1
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3 Human revealed stochastic preferences for multi-component economic choice options
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31
32 **Running Head:**
33 How we choose multi-component rewards
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36 Reward
37 Utility
38 Indifference curve
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41 **Abstract**
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Realistic, everyday rewards contain multiple components. An apple has taste and size.
However, we choose in single dimensions, simply preferring some apples to others. How can
such single-dimensional preference relationships refer to multi-component choice options?
Here, we investigated stochastic choices of two-component milkshakes. The revealed
preferences were intuitively graphed as indifference curves that indicated how much of one
component was traded-in for obtaining one unit of the other component without a change in
preference. Options on higher indifference curves were preferred to those on lower curves,
despite widely, and sometimes oppositely, varying individual components. These
characteristics deomonstrated the orderly integration of multiple components into single-
dimensional estimates. The systematic, non-overlapping curves satisfied leave-one-out tests,
followed decoder predictions and correlated with Becker-DeGroot-Marschak auction-like
bids. These single-dimensional estimates of multi-component options complied with rigorous
concepts of Revealed Preference Theory and encourage formal investigations of normal,
irrational and pathological decisions and their neural signals.
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58 **Keywords**: Bundles, decision-making, reward, stochastic choice, psychophysics

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Introduction

We like apples, in particular if they are sweet. We can state our preference in words, but this may not be accurate because of poor introspection, faulty memory or erroneous report. It would be more accurate to observe our choice between different apples by which we reveal our preference at that moment. But how do we make that choice? When choosing between apples, we may prefer a sweeter one even if it is a bit smaller; we would trade-in some size for more sweetness. As the trade-off illustrates, our preference among apples does not come from any component alone but from their combination. Every reward or economic good constitutes a bundle with multiple components, attributes or dimensions, and thus is formally a vector. The bundle components may be integral parts of a good, like the sweetness and size of the apple, or consist of distinct entities, like the steak and vegetable of a meal. Importantly, each component contributes to the choice of the bundle. Without considering the multi-component nature of choice options, we can only study gross choices or exchanges, like between an apple and a pear (not really a choice for an apple lover), or between a movie ticket and a meal (not good when hungry). Thus, to understand realistic, fine-grained choices, we should consider that choice options have multiple components.

In contrast to the multi-dimensionality of realistic, vectorial choice options, revealed preferences and subjective reward value ('utility') are single-dimensional. When faced with two options, a rational decision maker can only prefer one option or its alternative or be indifferent to the options (completeness axiom; see Mas-Colell et al. 1995). With repeated, stochastic choices, preferences are revealed by choice probability (McFadden 2004); the probability of choosing one option over its alternative varies in a graded, scalar manner. Correspondingly, the utility of one choice option can only be higher, lower or equal to that of its alternative. Further, neural signals reflecting choice options vary only along a single dimension at any given moment and thus are also scalar. Hence the question: how can single-dimensional revealed preferences, utility and neural signals concern vectorial, multi-component choice options? In particular, can we test the issue empirically using well worked-out theoretical concepts that should reduce possible confounds?

The issue of vectorial-to-scalar transformation can be formally approached by considering that the same scalar measure may arise from oppositely varying vector components. I may trade-in a larger, less sweet apple for a smaller but sweeter one without loss or gain in utility (I do not want to lose anything, nor is it easy to gain anything). Thus, the increase in sweetness is compensated by the decrease in size. This trade-off is captured by Revealed Preference Theory, whose two-dimensional indifference curves (IC) provide a convenient graphic formalism (Fisher 1892; Samuelson 1937; 1938). Equally revealed preferred but differently composed multi-component bundles are plotted on one and the same IC. The continuous nature of these ICs reflects the trade-off that characterises the well-ordered integration of multiple bundle components into a single-dimensional preference relationship and utility. For these reasons, ICs are a mainstay of economic theory and textbooks (Laidler & Estrin 1989; Kreps 1990; Varian 1992; Mas-Colell et al. 1995; Perloff 2009).

Previous research investigated multi-component choice options in several ways. Without referring to the IC graphism, some studies tested quality and price of television sets (Simonson 1989), comfort and fuel consumption of cars (Simonson 1989), payoff amount and probability (Tversky 1969; Soltani et al. 2012; Levy et al. 2012), shape and color of cards (Fellows 2016), artificial objects and appendages (Pelletier & Fellows 2019), various food components (Suzuki et al. 2017; DiFeliceantonio et al. 2018; Harris et al. 2018), hats and shoes (Thurstone 1931), pastries and payoffs (MacCrimmon & Toda 1969), and monetary token from two accounts (Choi et al. 2007; Kurtz-David at al. 2019). Without using real outcomes, some studies used ICs for hypothetical outcomes, such as hats and shoes (Thurstone 1931), pencils or pastries and payoff (MacCrimmon & Toda 1969), and monetary token (Choi et al. 2007; Kurtz-David et al. 2019). Without actual experimentation, conceptual

studies used the IC scheme for explaining inconsistencies of economic choices, such as
violations of the Independence of Irrelevant Alternatives (IIA) (Rieskamp et al. 2006; Chung
et al. 2017; Gluth et al. 2017). Despite the importance of ICs for economic concepts and
research, what is missing are empirical tests of ICs in controlled laboratory situations using
actual, tangible outcomes. Such tests in humans would ideally be suitable also for
neuroeconomic research using neuroimaging. The tests require multiple trials of measurable,
operant choices between bundles composed of immediately consumable rewards (rather than
single-shot, verbally expressed choices of fictive goods). The designs could also make links
to neuronal investigations of economic decisions in monkeys with their superb and
understandable cognitive and behavioural abilities. All tests should benefit from well-
conceptualised IC formalisms that constitute the basis of Revealed Preference Theory.
The purpose of the present study was to empirically estimate formal ICs of revealed
preference relationships. Based on rigorous concepts of Revealed Preference Theory, our
design was aimed to provide empirical tests of revealed preferences in normal humans, for
detecting fine-grained 'irrational' IIA anomalies in normal and brain-damaged humans who
might fail to properly consider all components of choice options, for studying underlying
normal and pathological brain processes, and for comparing the data with behavioural
choices and high-precision single-neuron recordings in animals. These aims imposed
substantial constraints, including specific event timing and the use of types and amounts of
immediate payouts that are comparable to the tangible liquid and food rewards of animal
studies (Kagel et al. 1975; Pastor-Bernier et al. 2017). The repeated trials necessary for
statistical data analysis of human imaging (and animal neurophysiology) required us to study
stochastic preferences (McFadden 2004; Stott 2006). Sitting in front of a computer monitor,
our participants chose between two visually presented, rather than verbal stated, options that
contained the same two fatty and sugary milkshakes in independently varying quantities.
After extended piloting, we obtained reliable estimates of revealed preference that satisfied
statistical scrutiny by leave-out and decoder tests and were mechanism independent (as
assessed by Becker-DeGroot-Marschak, BDM). Thus, the empirical choice behaviour of
human participants conforms with the ICs of economic theory that represent revealed
stochastic preferences for multi-component choice options.
144
145 **Methods and material**
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Participants. A total of 24 participants (11 males, 13 females; mean age 25.4 years, range
19-36 years) completed a binary choice task for measuring revealed preferences and
performed a Becker-DeGroot-Marschak (BDM) control task. None of the participants had
diabetes or lactose intolerance, nor did they require specific diets, to avoid medical and
cultural interference. All participants had a known appetite for milkshakes and provided
written informed consent based on a detailed information sheet. The Local Research Ethics
Committee of the Cambridgeshire Health Authority approved the study.
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Implementation of concepts. Revealed Preference Theory conceptualizes observable
preference relationships between multi-component choice options. Decision makers reveal
their preference at the moment of choice. We respected general notions of discrete choice
models: the employed choice sets were exhaustive and had finite numbers of options (two),
and their options were mutually exclusive. The statistical analysis of neural responses in
humans and animals requires the use of multiple trials. Therefore, we used a version of
Revealed Preference Theory in which preferences are stochastic (McFadden 2004). Thus, we
assessed revealed preference from the probability of multiple choices, rather than by
traditional single-shot economic tests. The probability of choosing one option over its
alternative varies in a graded manner and thus is scalar. As in our parallel behavioural and
neurophysiological work on monkeys (Pastor-Bernier et al. 2017; 2019), we implemented
these concepts using the following notions (Figure 1):

(1) Participants made choices between two bundles that contained the same two distinct milkshakes in independently varying scalar amounts (component A, component B, each measured in ml; Figure 1a). Thus, each choice option consists of a two-dimensional bundle vector. We kept effort cost for obtaining any bundle and budget constant for all choice options for better task control and data interpretation (single keyboard button press).

