fig. S10). Figure 5D

shows the DF and DP curves from a MDCK cyst. We found a DEB in the DF curves,

implying that a DEB is generally involved in morphogenesis of cavity-bearing structures.

Simulations based on inferred DF curves revealed that, a cavity was stably

maintained in MDCK cells (Fig. 6C and S11). Moreover, the DF curves from which the

DEB was artificially eliminated could not maintain the cavity-bearing structure (Fig. 6C).

Taken together, these findings indicated that the DEBs in the blastocyst and the MDCK

cyst play roles in the formation and maintenance of a cavity, respectively.

Modeling of distance–force curves

The analyses from the blastocysts and MDCK cysts suggest that the effect of

cavities can be incorporated into effective potential for each cell modeled as cell–cell

interactions. We hypothesized that the DF curves incorporating such external factors

represent a rule capable of explaining various morphogenetic events. Moreover, the

profiles of the DF curves in Figure 3-5 were quantitatively different each other (Table S2),

suggesting the quantitative differences are critical for morphogenesis. To

comprehensively understand the effect of the profile of the DF curves on morphogenesis,

it is necessary to model the DF curves as a simple mathematical equation. We selected

the following equation as described in SI (Section 6):

$$F(D) = \varepsilon (D - D_0)^{-N} \cos \left\{ \frac{2\pi(D - D_0)}{\delta} \right\}, \text{ for } D > D_0 \qquad \text{(Eq. 3)}$$

Here, F is the force, D is the distance, and N is the exponent of D, affecting the decay of

the DF curves along D (X-axis). D_0 and δ affect the profile of the DF curves as shown in

Figure 7A. Specifically, D_0 can transform the DF curves along D, and δ affects the

wavelength of the cosine function. All parameters and variables related to

lengths/distances were normalized by ($\delta/4$), leading to generation of dimensionless

parameters and variables such as $D_0^* = D_0/(\delta/4)$; usage of dimensionless parameters and

variables is general technique to reduce the number of parameters. This enabled us to

present simulation outcomes in two-dimensional space (D_0^* and N) in a manner similar

to a phase diagram (Fig. 7).

Systematic analyses of the roles of the distal energy barriers

Using the DF curves defined by Equation 3, we systematically investigated what

kind of morphologies could be generated or stably maintained, and what kind of

morphological transformation could be achieved. Here, we tried to identify possible

stable states, including metastable states. We set various initial configurations of particles,

performed simulations under various DF curves, and then searched for local minimum

states. A cavity-bearing structure was set as the initial configuration (Fig. 7B). The

simulation outcomes were categorized by their morphologies and plotted on a two-

dimensional space (Fig. 7B). Examples of the morphologies and DF curves are also

shown. We identified cavity-bearing structures ("cavity" and "cavity with extra

particles"), cups, tubes, and lattices / aggregates as possible stable states. Multiple clusters

of particles were formed or disorganized morphologies were generated which include

distorted rings, etc. Importantly, when the DEBs were eliminated from the DF curves, one

of the cavity-bearing structures ("cavity"), cups, or tubes were not generated (Fig. 7C),

indicating that the DEBs are essential for the formation or maintenance of these structures.

Next, we tried other initial configurations of particles. A two-dimensional sheet was

applied as the initial configuration which corresponds to a monolayered but not multi-

layered cell sheet (Fig. 8). The simulation outcomes included spherically aggregated

structures, cavity-bearing structures, and two-dimensional sheets (Fig. 8A). In contrast to

Figure 7B, the cavity-bearing structures were newly generated even in the absence of the

DEBs (Fig. 8B). DF curves were found, which can maintain the two-dimensional sheet,

and the maintenance was largely dependent on the DEBs (Fig. 8A vs. 8B). We confirmed

that the generated two-dimensional sheets were not broken by slight positional

fluctuations (Movie S7), indicating that the structures were stable. Thus, the DEBs are

essential for the maintenance of two-dimensional sheet. Other analyses considering

different initial configurations and/or different particle numbers are shown in Figure S12;

tubes were maintained by DEBs, and cell sorting in systems containing multiple types of

cells, which is an important phenomenon in developmental biology[3], was observed.

Possible transitions among various structures

The results presented above demonstrate that various structures can be formed

or maintained by the DF curves harboring DEBs. We next investigated whether

modulation of the profiles of the DF curves can induce transitions among the structures.