(2) Stochastic preference is revealed by the measurable probability of repeated choice. The probability of choosing a given bundle depends on all components of all bundles present on that trial (as opposed to 'lexicographic' preferences for only one component) (Figure 1b, c).

(3) Equal choice probability for two bundles indicates equal revealed stochastic preference ($P = 0.5$ each bundle). The point at which both bundles are chosen with equal probability (point of subjective equivalence or choice indifference point, IP) is estimated from an S-shaped psychophysical function fitted to the probabilities of repeated choices while varying one component of one bundle and keeping all other components constant (Figure 1c).

(4) Every bundle has a subjective value ('utility') for the decision maker that depends only on the reward amounts of both milkshakes. A bundle is chosen with a higher probability than any other bundle in the same option set if and only if its utility is higher than in any other bundle in that choice set. In other words, the preference relationship between two bundles is monotone if the higher utility of one bundle implies that it is preferred to its alternative (Mas-Colell et al. 1995). A bundle is chosen with equal probability of $P = 0.5$ against another bundle if the two bundles have the same utility. Notions (2) - (4) are compatible with basic assumptions of stochastic choice theories (Luce 1959; McFadden & Richter 1990; McFadden 2004; Stott 2006) (although we analysed the choices with the probit model rather than McFadden's logit model because of its less restrictive assumptions; see below).

(5) Each bundle is graphically displayed at the intersection of the x-coordinate (component A) and y-coordinate (component B) of a two-dimensional graph (Figure 1d). A bundle that is as revealed preferred as its alternative is graphically represented as a two-dimensional IP. Multiple IPs align on a single, continuous indifference curve (IC) (Figure 1e). Thus, an IC reflects scalar utility and choice probability for the vectorial bundle in an orderly and systematic manner.

(6) Bundles can be equally revealed preferred despite variation in bundle composition, such that some amount of one component is given up in order to gain one unit of the other component without change in preference (marginal rate of substitution, MRS). This trade-off is graphically characterised by two parameters: (i) the IC slope, which reflects the relative utility (currency) of the two bundle components and can be variable and asymmetric between x-axis and y-axis; (ii) the curvature, which captures any slope change between IC center and IC periphery (Figure 1d).

(7) Bundles with larger amounts of both components are revealed preferred to bundles with smaller amounts (as long as the 'value function' for each component does not decrease with amount, indicating that 'more is better') (Figure 1e). Any bundle that lies above an IC (farther away from the origin) is revealed preferred to any bundle that lies on that IC, any bundle that lies on an IC is revealed preferred to any bundle that lies below that IC (closer to the origin), and thus any bundle on a higher IC is revealed preferred to any bundle on a lower IC.

(8) The preference relationship between two bundles may hold even when one component of the revealed preferred bundle comes with a smaller amount than the alternative bundle (physical non-dominance, requiring overcompensation by the other bundle component) (Figure 1e, arrows). This aspect, together with the equal-preference trade-off, is a crucial characteristic for the integration of the physical values of both bundle components into single-dimensional preference relationships and utility.

Figure 1. Binary choice task and experimental design.
(a) Bundle stimuli for binary choice. Each bundle contained two components with amounts between 0.0 and 10.0 ml. Component (Comp) A: low-sugar high-fat milkshake, Comp B: high-sugar low-fat milkshake. Participants chose between the Reference Bundle and the Variable Bundle, whose locations on a computer monitor alternated pseudorandomly between fixed left and right positions.
(b) Psychophysical test design. In the Reference Bundle, components A and B were set to 0.0 ml and 8.0 ml, respectively; in the Variable bundle, component A was set to a specific test amount while the amount of component B was psychophysically varied (dashed arrows).
(c) Psychophysical assessment of two example choice indifference points (IP; choice probability P = 0.5 each option; yellow and green) in a typical participant. During repeated trials, the participant chose between the preset Reference Bundle and the Variable Bundle. Component A of the Variable Bundle was set to specific test amounts (here 0.5 ml and 2.0 ml, respectively). Component B of the Variable Bundle varied between seven randomly selected amounts of component B. IPs were estimated from choice probabilities, p (Bundle 1) and p (Bundle 2), respectively, using the probit model (Eqs. 1, 2).
(d) Schematic indifference curve (IC), fitted by a hyperbola (Eqs. 3, 3a) to all equally revealed preferred but differently composed bundles. Yellow and green dots show IPs relative to the bundle marked by blue dot, as estimated in B, C.
(e) Schematic map of three ICs. Increasing distance from origin represents higher utility; all bundles on higher ICs are revealed preferred to all bundles on lower ICs. Heavy dots denote a preference relationship between two bundles with oppositely varying physical amounts of component A (top-IC bundle is revealed preferred to mid-IC bundle despite lower physical amount of component A; partial physical non-dominance).

Experimental design. We measured repeated choices between two options, each of which contained two milkshakes; the liquids constituted rewards, as evidenced by the participants' voluntary consumption. A more complete and traditional description of Revealed Preference Theory includes budget constraints (Mas-Colell et al. 1995); however, in this initial empirical study, we aimed to obtain as straightforward and easily interpretable empirical data as possible and therefore did not test aspects of budget constraint. In particular, the acquisition of any chosen bundle always required exactly one single finger movement (button press on a computer keyboard).

We used quantitative stimuli to represent the two milkshakes and their amounts in each of the two bundles (Figure 1a). Each bundle stimulus consisted of two vertically aligned