We modulated the profile of the DF curves by modifying the values of the variables in

Equation 3. Through trial-and-error of the modulation of the profile, we found that these

structures transformed each other as shown in Figure 9 and Movie S6 (SI, section 7-3):

e.g. aggregate → cavity → cup →tube → aggregate, etc.

Combination of inference method with traction force microscopy

We have been showing that some external factors, such as cavities, can be

incorporated into effective potentials for each cell modeled as cell–cell interactions. An

alternative approach is to subtract external effects prior to inference of effective potentials,

which can lead to acquisition of potentials derived from direct effects of cell–cell

interactions. In other words, external effects are quantitatively measured to exclude them

from inference of effective potentials. However, this kind of quantitative measurements

is still challenging in biology especially in three-dimensional situations. Alternatively, we

tested this approach in two-dimensional situations where traction forces are exerted

between cells and substrate, because there is a well-established method to quantitatively

measure traction forces, traction force microscopy (TFM) [47–50].

First, we tried this approach by using artificially generated simulation data where

traction forces were provided (Fig. S13 and Movie S9). By subtract traction forces prior

to inference, we obtained DF curves consistent with the given DF curve derived from the

LJ potential (Fig. S13). Finally, we tried this approach in two-dimensionally cultured

MDCK cells with TFM measurement (Fig. S14). Without the subtraction of traction

forces, the inferred forces of cell–cell interactions were shifted toward repulsive forces.

On the other hand, by subtracting traction forces, we obtained typical DF curves with

repulsive forces at shorter distance and attractive forces at middle distances (Fig. S13 and

S14). These results suggest that, by combination with TFM and possibly other techniques,

our inference method can yield potentials derived from direct effects of cell–cell

interactions.

Extraction of information of cell polarity

We have been inferring effective potentials for each cell modeled as cell–cell

interactions without explicitly considering cell polarities such as apico-basal polarities

and polarities on cell sheet including planar cell polarity (PCP). We tried to extract these

polarities from the inferred effective potentials by considering directions of cell–cell

interactions.

First, we analyzed the inference results derived from artificially generated

simulation data with unidirectional traction forces under two-dimensional situations as

previously shown (Fig. S13G and S13H). In this simulation, the leading cells exerted the

largest traction forces among all cells, and the traction force values were gradually

decreased from the leading cells to the followers. This situation resembles wound healing.

Thus, the cells have polarities on the cell sheet: the direction of collective migration vs.

its vertical direction. We divided the directions of cell–cell interactions into three

categories according to their angles, and tested whether DF curves obtained from these

categories became different each other. Note that we did not subtract traction forces in

this analysis. As shown in Figure S15, the DF curve along the vertical direction of

collective migration was equivalent to the DF curve derived from the LJ potential,

whereas the DF curve along the direction of collective migration was significantly

changed, resulting in a repulsive force-dominant profile. This is intuitively reasonable,

because cell–cell distances along the direction of collective migration are increased

during wound healing, which corresponds to existence of apparent repulsive forces. These

results suggest that cell polarity can be extracted after inferring effective potentials. Note

that the inferred effective potentials contain parts of the effects of external forces but not

all: external forces such as traction forces can move the centroid of cell populations but

internal forces such as the effective potentials modeled as cell–cell interactions cannot

due to the law of conservation of momentum.

We also tried to extract cell polarity in mouse embryos. During the compaction

stage, the cells, which are located on the outer surface of the embryos, gradually form cell

polarity [51], leading to the differences in protein localization between the junctions of

outer–outer cells and of outer–inner cells. We found that the DF curves of these two

interactions were different each other (Fig. S7A). Finally, we analyzed apico-basal

polarity in TE cells of mouse blastocyst. We tried to extract DF curves in the cell sheet of

TE and along the direction vertical to the cell sheet corresponding to the apico-basal

direction. However, there were essentially no cell–cell interactions detected along the

apico-basal direction, as shown by the probability of cell distribution against the

directions (Fig. S9A-C). Thus, we cannot infer DF curve along the apico-basal direction.

In general, probability of states is linked to potential energy (SI, Appendix section 12-1).

Thus, we calculated potential energies against the angular direction, and found that the

potential energy along the apico-basal was expected to be extremely large compared with

the direction on the cell sheet (Fig. S9D and S15D).

Influence of experimental noise on inference result

609   In general, accuracy of inference methods can be affected by observation noises.

In our method, noises of nuclear positions during image recognition may affect the

accuracy. We theoretically assessed the influence of the noises of nuclear positions (SI,

Section 4-5). Using artificially generated simulation data, we added noises of nuclear

positions as Gaussian noises, and inferred effective potentials for each cell modeled as

cell–cell interactions. The averaged distances between nuclear positions before and after

adding Gaussian noises was set variously from 0.16 to 1.0μm (Table S1-i-iv). We found

that the increase in the Gaussian noises caused modulation of inferred DF curves (Table

S1-i-iv and Movie S10). There were two major modulations: an emergence of

disorganized patterns similar to that from random walk and an increase in maximum

absolute values of attractive forces (Table S1-i-iv and SI, Section 4-5). In spite of the

latter modulation, the peaks of attractive forces were not significantly changed along the

distances, suggesting that overall patterns were almost conserved except for the absolute

values of attractive forces. It is difficult to correctly estimate actual experimental noises,

but residuals from smoothing spline interpolation along time for positions of each nucleus

might roughly correspond to experimental noises (Fig. appx 4 and SI, Appendix section

12-4). We found that the values of the residuals ranged from 0.2 to 0.4μm in the *C. elegans*

embryo, the mouse embryos, and the MDCK cysts (SI, Section 4-5-2). As shown in Table

S1-i-iv, these *in vivo* situations fell within acceptable conditions expected from the

artificial data: the patterns of DF curves would not be disorganized/randomized and the

orders of the absolute values of attractive forces are expected to be sustained.

Discussion

In this study, we developed a method for statistically inferring effective forces

of cell–cell interactions using nuclear tracking data obtained from microscopic images.

We then demonstrated that effective potentials for each cell modeled as cell–cell

interactions can be extracted from the inferred forces in various living systems. Our

findings provide for the first time the experimental quantification of effective potentials,

which have been recognized conceptually [11,17]. We also showed that, in the coarse-grained

cellular model, the effective potentials can partially contain information of cell polarities

and of external factors such as cavities, which can be decoded after inference, suggesting

that the effective potential is an integrative parameter. By considering the effective

potentials, we successfully reproduced various three-dimensional structures, including

cell aggregates, cavity-bearing structures, cups, tubes, two-dimensional sheets, and sorted

cells. These structures are often observed during embryogenesis, morphogenesis, and

organoids prior to the formation of complex tissue structures. For instance, cell aggregates

are observed in early embryos in various species and spheroids [20,32,52–57]; cavity-bearing

structures in early embryos of various species, organoids, and cystic diseases [46,52,58–61];

cell sheets in epithelia and germ layers [20,56,62–65]; tubes in tubular organs [66–68]; and cups in

retina (i.e. optic cup) and murine epiblast [53,69]. Thus, we consider these structure as basic

structural units for morphogenesis. These results suggest that the effective potentials

alone are powerful to explain diverse three-dimensional morphologies. Although cell–

cell interactions are affected by many parameters including proteins such as cadherins

and cellular structures such as actomyosin cytoskeletons (Fig. S1 and S5), we suppose

that the effective potentials in the coarse-grained model can be an integrative and unifying

mesoscopic parameter to explain morphogenetic events. Our goal is to develop a

framework for explaining diverse morphogenesis in a minimal model. The model based

on effective potential represents such a framework with few degrees of freedom (Fig.

S1A), and this provides a novel paradigm for understanding the morphogenesis of multi-

cellular systems and their morphological evolution. We believe that effective potentials

and the inference method constitute a powerful approach for exploring principles for

diverse morphogenesis.

Profile of effective potentials of cell–cell interaction

We discovered distant energy barriers (DEBs) in the effective potentials obtained

from blastocysts and MDCK cysts, which would be derived from indirect effect of

cavities. These energy barriers contributed significantly to the generation and

maintenance of cavity-bearing structures, tubes, and sheets. Such energy barriers are well-

known in molecular and colloidal sciences, as mentioned in SI (Section 5-4-1). We

speculate the physical interpretation of these energy barriers in cells below. In the case of

the blastocysts, the trophectoderm cells form tight junction to seal gaps between the cells

and transport liquid from the outside of the embryos to the cavities, resulting in increased

hydrostatic pressure, fracturing of the cell–cell contacts of the inner cells, and subsequent

expansion of the cavities [70–72]. Therefore, the TE cells push each other through the liquid

in the cavities. This indirect mechanical repulsion might be reflected in the DEBs (Fig.