rectangles. The colour of each rectangle indicated the bundle component. After extensive
piloting with various liquids and liquidised foods, we found milkshakes with a controlled mix
of sugar and fat to give the most reliable behavioural performance. As the milkshakes were
delivered separately, with a 500 ms interval (see below), drinks containing either only sugar
or only fat were deemed to be too unnatural. Thus, in our bundles, component A (top, blue)
consisted of a low-sugar high-fat milkshake (25% whole milk, 75% double cream, no sugar),
and component B (bottom, red) consisted of a high-sugar low-fat milkshake (10% sugar in
skimmed milk). The vertical position of a bar in each rectangle indicated the physical amount
of each component (higher was more). The milkshakes were delivered using a custom-made
syringe pump system with silicone tubing approved for delivery of food stuffs (VWR
International Ltd). The pump was controlled using a National Instruments card (NI-USB-
6009) via the Matlab Data Acquisition Toolbox. The Psychtoolbox in Matlab (Version
R2015b) running on a Dell Windows computer served for stimulus display and recording of
behavioural choices.
In the choice task, each participant revealed her preference in choices between the two
composite stimuli representing the two bundles (Figure 1a). The two bundle stimuli appeared
simultaneously at pseudorandomly alternating fixed left and right positions on a computer
monitor in front of the participant; each bundle stimulus contained the same two milkshakes
with independently set amounts. Both bundle stimuli appeared after a short, pseudorandomly
varying interval (mean 0.5 s) following an initial central fixation cross. The participant made
a choice between the two bundle stimuli by pressing a single button once (left or right
computer keyboard arrow for corresponding choice of left or right bundle). Reaction time
was defined as the interval between the appearance of the two bundle stimuli and the
participant's key press. The two milkshakes from the chosen bundle were delivered together
with a probability of $P = 0.2$; i. e. every fifth chosen bundle was paid out on average using a
Poisson distribution, and there was no payout of any milkshake on the remaining trials.
Component B was delivered after a constant interval of 500 ms after component A. This
constant delay, rather than simultaneous delivery or pseudo-randomly alternating sequential
liquid delivery, prevented uncontrolled liquid interactions, maintained discriminability of the
liquids and amounted to constant temporal discounting. Thus, the utility for component B
reflected not only the subjective value of the milkshake alone but also a temporal discount
due to longer delay. Although participants were instructed to not eat or drink up to four hours
prior to the testing, satiety was a concern due to the high sugar and fat content of the
milkshakes. We addressed the issue by the $P = 0.2$ payout schedule, by limiting each payout
to maximally 10.0 ml, and by delivering not more than 200 ml of total liquid to each
participant on each session.
We used a standard psychophysical staircase procedure (Green & Swets 1966) to
estimate the IP at which each of the two bundles were chosen with equal probability ($P = 0.5$
each option), revealing equal preference for each option. The procedure required repeated
testing, which was also required for the subsequent neuroimaging experiment with the same
participants (to be reported elsewhere).
We started the psychophysical procedure by setting, in the Reference Bundle,
component A to 0.0 ml and component B to 2.0 ml, 5.0 ml or 8.0 ml (Figure 1b). We then
varied the alternative Variable Bundle in a systematic fashion; first, we set its component A
one unit higher (mostly 0.5 ml, 1.0 ml or 2.0 ml higher), thus specifying the amount of
component A gained by the participant from the choice; then we selected randomly (without
replacement) one amount from a mean of 7 fixed amounts of component B (multiples of 0.5
ml), spanning the whole, constant range of tested amounts; we repeated the selection until all
7 amounts had been tested once (Figure 1c). Then we estimated each IP from 6 repetitions
using sigmoid fitting (see Eqs. 1, 2 below), requiring 42 choices per IP. At the IP, the amount
of component B was usually lower in the Variable Bundle compared to the Reference
Bundle. In this way, we assessed at each IP how much of component B the participant was

willing to give up in order to gain one unit of component A, always relative to the constant
Reference Bundle.
We obtained further IPs in choices between the constant Reference Bundle and the
Variable Bundle whose amounts of component A increased step-wise, thus advancing from
top left to bottom right on the two-dimensional x-y indifference map. We are aware that the
unidirectional progression of testing may lead to somewhat different IP estimates than testing
in the opposite direction or in random sequence (Knetsch 1989). However, in this initial
study, we were primarily interested in the systematic assessment of consistent IPs rather than
exploring potential pitfalls.
To obtain three levels of revealed preference, we used three starting amounts of
component B in the Reference Bundle (2.0 ml, 5.0 ml, or 8.0 ml). In total, we estimated 4 IPs
(from 5 test bundles) at each of 3 levels of revealed preference, resulting in a total of 12 IPs,
derived from 504 choices among 84 different option sets in each participant (7
psychophysically tested amounts for each of 12 IPs; 6 repetitions).
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Statistical analysis. In order to estimate the choice IPs numerically, we obtained a sigmoid
fit to the empirically assessed choice frequencies via a general linear regression. To do so, we
used the Matlab function *glmfit* (Matlab R2015b; Mathworks) on a binomial distribution with
a probit link function, which is the inverse of the normal cumulative distribution function (G).
Specifically, the generalised linear regression $y = \beta_0 + \beta_1 B_{var} + \varepsilon$ can be rewritten after
applying the link function as:
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329     $G(y) = \beta_0 + \beta_1 B_{var} + \varepsilon$ Eq. 1
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with y as number of times the subject chose the Variable Bundle in the current block from a
series of six repetitions, β_0 as offset constant, β_1 as regression slope coefficient, B_{var} as
reward amount (ml) of component B in the Variable Bundle, and ε as residual error. We
choose the probit model because it assumes that random errors have a multivariate normal
distribution, which makes it attractive as the normal distribution provides a good
approximation to many other distributions. The model does not rely on the assumption of
error independence and is used frequently by econometricians (Razzaghi 2013). By
comparison, the logit model is computationally simpler but has more restrictive assumptions
of error independence. As preliminary data had revealed a similar fit for the logit as for the
probit model we used the probit model due to its less restrictive assumptions. Thus, we
estimated the IPs from the sigmoid fit provided by the probit model, using the following
equation:
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344 Indifference Point = - (β_0 / β_1) Eq. 2
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with β_0 and β_1 as coefficients of the general linear regression (Eq. 1). By using a probit
instead of a logit function, we obtained the coefficients from the output of the probit analysis
(Amemiya 1981).
We obtained single ICs, separately for each individual participant, from a set of
individual IPs by weighted least-mean-square, non-linear regression (as opposed to the probit
regression for estimating each IP). We applied a weight in order to account for within-
participant choice variability; the weight was the inverse of the standard deviation of the
titrated amount of the B-component at the corresponding IP (the IP having been estimated by
the probit regression analysis). We estimated the best-fitting β coefficients from least-mean-
square fitting to obtain the equal-revealed-preference IC (which is a utility level) and wrote
the basic hyperbolic equation in our notation as:
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358 IC = $\beta_0 + \beta_1 B + \beta_2 A + \beta_3 BA + \varepsilon$ Eq. 3
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with A and B as amounts of components A and B (ml) referring to the x-axis and y-axis, respectively, β_1 and β_2 as slope coefficients of the least-mean-square, non-linear regression, and β_3 as curvature coefficients. As IC is a constant, we merged the other constants offset (β_0) and error (ϵ) into a common final constant k. To draw the ICs, we computed the amount of component B as a function of component A from the derived equation:

$$B = k - (\beta_2/\beta_1)A + \beta_3A \qquad\qquad \text{Eq. 3a}$$

To graphically display a fitted IC (Figure 2a, b), we plotted the preset amount of component A on the x-axis, and the computed fitted amount of component B (Eq. 3a) on the y-axis. The error on the hyperbolic curve was measured as 95% confidence interval. The higher the error around an IP the less weight was given to this point when the IC was calculated. This model resulted in good fits in earlier work (Pastor-Bernier et al. 2017). In this way, the IPs of 5 equally revealed preferred but differently composed bundles aligned as a single fitted IC. The three ICs representing increasing revealed preference levels (low, medium, high) were located increasingly farther away from the origin (Figs. 1e; 2a, b). The indifference map of 3 x 5 IPs was unique for each participant (Figure 3).

The IC shape is derived from hyperbolic fit and is quantified by two coefficients: slope and curvature. The IC slope coefficient, derived from the ratio of regression slope coefficients (β_2 / β_1), reflects the currency relationship between the components and describes the participant's preference for component A relative to component B. For example, an IC slope of -60^0 indicates that component A is valued twice as much as the same ml amount of component B. The curvature coefficient (β_3) quantifies the constancy in the trade-off between bundle components. A linear curve (curvature coefficient = 0) indicates a constant rate of exchange for the bundle components, indicating that the components are perfect substitutes. A more convex IC (curvature coefficient > 0) indicates a varying rate of exchange, suggesting that the participant is giving up lesser amounts of component B to obtain one unit of component A when advancing on the IC from top left to bottom right. For a more intuitive measure, we quantified the curvature by measuring the largest perpendicular distance between the IC and the line between the x-axis and y-axis intercepts (Figure 2e):

$$d = \max (B_{IC} - B_{linearIC}) \qquad\qquad \text{Eq. 4}$$

with d as maximal perpendicular distance (ml) (whereas β_3 is a best-fitted, estimated parameter, and thus less conservative), B_{IC} as amount of component B on the IC (ml), and $B_{linearIC}$ as amount of component B at the line connecting the x- and y-intercepts (constant amount of component A, x-axis; ml). This simplified curvature measure reflects the change in trade-off between the two components across the tested range of reward amounts, in ml of component B.