S16A). In the case of the MDCK cysts, tubes, and sheets, the DEBs may reflect properties

of cell sheets as discussed in Figure S16B.

Potentials of cell–cell interactions have been widely considered in various

materials such as active matter including self-migratory cells. In these studies, however,

the profiles of the distance–potential curves are assumed without experimental bases. For

instance, hard- or soft-core, square-well, JKR, and other potentials have been considered

(Fig. appx 5 and SI, Appendix section 12-5) [19,20,25,73,74]. Animal and human behaviors can

also be modeled as active matter governed by attractive and repulsive interactions [75–78].

Our inference method can be used to obtain the profiles of the potentials in these materials,

possibly leading to new finding of the dynamics of these materials. Moreover, the material

properties of particulate soft matter can be estimated using inferred effective potentials as

demonstrated in SI (Fig. S17, Table S2, and SI, Section 4-6-2 and 5-4-2).

Usefulness, application, and limitation of inference method

Our *in situ* inferences provide information about mechanical states, or maps, of

living systems. Various methods have been developed for measuring or inferring

mechanical maps in living systems such as TFM (Fig. S14) [17,37,79–82]. Our method is

advantageous for obtaining mechanical states in three-dimensional situations with

temporal evolution. To understand mechanical states, it is necessary to complementary

utilize these methods with various temporal and spatial resolutions, as demonstrated by

combining our method with TFM (Fig. S14).

To understand morphogenetic events, mechanical states and simulations based

on those states are essential. Models such as vertex and Cellular Potts models are often

utilized, especially for epithelial cells, where the cells are modeled as polygonal or

polyhedral shapes with adhesion energy and surface tension [5,52,58]. These models

successfully recapitulated various morphogenetic events, including the formation of the

mouse blastocyst [52], very early embryogenesis in *C.elegans* [83], and the formation of cystic

structures in kidney [58]. These models often contain non-cellular components such as

liquid in cavities and egg shells. However, it is still challenging to measure cellular and

non-cellular parameters in three-dimensional systems with high spatiotemporal resolution

(e.g. spatiotemporal information of pressure of cavities, and traction forces in three-

dimensional situations), although high-resolution inference has been performed in two-

dimensional situations [37,82]. This means that there is no clear way to connect the model

parameters to *in vivo* ones in three-dimensional systems. On the other hand, particle-based

cellular models can also assume parameters other than the potential of cell–cell

interactions, leading to recapitulation of complex three-dimensional structures such as the

blastocyst [84–86]. In particular, a recent study reported a model that considered cell

polarities, in which transformation from cell aggregates to cell sheets and bending of cell

sheets were simulated [86]. Particle-based models have also been expanded by considering

cell shapes, where a cell is composed of two particles or a Voronoi tessellation is

combined to implement multibody effect [22,28,31,57,63,87,88], and mechanical properties and

dynamics of self-propelled cells or cell aggregates were investigated. These observations

suggest that particle-based models are potentially applicable to very complex structures,

although it has not been established whether highly deformed cells and non-cellular

structures such as extracellular matrix can be implemented simply in particle-based

models. In our strategy, model parameters inferred from *in vivo* systems can be used for

simulations. Our method provides a framework for quantitatively connecting model

parameters to *in vivo* parameters under three-dimensional situations, and will thus be

complementary to other models.

725    Investigations of the origins of the effective potentials are important for a

726 complete understanding of morphogenesis. Various proteins including cadherins and

727 actomyosins should be involved in the potentials. In fact, disruption of the gene encoding

728 α-catenin, which is essential for the function of cadherin proteins, leads to the changes in

729 the profile of effective potentials (Fig. S5). Our framework enables us to relate proteins

730 to the profile of potentials by evaluating the effect of gene disruption or inhibition on the

731 profile. Comprehensive evaluation of the effect of various proteins will be valuable, and

732 will provide a bridge between the physics of morphogenesis and molecular biology.

734 *Toward understanding of a later stage of embryogenesis and organogenesis*

735    In this study, we used relatively small embryos: mouse pre-implantation embryos

736 (several tens of cells) and the *C. elegans* embryo (< 350 cells). To apply our inference

737 method and simulation model to later embryonic stages, new components should be

738 considered, such as the increase in the number of cell types, cell polarities, and

739 chemotaxis. The number of cell types increases through cell differentiation, in which

740 mesenchymal cells as well as blastomere cells and epithelial cells are included. The

741 blastocyst contains at least two cell types (ICM and TE), and we successfully detected the

742 differences in the DF curves between these two cell types (Fig. S7). In later stages of

embryogenesis, such as gastrulation, the embryos are usually composed of mixtures of

mesenchymal and epithelial cells, in which the epithelial-to-mesenchymal transition

occurs. In contrast to the vertex model, where mesenchymal cells cannot be considered,

particle-based models may simulate the mesenchymal cells and mixtures of mesenchymal

and epithelial cells [24,25]. Mesenchymal and epithelial cells are separated each other (i.e.

cell sorting), which has been explained by differential cell–cell adhesiveness [3]. Other

group and we demonstrated that particle-based models can reproduce cell sorting by

considering cell–cell adhesiveness as potentials (Fig. S12-III-B) [89]. Apico-basal and

planar cell polarities are critical parameters for morphogenesis [86]. We speculate that these

polarities can be implemented in the particle model by implementing two particles for a

cell where cell shapes become anisotropic [28,31], or by considering directional differences

in the profiles of effective potentials, as recently proven by other group [86] and shown in

Figure S7A and S15D. Cell–cell interactions are regulated not only by mechanical forces

but also by chemical signaling such as chemotaxis and ephrin proteins (Fig. S1B). By

expanding the concept of effective potentials, we think that attractive and repulsive effects

of chemical signaling can be implemented as the profile of the effective potentials,

possibly with non-reciprocal interactions [90–94]. It is possible that by analyzing inferred

cell–cell interaction forces, we could extract differences in cell types, directional

differences in the profile of the effective potentials, and the effects of chemotaxis.

At this moment, we do not know whether the particle-based model is useful for

later stages of embryogenesis or larger tissues to which continuum models are usually

applied [95]. Multi-scale frameworks may be required to connect particle-based models with

continuum models based on macroscopic parameters. In any case, our inference method

may be useful for evaluating material properties of larger tissues, which are essential for

developing continuum models.

Conclusion

Our framework, composed of the inference method and the particle-based model,

has various potential uses for investigating the properties of cell–cell interaction, single-

cell analysis of mechanics, material properties, mechanical maps, three-dimensional

morphogenesis, relationships between mechanical parameters and gene products, and

comparisons of mechanical parameters and behaviors among different living species and

non-living systems. Although some of these issues will be examined in the future, we

propose that our framework provides a novel physical scenario for understanding the

biophysics and physical biology of diverse morphogenetic events.

Experimental materials and methods

Mouse embryos were obtained by mating Rosa26-H2B-EGFP knock-in male mice [36] with

ICR female mice. Animal care and experiments were conducted in accordance with the

Guidelines of Animal Experiment of the National Institutes of Natural Sciences.

Experiments were approved by the Institutional Animal Care and Use Committee of the

National Institutes of Natural Sciences. MDCK transgenic cells bearing H2B-EGFP were

generated using a piggy-bac plasmid [96,97]. α-catenin mutant cells were constructed

previously [98]. The detailed procedures for cell and embryo cultures, microscopic imaging,

nuclear tracking, force inference, simulations, and data analyses are described in SI. The

nuclear tracking data, inferred forces, and profiles of distance–force curves are provided

in SI. TFM was performed as described previously [48,49].

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Conflicts of interest

The authors declare no competing financial interests.

Author contribution

H.K. designed the work, H.K., A.M.I., and H.O. contributed to the conception. H.K. and

T.F. designed experiments, T.O. provided experimental materials, and H.K. performed

experiments and image processing. H.K. and H.O. designed models, H.K. developed

computational algorithms, and H.K. and K.N. statistically analyzed the data. H.K., T.O.,

H.O., A.M.I., K.N., K.K., and T.F. wrote the manuscript, and all authors contributed to

the interpretation of the results.