We used logistic regression on trial-by-trial choices to confirm that the measured choices were explained by bundle components rather than other factors. In a random-effects analysis, we fitted a logistic regression to the data from each individual participant and then averaged the obtained β coefficients and p-values across all participants. We used the following regression:

$$y = \beta_0 + \beta_1 RefB + \beta_2 VarA + \beta_3 VarB + \beta_4 RT + \beta_5 VarPos + \beta_6 PChoice + \epsilon \quad \text{Eq. 5}$$

with y as either 0 or 1 (not-choice or choice of Variable Bundle), A and B as amounts of bundle components A and B (ml), RefB as amount of component B in the Reference Bundle (ml), VarA and VarB as amount of components A and B in the Variable Bundle (ml), RT as reaction time (ms), VarPos as left or right position of the Variable Bundle on the computer monitor relative to the Reference Bundle (0 or 1), and PChoice as choice in the previous trial (0 or 1) (Figure 2g). Each β coefficient was standardised by multiplication with standard

deviation of the respective independent variable and division by standard deviation of y
(dependent variable). A subsequent one-sample t-test against 0 served to assess the
significance of the beta (β) coefficients in the population of the 24 participants.
Satiety may have occurred and could have affected the preferences for the two bundle
components in an uncontrolled manner, even though the bundle rewards were only paid out
on every fifth trial on average and were limited to a total of 200 ml. A prime suspected effect
might have been a differential devaluation between the two bundle components that would
result in changed currency relationship between the two components. Such change between
the two components should be manifested as gradual change in instantaneous choice
probability near the IPs over repeated test steps of 42 trials each (Figure S1). We calculated
the instantaneous choice probability as:

$$y = \Sigma \, (n=1 \text{ to } 6) \, (CV \, / \, TS) \qquad\qquad\qquad\qquad \text{Eq. 6}$$

with y as instantaneous probability ($P = 0.0$ to 1.0), CV as not-choice or choice of Variable
Bundle (0 or 1), and TS as test step (1-6).
We used a leave-one-out analysis to assess the meaningful representation of revealed
preferences by the fitted ICs and to test the accuracy of the hyperbolic IC fit to the IPs. In this
analysis, we removed one IP per IC (but not the initial Reference Bundle set at $x = 0$), fitted
the curve again with the hyperbolic model and assessed the deviation between the original IC
and the IC with the left-out IP. We defined the deviation as the difference of component B
between the original, left-out IP and the refitted IC (Figure 4b):

$$d = B_{IP} - B_{refit} \qquad\qquad\qquad\qquad\qquad\qquad \text{Eq. 7}$$

with d as difference (in ml; y-axis), B_{IP} as amount of component B of the left-out IP (ml), and
B_{refit} as amount of component B on the refitted IC (ml). Thus, a difference of 0 ml suggested
that removal of one IP did not affect the shape of the IC at all, whereas any difference
unequal to 0 ml quantified the violation of this assumption.

Decoding of preference levels. To confirm the contribution of each IP to the two-
dimensional representation of revealed preferences, we determined the accuracy (in percent
correct) with which a randomly selected bundle, defined by the amounts of the two
components A and B (in ml), could be assigned to its original revealed preference level as
opposed to any one other level (binary distinction). By definition, each bundle that was
psychophysically estimated to be as much revealed preferred as the Reference bundle
constituted an IP; all bundles to which participants were choice indifferent against the same
Reference Bundle constituted a series of IPs. In our experiment, three different Reference
Bundles defined three preference levels (low, medium, high: component B: 2.0 ml, 5.0 ml or
8.0 ml, respectively; component A was always 0.0 ml; Figure 5a). The decoder used as inputs
only bundles at the psychophysically estimated IPs (to which an IC was fitted using Eqs. 3,
3a), rather than bundles positioned on the fitted ICs.
Our main test employed a binary support vector machine (SVM) decoder separately on
each individual participant. We used similar methods as previously described for predicting
choice from neuronal activity (Tsutsui et al. 2016). The SVM algorithm considered 5 IP
bundles from each of 2 revealed preference levels (total of 10 IPs that had been assessed 12
times at each position in each participant) (Figure 5a). Each of the 2 preference levels was
associated with a matrix of 2 columns (containing the x- and y-coordinates of bundle
components A and B, respectively) and 5 rows (containing the 5 bundles). The 5 bundles
were randomly selected (with replacement) from 60 bundles on each level (due to the random
procedure with replacement, some bundles may have entered the algorithm multiple times,
and not all five bundles may have been used for a given analysis). We left out 1 randomly
selected bundle from the 10 bundles, trained the SVM algorithm with the remaining 9

bundles, and assessed whether the SVM decoder assigned the left-out bundle to its original
revealed preference level or to another level. Thus we used 90% of the data for training the
decoder and 10% for testing its classification performance. We repeated this procedure 10
times with the same selected 2 x 5 bundles but with a new randomly selected left-out bundle
and calculated decoder accuracy as percent correct classification in these 10 trials. We
repeated the random selection of the 2 x 5 bundles and the 10-trial accuracy assessment 150
times. For final decoding accuracy, we averaged the percentages from these 150 iterations
(Table 1). We applied this procedure separately to all three possible combinations of two
revealed preference levels (i. e. low and medium, medium and high, low and high). The SVM
was implemented with custom written software in Matlab R2015b (Mathworks) using the
functions *svmtrain* and *svmclassify* with linear kernel (our previous work had shown that use
of nonlinear kernels did not improve decoder performance; Tsutsui et al. 2016).
We supplemented the SVM procedure with binary linear discriminant analysis (LDA)
that provided visualization of the different levels of revealed preference (Figure 5). We used
the same IPs and the same data matrices as for the SVM analysis (and the same IPs as used
for the hyperbolic fitting of the three indifference curves, ICs). We obtained two variances;
the discriminant 1 eigenvector captured the best separation between the three revealed
preference levels as 'across-level variance' (colors in Figure 5); the discriminant 2
eigenvector captured the best within-level separation between five bundles on each of the
three preference levels as 'within-level variance' (symbols in Figure 5). The results indicate
visually the discrimination accuracy on the two axes of the two-dimensional plots. We also
assessed the numeric accuracy of decoding as percent of correctly assigning a randomly
selected bundle to its original revealed preference level. The decoder used the Matlab
functions *fitcdiscr* and *predic* on z-normalised data from individual participants. For the
LDA, our limited data required pooling from multiple participants. As revealed preferences
are private and subjective, and therefore difficult to compare between individual participants,
the LDA results should be considered as merely supportive and not as stand-alone data.
493
Mechanism-independent validation. To relate the estimated revealed preferences to
inferred utility, we implemented a Becker-DeGroot-Marschak (BDM) mechanism akin to a
second price auction (Becker et al. 1964). The BDM value bids were then compared with the
three levels of revealed preference within each participant.
In the BDM, the participant received a fresh monetary endowment (20 UK pence) on
each trial. The participant bid for a bundle against a pseudorandomly set computer bid
(retrieved from a normal distribution with replacement). If the participant's bid was higher
than or equal to the computer bid then she received both component rewards of the bundle
and paid an amount equal to the computer bid. If her bid was lower than the computer bid,
she lost the auction, paid nothing and did not receive any bundle reward. The participant was
informed about a win or a loss immediately after placing the bid; when winning the bid, the
participant received the bundle rewards in the same sequence and frequency (every fifth trial
on average) as in the choice task assessing revealed preferences.
We showed each participant single bundles that were randomly selected (without
replacement) from a set of 15 bundles (5 equally revealed preferred bundles at each of 3
levels; the 15 bundles had been used to fit the 3 ICs shown in Figure 2a, b and Figure 3). A
given bundle was set to the participant's psychophysically estimated IP (Figure 6a). We
presented each of the 15 bundles 12 times, resulting in 180 trials in total, and considered the
mean of these bids as the BDM-estimated utility. The participant indicated her bid by moving
a cursor horizontally on the computer monitor with left and right keyboard arrows (Figure 6a
inset). The BDM bid was registered from the cursor position at 5.0 s after onset of
presentation of the horizontal bidding scale.
We first assessed the basic question whether the monetary bids increased for higher
valued bundles but were similar for equally valued bundles (which constituted IPs), using
Spearman rank correlation analysis confirmed by one-way Anova with post-hoc Tukey-