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Figure 1: Overview of strategy for inferring the effective potential of cell–cell interactions

A. Cells were modeled as particles. A blue sphere corresponds to a cell. B. In the particle-

based model, distance–force (DF) and distance–potential (DP) curves are considered.

Typical profiles of the curves, their physical meaning, and their origins are shown. DF

and DP curves are mathematically transformed each other. The distance with force = 0

corresponds to the distance with energetic minimum. C-I. Strategy for inferring effective

forces of cell–cell interactions is described. C. Mechanics of cell–cell interactions is

coarse-grained, leading to effective forces of cell–cell interactions as shown in D. There

are many components related to cell–cell interactions, which are not explicitly considered

in this coarse-grained model. D. In the coarse-grained model, attractive or repulsive force

between the particles (blue spheres #1–4), was considered; force vectors (F_i) are

illustrated by black arrows in the case of particle #3. The net force (F_p) is shown by a red

vector, and the relationship between F_p and velocity (V_p) is described. E. From a time

series of microscopic images of nuclei, nuclear tracking data were obtained. An image

derived from a cyst formed by MDCK cells expressing H2B-EGFP is shown as an

example. F. Effective forces of cell–cell interactions were inferred by solving an inverse

problem: we statistically searched for force values in the model (D) that could reproduce

the nuclear tracking data (E). G. The inferred force values (F) were plotted against the

distance between cell–cell interactions, and   whether DF or DP curves are emerged from

the plots were examined. H. Whether the DF or DP curves can reconstitute tissue shape

or cell dynamics was examined. I. Whether components related to cell–cell interactions

described in C can be decoded from the DF or DP curves was tested. In addition,

influences of external factors on the profiles of the DF or DP curves was assessed.

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Figure 2: Validation of inference method using artificially generated data

Simulation data were used to validate our method for inferring effective forces of cell–

cell interactions. The simulations were performed on the basis of a distance–force (DF)

curve obtained from the Lenard-Jones potential (LJ). The simulation conditions contain

"Steady state" (A), "Cell proliferation" (B), "Egg shell-like spatial constraints" (C), and

"Cavity-like spatial constraints" (D). The left panels in A-D are snapshots of the

simulations. The right panels in A-D are DF curves obtained from the inference. Effective

forces of interactions for each pair of cells for each time frame were plotted against the

distance of the cell–cell interactions. The graph space was divided into 64×64 square

regions, and the frequencies of the data points plotted in each region were calculated. The

mean value of the frequencies was calculated for each of the 64 columnar regions along

the distance. Frequency index (FI) is defined so that the mean value of the frequencies is

1, and a colored heat map was generated according to the FI. White corresponds to an FI

of 0. Averaged values of the forces were calculated for 64 columnar regions along the

distance, and are shown in yellow (binned average). Detailed procedures are described in

SI (Section 4). In some graphs, the DF curves from the LJ potential are overlaid by orange

broken lines (LJ). In A and B, simulation data with different sampling intervals were

applied. In C, under the cylindrical constraint with a shorter length where particles are

highly compressed, the DF curve was shifted leftward along the distance (white arrow).

In D, on the expanding cavities, repulsive forces at distant regions were detected, which

we called distant energy barriers (DEB). Related comprehensive analyses are shown in

Figure S4 with simulation movies (Movies S8A-K). Detailed assumptions and conditions

of the simulations are described in SI, Section 7-4.

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Figure 3: Inference of effective force of cell–cell interaction in *C.elegans* embryos

A. Snapshot of the nuclear positions in the *C.elegans* embryo (left panel) and snapshots

with inferred effective forces (right three panels) were three-dimensionally visualized at

the time frames as described (t16, t76, and t195); the interval between time frames is 1

min. The spheres correspond to cells whose colors represent cell lineages, including AB,

C, D, E, MS, and P & Z. The lines represent cell–cell interactions; the colors indicate the

values of the effective forces (red, attractive; blue, repulsive). Forces are depicted in

arbitrary units (A.U.); 1 A.U. of the force can move a particle at 1μm/min as described in

SI (Section 4-4). The nuclear tracking data were obtained from a previous report [32].

Related figures and movies are provided (Fig. S6A, B, Movies S3A and B). B. Uniqueness

of solution of effective force inference was examined. The minimizations of Equation S6

were performed from different initial force values as described in the x- and y- axes, and

the inferred values of each cell-cell interaction were plotted by crosses. The inferred

values from the different initial force values were absolutely correlated. Values from all

time frames (t1-195) were applied. C. The inferred effective forces of cell–cell

interactions were plotted against the distance of cell–cell interactions in a manner similar

to Figure 2. The graphs for time frame 76–115 and 156–195 are shown. Data at other time

frames are provided in Figure S6C. D. Distance–force (DF) and distance–potential (DP)

curves were estimated. The averaged values of the effective forces in (C) were smoothed

over the distance, and the potentials were calculated by integrating the forces. Details are

described in SI (Section 5-3). The curves for each time frame are shown. E. Mean

diameters of cells at each time frame were estimated from cell numbers and the DP curves.

Given that the volume of the embryos is constant ($= \mathrm{Vol}$) during embryogenesis, the mean

diameters were estimated from the cell numbers (N_c) at each time frame as follows: mean

diameter $= \{\mathrm{Vol} / (4/3 \ \pi \ N_c)\}^{1/3}$. The diameters relative to that at time frame $= 16$ are

shown with cell numbers (parentheses). The sizes of the circles reflect the diameters,

whose colors roughly correspond to the colors in the graph in D. The diameters were also

estimated from the DP curves in D; the distances with minima of the DP curves roughly

reflect the diameters (Fig. 1B and S2D).

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Figure 4: Inference of the effective force of cell–cell interaction in mouse pre-

implantation embryos

A. Eight-cell, compaction, and blastocyst stages of mouse embryo are illustrated, and

their confocal microscopic images are shown: bright field, maximum intensity projection

(MIP) and cross-section of fluorescence of H2B-EGFP. Snapshots of nuclear tracking are

also shown; blue spheres indicate the detected nuclei. In the illustration of the blastocyst,

ICM and TE cells are depicted by orange and blue, respectively, and a cavity is also

described. In the nuclear tracking image of the blastocyst, ICM cells are located around

the bottom region of the image, as indicated by *. Scale bars = 15μm. Related movies are

provided (Movie S4A–C). B. Snapshots of nuclear positions with inferred effective forces

were visualized three-dimensionally for the three embryonic stages in a manner similar

to Figure 3A. Blue spheres correspond to cells. The lines represent cell–cell interactions,

and the colors indicate the values of the effective forces [red, attractive (attr.); blue,

repulsive (repl.)]. Related movies are provided (Movie S4D-G). The uniqueness of the

solutions was confirmed in Figure S10B. C. The inferred effective forces of cell–cell

interactions were plotted against the distance of cell–cell interactions in a manner similar

to Figure 2, except that the graph space of the 8-cell stage was divided into 32×32 square

regions, and the averaged values were calculated for the 32 columnar regions. D.

Distance–force (DF) and distance–potential (DP) curves were estimated from (C) in a

manner similar to Figure 3D. A distant energy barrier (DEB) was detected, as indicated

by arrows. Data from other embryos are provided in Figure S6D. The differences in DF

and DP curves between ICM and TE cells and between the outer and inner cells in the

compaction state were described in Figure S6. DP curves between adjacent cells are

provided in Figure S8A-C. Directional differences of DP curves, which may correspond

to cell polarity, are discussed in Figure S9 and S15D.

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Figure 5: Inference of effective force of cell–cell interaction in cysts formed by MDCK

cells

A. MDCK cysts formed under suspension conditions are illustrated, and confocal

microscopic images are shown: bright field, maximum intensity projection (MIP) and

cross-section of fluorescence of H2B-EGFP. A snapshot of nuclear tracking is also shown;

blue spheres represent nuclei. Scale bars = 10μm. A related movie is provided (Movie

S5A). B. A snapshot of the nuclear positions with inferred effective forces was three-

dimensionally visualized in a manner similar to Figure 3A. The blue spheres correspond

to cells. The lines represent cell–cell interactions and the colors indicate the values of the

effective forces [red, attractive (attr.); blue, repulsive (repl.)]. A related movie is provided

(Movie S5B). The uniqueness of the solutions was confirmed in Figure S10C. C. The

inferred effective forces of cell–cell interaction were plotted against the distance of cell–

cell interaction in a manner similar to Figure 3C. D. Distance–force and distance–

potential curves were estimated from (C) in a manner similar to Figure 3D. DEB was

detected as indicated by arrows. Data from other cysts are provided in Figure S6E. DP

curves between adjacent cells are provided in Figure S8D.