Kramer test. Then we performed a random-effects analysis with a general linear regression
with a normal (Gaussian) link function on separate data from each participant and averaged
the obtained β coefficients and their p-values across participants. We used the following
regression:

$y = \beta_0 + \beta_1 \text{PrefLev} + \beta_2 \text{AmBundle} + \beta_3 \text{TrialN} + \beta_4 \text{PrevBid} + \beta_5 \text{Consum} + \varepsilon$ Eq. 8

with y as monetary bid, PrefLev as revealed preference level (low, medium, high),
AmBundle as summed ml amount of components A and B in the currency of component A
$(A + (k - (\beta_2/\beta_1)A + \beta_3 A)$ as in Eq. 3a), TrialN as trial number, PrevBid as BDM bid in
previous trial (UK pence), and Consum as accumulated drinks consumption for component A
and component B up to this point in the experiment (ml) (Figure 6c). Each β coefficient was
standardised by multiplication with standard deviation of the respective independent variable
and division by standard deviation of y (dependent variable). A subsequent one-sample t-test
against 0 assessed the significance of the beta (β) coefficients in all 24 participants.
To assess the internal consistency of BDM bids, we used binary SVM analysis on bids
from individual participants in analogy to SVM decoding of preference levels. We tested the
same IPs as used for hyperbolic IC fitting (Figure 6a). Each of the 2 preference levels was
associated with a matrix of 1 column (containing the bids to each bundle) and 5 rows
(containing the 5 bundles). The remainder of the bundle selection, leave-out and repetition
procedure was identical to that used for the SVM decoding of preference levels (see above).
Thus, the SVM decoder for BDM bids assessed the accuracy with which the left-out bundle
belonged to its original revealed preference level. We supplemented the SVM analysis of the
BDM bids with analogous LDA for supportive visualization.
Finally, we compared hyperbolically fitted BDM isolines directly with hyperbolically
fitted revealed preference ICs (rather than with revealed preference levels just described),
separately for each individual participant. This procedure required to present BDM bids on
the same scale as revealed preference ICs. To this end, we fitted isolines of same BDM-bids
in analogy to fitting same-preference ICs. We fitted a hyperbolic function to the measured
mean BDM bids in analogy to Eq. 3:

$\text{BDMBid} = \beta_0 + \beta_1 B + \beta_2 A + \beta_3 BA + \varepsilon$ Eq. 9

with β_1 and β_2 as regression slopes, and β_3 as curvature coefficients, and A and B as amounts
of components A and B (ml), respectively. Coefficients β_1 and β_2 were standardised by
multiplication with standard deviation of components B and A, respectively (independent
variables), and division by standard deviation of BDMBid (dependent variable). We obtained
separate β coefficients from all participants and averaged them and their p-values across
participants. A subsequent one-sample t-test used the individual beta (β) coefficients from all
24 participants to test overall significance against 0.
To compare BDM bids with ICs, we graphically displayed BDM isolines along which
all mean BDM bids were equal. As a BDM isoline is a constant, we merged the constants
offset (β_0) and error (ε) into a common final constant k. To draw the BDM isolines, we
computed the amount of component B as a function of component A from the derived
equation:

$B = k - (\beta_2/\beta_1)A + \beta_3 A$ Eq. 9a

To display a three-dimensional map, we graphed colored BDM isoline zones on the z-axis as
a function of the amounts of components A (x-axis) and B (y-axis) (Figure 7a). For a two-
dimensional map of BDM isolines, we plotted the preset amount of component A on the x-
axis and the amount of component B computed from the isolines (Eq. 9a) on the y-axis
(Figure 7b). For comparison, we plotted the revealed-preference ICs on the same two-

dimensional map using the same scale. We also compared numerically, separately for each
participant, confidence intervals and slope and curvature coefficients between hyperbolically
fitted BDMBids (Eq. 9a) and hyperbolically fitted revealed preference ICs (Eq. 3a), using the
paired Wilcoxon test (Figure 7c-e).

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Results
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Basic properties of indifference curves. Participants repeatedly chose between two
composite visual stimuli that represented two-component bundles (see Methods notion 1;
Figure 1a). Each bundle stimulus consisted of two colored vertical rectangles; blue indicated
component A, a low-sugar high-fat milkshake; red indicated component B, a high-sugar low-
fat milkshake; a vertically positioned bar inside each rectangle indicated the amount of each
milkshake (higher was more). We assessed choices between a preset Reference bundle and a
Variable bundle whose component A was set to a specific test amount and whose component
B varied pseudorandomly across a wide range (Figure 1b). The choice probabilities in all 24
participants followed the change of component B in an orderly fashion, thus revealing
monotonic preference relationships (notions 2 - 4; Figure 1c). We estimated the indifference
point (IP; $P = 0.5$ for each bundle) from six repetitions using the probit choice function (Eqs.
1, 2); each IP required 42 choices. Thus, a two-dimensional IP indicated the amounts of the
two components of the Variable Bundle that was as much revealed preferred, and had the
same utility, as the constant Reference Bundle. To obtain a series of IPs, we repeated this
procedure by setting increasing amounts of component A in the Variable Bundle while
keeping the Reference Bundle constant. All IPs of such a series were as much revealed
preferred as the constant Reference Bundle and, by transitivity, equally revealed preferred to
each other.
We obtained single ICs from such a series of five equally revealed preferred IPs by
hyperbolic fitting (Eqs. 3, 3a) (notions 5 and 6; Figure 1d, e). Such an IC defined the trade-
off between the two components of an equally preferred bundle: it indicated how much of
component B a participant gave up for obtaining one more unit of component A without a
change in utility. Thus, the IC characterised the orderly integration of both bundle
components into a single-dimensional estimate. The continuous ICs were asymmetric
between the x-axis and y-axis, indicating different subjective weighting of the two
milkshakes; the convex IC curvature suggests that lower amounts of both milkshakes together
were as much preferred as higher amounts of singular milkshakes (possibly reflecting
gradually flattening, concave utility functions and/or complementarity between high-sugar
and high-fat components).
We estimated three sets of 5 IPs with their respective, hyperbolically fitted ICs by
presetting the Reference Bundle to three different amounts of component B (2.0 ml, 5.0 ml or
8.0 ml; component A being always 0.0 ml). Figure 2a (inset) shows the psychophysical
assessment of an example IP. Although the ICs varied in slope and curvature between
participants (for two example participants, see Figure 2a, b; for all participants, see Figure 3),
the ICs of bundles with larger reward amounts were located farther away from the origin,
thus representing revealed preference for larger rewards (notion 7). The three ICs were well
ordered and failed to overlap. Only the 95% confidence intervals overlapped partly in four of
the 24 participants (17%). Closer inspection of Figs. 2a, b and 3 shows that some bundles on
higher ICs had a smaller amount of component B than bundles on lower ICs, indicating
revealed preference despite partial physical non-dominance (notion 8; involving
overcompensation by higher amount of component A). As with the trade-off, such
preferences for bundles with one physically lower component indicate the integration of the
values of both bundle components into single-dimensional utility.
Taken together, the continuous nature of the ICs with their multiple IPs reflected the
orderly and systematic fashion of single-dimensional utility and stochastic preference for
differently composed vectorial bundles, in compliance with the basic concepts.

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Figure 2. Empirical indifference curves (IC) representing revealed preferences.
(a) Typical convex ICs from an example participant, as seen in 18 of the 24 participants. Component
A was a low-sugar high-fat milkshake; component B was a high-sugar low-fat milkshake. Solid lines
show hyperbolically fitted ICs, dotted lines show 95% confidence intervals of fits. Dots show bundles
that are equally preferred on the same IC (IPs). Inset: psychophysical assessment of indifference point
(IP) marked on highest IC (test points in blue, IP estimated by probit regression in red).
(b) Typical linear ICs from another example participant, as seen in 6 of the 24 participants.
(c, d) Distributions of slope and curvature, respectively, of hyperbolically fitted ICs from all 24
participants (coefficients β_2/β_1 and β_3 in Eq. 3, respectively). N = number of participants.
(e) Scheme of intuitive numeric assessment of IC curvature: maximal vertical distance (ml of
component B on y-axis) between fitted IC (curve) and a straight line connecting the x-axis and y-axis
intercepts. A distance of > 0.0 ml indicates convexity, whereas a 0.0 ml distance indicates perfect
linearity.
(f) Distribution of convex curvature, as measured using the scheme shown in E. The two peaks
indicate six participants each with similarly near-linear ICs and similarly convex ICs, respectively.
(g) Specificity of bundle choice, as opposed to unrelated parameters. Bar graph shows standardised
beta (β) regression coefficients for choice of Variable Bundle over Reference Bundle (Eq. 5), as
assessed for each individual participant and then averaged across all 24 participants. RefB, component
B in Reference Bundle; VarA and VarB, components A and B in Variable Bundle; RT, reaction time;
VarPos, left-right position of Variable Bundle stimulus; PChoice, choice in previous trial. Error bars
show standard error of the mean (SEM). *$P \leq 0.02$.

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Figure 3. Empirical indifference curves (IC) from all 24 participants. Note the fanning out of the confidence intervals towards the bottom right in each graph (thin lines), which likely reflects the progression of choice testing: the Reference Bundle was kept constant at the y-axis intercept (x = 0), whereas testing with the Variable Bundle progressed from top left to bottom right. Same conventions as for Figure 2.

IC coefficients. The shape of ICs reflects the trade-off between the components and can be quantified by slope and curvature coefficients of hyperbolic fits (Eq. 3a). The global IC slope, between y-axis and x-axis intercepts, was measured as ratio of the two regression coefficients β_2 / β_1 in Eq. 3; it indicated how much the participant was globally willing to give up in order to obtain one unit of the other component; the measure reflected the relative utility (currency) of the two bundle components. The IC slopes steeper than -45° indicate that a participant gave up a higher amount of component B (high-sugar, y-axis) for a smaller amount of component A (high-fat, x-axis), thus indicating higher subjective value of fatty than sugary milkshake. The IC slope was -71° ± 6.5° (mean ± standard error of the mean,

SEM; range -45° to -76°; N = 24 participants; Figure 2c). The higher valuation of high-fat
component A over high-sugar component B amounted to a factor of 3:1 in 18 of the 24
participants (75%). The predominantly asymmetric trade-off between the two milkshake
components documents that each component contributed to bundle preference in its own
distinct way.
The IC curvature showed substantial convexity in 18 of the 24 participants (75%), as
indicated by β_3 coefficients from the hyperbolic fit (Eq. 3) that were significantly larger than
0.0 (8.89; mean ± 5.9 SEM; p<0.05, t-test; Figure 2d). For graphically assessing IC
curvature, we measured the distance between the IC center and a straight line connecting
equally revealed preferred bundles at the x-and y-intercepts, in units of ml on the y-axis
(Figure 2e). This distance ranged from 0.09 ml (quasi-linear IC) to 3.76 ml (most convex IC)
(mean of 1.28 ml ± 0.19 SEM; Figure 2f). The distribution of the IC distance was overall
similar to that of the β_3 curvature coefficient from Eq. 3. The two highest histogram bars in
Figure 2f show data from six participants with rather similar, considerably convex IC, and
from six other participants with rather similar but quasi-linear IC. Thus, the coefficients
confirmed numerically the well-ordered nature of the representation of revealed preferences
by the ICs.
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Control for other choice variables. To test whether the choices reflected the components of
the bundles rather than other, unrelated factors, we performed a logistic regression analysis
separately for each of the 24 participants, using the following regressors: amount of each
bundle component, reaction time, Reference Bundle position on participant's monitor, and
previous trial choice (Eq. 5). The standardised beta (β) coefficients and p-values were
assessed for each participant and then averaged across all 24 participants; they demonstrated
that the choice of the Variable Bundle was negatively correlated with the amount of
component B in the Reference Bundle (RefB: β = -0.43 ± 0.16, mean ± SEM; P = 0.020 ±
0.005) (component A was constant 0.0 ml, see Methods) and positively correlated with both
components A and B in the Variable Bundle (VarA: β = 0.67 ± 0.16; P = 0.009 ± 0.004;
VarB: β = 0.94 ± 0.33; P = 0.012 ± 0.009) (Figure 2g). The beta (β) coefficients for these
three variables differed significantly from 0 (P = 0.012, P = 0.00088 and P = 0.00028,
respectively; one-sample t-test), confirming the validity of the β's. Thus, more frequent
choice of the Variable Bundle correlated with lower amounts in the Reference Bundle and
with higher amounts of either component in the Variable Bundle. The result suggests that
both bundle components, rather than a single component alone, were important for the
observed preference relationships. The remaining variables, including reaction time, position
of Reference Bundle on the monitor and previous trial choice, failed to significantly account
for current choice of the Variable Bundle (P = 0.754 – 0.988 ± 0.003 – 0.290). Thus, the
revealed preference relationships concerned the bundles with their two components rather
than other task factors.
To assess potential consumption effects, we searched for signs of satiety. We followed
choice probability across the total test duration in each of the 24 participants. We selected
two bundles that were situated above and below the IP, respectively. These two bundles
contained high-sugar low-fat (above IP) and low-sugar high-fat (below IP) milkshakes. We
plotted choice probability over six repeated test steps (Figure S1). Choice probabilities
fluctuated only insignificantly, without conspicuous upward or downward trend (above IP: F
(5, 41) = 0.28, P > 0.05; below IP: F (5, 41) = 1.53, P > 0.05; 1-way repeated measures
Anova with post-hoc Tukey Test). Even at the final, sixth step, choice probability differed
only insignificantly from any other step. Thus, the revealed references did not seem to be
importantly confounded by satiety for neither sugar nor fat within the amounts and
concentrations used in our experiment.
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Internal validation of indifference curves. We performed a leave-one-out analysis to assess
the contribution of individual bundles to the ICs obtained from hyperbolic fits to the