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Figure 6: Outcomes of simulations based on inferred distance–force curves

Simulations results under the DF curves derived from *C. elegans* (A) and mouse embryos

(B), and the MDCK cyst (C) are shown. Blue spheres correspond to cells. In the *C.*

elegans embryos, the DF curves were derived from time frames 36–75 and 156–195 in

Figure 3D. In the mouse embryos, the DF curves were derived from the compaction and

blastocyst stages in Figure 4D. Cross-sections are also visualized. In the case of the mouse

blastocyst, two parallel cross-sections (shown in blue and green) are merged. In the

MDCK cyst, the DF curves are derived from Figure 5D. In the blastocyst and the MDCK

cyst, the DF curves without DEB were also applied (w/o DEB). The diameters of the blue

spheres are set to be equivalent to the distance with the minimum potential (Fig. S2D).

The initial configurations of the simulations are described in Figure S11, along with other

simulation results.

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Figure 7: Modeling of distance–force curves and diagrams of simulation outcomes

A. DF curves were mathematically modeled using four parameters: N, D_0, δ, and ε. In the

case of the DF curves without the cosine term, the curves ($F = \varepsilon (D - D_0)^{-N}$) are always

>0 (repulsive) and gradually approach 0 as D increases (green broken line). Upon

introduction of the cosine (purple broken line), the curves acquire regions with <0

(attractive) and DEB (red line). Upon introduction of D_0, the curves are translated along

the distance (blue line). N affects the gradient of the curves and the height of DEB (right

panel). DF curves from $D = 0$ to D at the end of DEB (w/ DEB) or the curves from $D = 0$

to D before starting DEB (w/o DEB) were used in the following simulations. The physical

meaning and interpretation of these parameters are described in SI, Section 6. B.

Simulations were performed starting from an initial configuration, and the outcomes

under various values of N and D^* are shown in the diagrams. A cavity-bearing structure

was given as the initial configuration; the blue spheres correspond to cells, and a cross-

section is visualized. According to the conditions, a cavity was maintained, whereas cups,

tubes, etc. were generated. Some conditions on the diagram were designated by symbols

(circle, square, and triangle), and their simulation results are visualized (blue spheres).

Example profiles of the DF curves are shown. The diameters of the blue spheres are set

to be 0.5× the distance with the minimum potential, which corresponds to the distance

with $F = 0$ (arrows in the profiles of the DF curves, Fig. S2D). In the case of the cup, the

mouth of the cup is marked by orange rings. In the case of the tube, the open ends of the

tube are shown by orange arrows. In the case of the cavity with extra particles, the extra

particles are shown by arrows. C. Simulation outcomes are shown on diagrams similar to

B, except that the simulations were performed without DEBs. Other diagrams are

provided in Figure S12.

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Figure 8: Simulation outcomes and their diagrams, based on distance–force curves; starting from 2D sheet

A. Simulations were performed from a two-dimensional (2D) sheet with slight positional fluctuations under various distance–force curves defined by Equation 3 (Fig. 7A), and a diagram of the outcomes is shown. The number of particles was set to 25. Cross-sections are also presented (blue circles). According to the conditions, the 2D sheet was maintained, whereas aggregates, cavity-bearing structures etc. were generated. The diameters of the blue spheres are set to be $0.2\times$ the distance with the potential minimum.

B. Similar to A, except that the simulations were performed without distant energy

1250 barriers (DEBs). 2D sheets were maintained, but the area in the diagram became

1251 significantly smaller than that in A. Thus, the maintenance of 2D sheet is enhanced by

1252 DEBs. In addition, cavities were generated without DEBs. Other diagrams are provided

1253 in Figure S12.

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Figure 9: Morphological transitions achieved by distance–force curves

Possible morphological transitions that can be achieved by modulating the values of three

variables: N, D_0, and δ in Figure 7. The values of the variables are presented. Conditions

with DEB are indicated as DEB (+). The analysis focused on specific structures: cavity-

bearing structures, cup, tube, spherical aggregate, two-dimensional sheet, and disk. The

cup is a transition state from the cavity-bearing structure to the tube. The disk is also a

transition state from the two-dimensional sheet to the cavity-bearing structure. A related

movie is provided (Movie S6).

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