empirically estimated IPs. We removed one IP at a time (except the initial Reference Bundle
at x = 0.0) from the set of five IPs to which a given IC had been fitted, and then refitted the
IC with the remaining four IPs using the same hyperbolic model (see Methods, Eqs. 3, 3a).
This was done separately for each of the three ICs, separately in all 24 participants (total of
288 IPs removed from 72 ICs). We found consistency in the refitted ICs in four measures
(Figure 4a). First, none of the 72 refitted ICs overlapped with the refitted ICs at different
levels in the same participant, thus demonstrating maintained IC separation despite one left-
out IP. Second, none of the 72 refitted ICs overlapped with the 95% confidence intervals of
original ICs at different levels, confirming IC separation despite one left-out IP. Third, most
refitted ICs (66 of 72 ICs, 92%) fell inside the 95% confidence intervals of the original ICs,
and the remainder curves (6 of 72 ICs, 8%) showed only some portions outside the 95%
confidence intervals of the original ICs, thus refuting possible overweighted influence of
individual IPs on ICs. Fourth, numeric comparisons showed only insignificant deviations
between refitted ICs and the IPs that had been left out when refitting the curves (vertical
distance of 0.05 ± 0.13 ml in all 24 participants; mean ± standard error of the mean, SEM; N
= 336; P = 0.98 against normal distribution; t-test) (Figure 4b, c), confirming absence of
overweighted IP influence on ICs. These four results suggest that the hyperbolically fitted ICs
captured the IPs consistently and provided valid representations of the revealed preferences.
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Figure 4. Leave-one-out validation of estimated indifference curves (ICs).
(a) Graphic assessment with an example participant: hyperbolically refitted ICs with one left-out
indifference point (IP) (solid lines), plotted together with 95% confidence intervals of the original
hyperbolically fitted ICs (dotted lines). The refitting resulted in 4 new ICs (partly overlapping) at each
of three levels. None of the refitted IPs fell outside the original confidence intervals.
(b) Scheme of numeric assessment: distance in ml (in ml on y-axis, red) between the left-out IP on the
original IC (heavy black dot on black curve) and the refitted IC (green).
(c) Histogram of distance between refitted ICs and left-out IPs across all subjects. Skewness was -
0.12, suggesting rather symmetric distribution around the mean (modeled in red).

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We used a linear support vector machine (SVM) decoder as different statistical
procedure to confirm the contribution of each IP to the two-dimensional representation of
revealed preferences by ICs. In each participant, we set a given test bundle to one of the
psychophysically determined IPs (Figure 5a) and assessed the accuracy with which the

decoder assigned that bundle to its original preference level (each preference level was
defined by a series of empirically estimated IPs but was not a fitted IC). SVM decoding
accuracy ranged largely from 70% to 100% ($P = 2.055 \times 10^{-101}$), although a few lower values
were observed (Table 1 left).
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Figure 5. Visualization of bundle decoding using Linear Discriminant Analysis (LDA).
(a) Schematics of bundle decoding at psychophysically estimated points of equal revealed preference
(indifference points, IPs, plotted along the dotted lines). Following the notions of revealed preference,
LDA should show accurate decoding across the three preference levels (green, blue, red) but not along
each level.
(b) LDA bundle distinction between three levels of revealed preference (81-100% binary numeric
decoding accuracy between two levels; first discriminant) but not along same preference levels
(second discriminant) in N = 80 bundles from six participants (6P) with similar convex ICs (same
convention applies to all panels). Bundles on the three preference levels are colored blue, green and
red according to distance from origin, red being highest. The five bundles on each preference level are
marked from top left to bottom right with 'o', '*', '+', 'x' and ' ' symbols. Due to the arbitrariness of
the scale, numbers are not indicated.
(c) As *(b)* but for partly physically non-dominating bundles (one lower component in preferred bundle
than in alternative bundle (97-100% accuracy; first discriminant).
(d) As *(b)* but for six participants with linear ICs (90-100% accuracy).
(e) As *(d)* but for partly physically non-dominating bundles (96-100% accuracy).
(f) As *(b)* but for all 24 participants (18 with convex IC, six with linear ICs) (83-100% accuracy).
(g) As *(f)* but for partly physically non-dominating bundles (97-100% accuracy).
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We supplemented the SVM analysis by visualization of decoding using linear
discriminant analysis (LDA). The considerable amount of data necessary for reasonable LDA
required us to pool data from several participants, which violates a basic tenet of economic
theory that prohibits pooling of subjective preferences across individual participants. To
somewhat contain expected inaccuracies, we normalised IPs across participants (z-score
normalization for reward B along the y-axis; reward A had been set experimentally to
identical values on the x-axis) and restricted the analysis to specific subsets of participants.
The LDA confirmed the SVM results in all subsets (Figure 5); the first linear discriminant
assigned bundles to the three revealed preference levels, as shown by spatial separation of the
three colored groups, with a numeric accuracy of 80-100% ($P = 1.148 \times 10^{-97}$). By contrast,
the second discriminant failed to accurately assign bundles to different positions on same
preference levels, as shown by the mix of the five shapes representing bundle position. These
characteristics were seen in six participants whose fitted ICs showed the highest similarity in

convexity (Figure 5b, c), in six participants with linear ICs (Figure 5d, e) and, for
comparison, in all 24 participants (Figure 5f, g) (for distinction of participants based on IC
curvature, see two highest bars in Figure 2f). Thus, LDA decoding followed the fundamentals
of ICs: preference for bundles on higher ICs but indifference along ICs.
Taken together, the two decoders confirmed three distinguishable levels of IPs, and
LDA confirmed indifference between IPs on same levels. As these IPs constituted the basis
for the hyperbolic fitting of the three ICs, the decoder results validated also the fitting
procedure and confirmed that the estimated ICs represented well the revealed preferences.
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Mechanism-independent validation of indifference curves. The ordered representation of
revealed preferences by our ICs can be further validated by comparison of the key features of
ICs with subjective values inferred from a different estimation mechanism. To this end, we
used a monetary Becker-DeGroot-Marschak (BDM) bidding task that is equivalent to a
second-price auction and estimates participants' truthful, 'incentive compatible' bids on a
trial-by-trial basis. The property of truthful revelation makes BDM an indispensable tool of
experimental economics and explains its increasing popularity in neuroscientific studies of
human decision making (Plassmann, O'Doherty & Rangel 2007; Medic et al. 2014).
For these comparisons, the same 24 participants made monetary BDM bids (UK pence)
for 15 bundles that had been set, individually in each participant, to five psychophysically
estimated IPs on each of the three preference levels (in ml of each of the two component
milkshakes). Each preference level was defined by IP bundles that were equally revealed
preferred to the initial Reference Bundle with component A set to 0.0 ml (Figure 6a; the same
15 bundles were used for hyperbolic IC fitting, but the preference levels used for this analysis
were defined by the IPs and not the fitted ICs). Although BDM is typically performed in
single shots, we obtained average BDM bids for each bundle from 12 repetitions, thus aiming
to approach the nature of data collection during stochastic choice with psychophysical
estimation of each IP (42 trials).
The BDM bids followed the ordered preference levels, as shown by significant positive
correlation between the bids for same-level bundles (means from all five bundles) and the
three levels (Spearman rank correlation: rho $= 0.60 \pm 0.05$; mean $\pm$ SEM; $N = 24$ participants;
$P < 0.01$; confirmed by one-way Anova with post-hoc Tukey-Kramer test between all
bundles across the three levels: $P < 0.01$) (Figure 6b; red, green, blue). A random-effects
analysis separately for each of the 24 participants (Eq. 8) demonstrated a relationship of
BDM bids to preference level (PrefLev: $\beta = 0.47 \pm 0.09$, mean across all 24 participants $\pm$
SEM; $P = 0.016 \pm 0.015$; β-coefficient difference from 0: $P = 0.000026$, one-sample t-test)
and bundle amount (AmBundle: $\beta = 0.15 \pm 0.13$; $P = 0.020 \pm 0.017$; $P = 0.0278$), rather than
trial number (TrialN: $\beta = -0.10 \pm 0.25$; $P = 0.726 \pm 0.354$), previous trial bid (PrevBid: $\beta =$
0.12 ± 0.11; $P = 0.676 \pm 0.427$) or consumption history (Consum: $\beta = 0.12 \pm 0.11$; $P = 0.224$
± 0.185) (all β's were standardised) (Figure 6c). A specific analysis (Eq. 9) demonstrated that
the BDM bids reflected both bundle components (component A: $\beta = 0.6534 \pm 0.0866$, mean
$\pm$ SEM; $P = 0.0324 \pm 0.0150$; β difference from 0: $P = 1.1613\ 0884 \times 10^{-7}$, one-sample t-
test; component B: $\beta = 0.6425 \pm 0.0585$, $P = 0.0289 \pm 0.0202$; β difference from 0: $P =$
1.2770×10^{-10}). Thus, the BDM bids followed well the revealed preference levels and took
both bundle components into account.
The BDM bids were similar for equally revealed preferred bundles. To comply with the
notion of trade-off between different bundle components, equally revealed preferred bundles
should elicit similar BDM bids despite varying bundle composition. We conducted a one-way
Anova on all BDM bids for bundles positioned on the same preference level, separately for
each level and each participant. The bids for bundles located on the same preference level
failed to differ statistically (low level: $P = 0.54 \pm 0.06$; medium level: $P = 0.69 \pm 0.06$; high
level: $P = 0.78 \pm 0.05$; mean $\pm$ SEM, $N = 24$ participants). Only three participants showed
significantly different bids for bundles on the medium preference level, and one participant
showed different bids for bundles on the high preference level ($P < 0.05$). Thus, most

participants showed similar BDM bids for equally revealed preferred bundles on same
revealed preference levels (while being sensitive to bundle amounts, as shown by increasing
bids across levels). The BDM bids confirmed the equal-preference trade-off between bundle
components independent of the binary choice mechanism used for assessing revealed
preferences.
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Figure 6. Characteristics of BDM bids for bundles at different revealed preference levels.
(a) Schematics of positions of bundles used for eliciting BDM bids at psychophysically estimated
points of equal revealed preference (indifference points, IPs, connected by dotted lines). Following
the notions of trade-off and increasing revealed preference, BDM bids should be similar for equally
valued bundles (along the dotted lines) but higher for revealed preferred bundles (farther away from
origin). We tested 5 bundles per level, 3 levels, 12 repetitions, total of 180 bids. Inset: BDM task.
Each participant bid for the visually presented two-component (A, B) bundle by moving the black dot
cursor using the leftward and rightward horizontal arrows on a computer keyboard. Numbers indicate
example bids (in UK pence).
(b) Mean BDM bids from a typical participant. The bids were rank ordered (Spearman Rho = 0.83, P
< 0.001) across bundles positioned on increasing indifference curves (blue, green, red) and failed to
overlap between lowest, medium and highest levels. There was a significant difference for BDM bids
between levels but not within levels (*P < 0.001; two-way Anova followed by Tukey-Kramer test).
Data are shown as mean ± SEM (standard error of the mean), N = 12 bids per bar.
(c) Specificity of monetary BDM bids, as opposed to unrelated parameters. Bar graph showing the
standardised beta (b) regression coefficients for BDM bids (Eq. 8), as assessed for each individual
participant and then averaged across all 24 participants. Abbreviations: PrevLev, revealed preference
level (low, medium, high); AmBundle, summed currency-adjusted amount of both bundle
components; TrialN, trial number; PrevBid, BDM bid in previous trial; Consum, accumulated drinks
consumption. Error bars show SEMs. * P ≤ 0.020.
(d) Visual decoding by Linear Discriminant Analysis (LDA) of BDM bids for bundles on different
preference levels (first discriminant; 88-100% numeric decoding accuracy; $P = 9.46 \times 10^{-13}$) and along
same preference levels (second discriminant; 43-51% accuracy) (N = 243 bundles; all 24
participants). The LDA used scalar BDM bids for bundles positioned at IPs shown in *(a)*. Same
conventions as for Figure 5a.
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The BDM bids were internally consistent. We used decoders to test the distinction of
bundle position across but not along preference levels. Using BDM bids, a binary SVM
decoder showed good accuracy of assigning a test bundle to its original preference level in

individual participants (mostly 50-70%; $P = 3.789 \times 10^{-9}$) but failed to distinguish bundles
along same levels (43-51%; $P = 0.1433$) (Table 1 right). We used an LDA, which is able to
decode several levels together, as supplementary test and found good visual assignment of the
test bundles (Figure 6a) to the three revealed preference levels in our population of 24
participants (first discriminant; numeric accuracy of 88-100%; $P = 9.46 \times 10^{-12}$; Figure 6d;
three colored symbol groups) but not to different positions on same preference levels (second
discriminant; numeric accuracy of 43-51%; mix of shapes) (note the reservations above when
combining data from multiple participants). Thus, the BDM bids followed well the two-
dimensional scheme of revealed preference represented by the ICs.
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Figure 7. Comparison between BDM bids and indifference curves (IC).
(a) Three-dimensional representation of hyperbolically fitted BDM isolines from a typical participant
(mean ± SEM). The BDM bids were made for bundles that had been placed on psychophysically
estimated revealed preference indifference points (black dots). The BDM isolines were similar along
same levels and increased across revealed preference levels.
(b) Match between hyperbolically fitted isolines of mean BDM bids (black; Eqs. 9, 9a) and
hyperbolically fitted revealed preference ICs (blue, green, red; ± 95% confidence intervals, CI,
shaded) from a typical participant. Thus, the BDM isolines fell within the respective CIs of the
revealed preference ICs.
(c) Confidence intervals of fitted BDM isolines (red) and revealed preference ICs (blue) from all 24
participants (averaged along each isoline/IC). The larger 95%CIs of BDM isolines (BDMiso) suggest
more variability compared to revealed preference ICs.
(d, e) Comparison of slope and curvature coefficient estimates, respectively, between BDM isolines
(red) and revealed preference ICs (blue; same data as shown in Figure 2c, d) from all 24 participants.
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The BDM bids matched well the revealed preference ICs in direct comparison. A
stronger mechanism-independent validation of the IC scheme may be achieved by direct
graphic comparison between BDM bids and hyperbolically fitted ICs. To this end, we
estimated isolines that were fit to BDM bids using Eqs. 9, 9a and compared them with ICs
that had been hyperbolically fitted to IPs (Eqs. 3, 3a). The BDM isolines represented BDM
bids from 12 averages, and the revealed preference ICs derived from fits to IPs estimated
from 42 stochastic choices. We found that the BDM isolines increased for IPs on increasing
preference levels (farther away from the origin), but were similar for IPs on the same
revealed preference level (same distance from origin) (Figure 7a). The BDM isolines

matched the revealed preference ICs within their 95% confidence intervals (CIs) in every one
of the 24 participants (Figure 7b). Statistical comparisons showed significantly higher CIs of
BDM isolines compared to revealed preference ICs (Figure 7c; $P < 0.0884 \times 10^{-8}$, Wilcoxon
paired test; $N = 24$ participants). Despite their larger variability, both BDM isoline slope and
curvature coefficients, derived from the respective β_2 / β_1-ratio and β_3 regression coefficient
in Eq. 9, failed to differ significantly from the respective slope and curvature coefficients of
revealed preference ICs (Eq. 3; Figure 7d, e; both $P > 0.05$, Wilcoxon test on BDM vs. IC
coefficients paired from each participant; $N = 24$). Thus, despite larger variability, the BDM
bids matched well the revealed preference ICs when assessed in a comparable way.

931   Taken together, the estimation mechanism of BDM bidding provided good validation
932 of revealed preferences graphically represented by the ICs.

Discussion

This study shows that actual, operantly measurable behavioural choices in humans reveal
preferences for multi-component choice options when small, tangible and immediately
consumable rewards are delivered in repeated trials in a controlled, non-verbal, laboratory
setting. The preferences followed critical assumptions of formal theory that conceptualise
orderly scalar utility and preference relationships (estimated from choice probability) among
vectorial choice options. The vectorial-to-scalar transformation rests in particular on the
continuous, graded trade-off between bundle components and the preference for bundles with
one physically lower component (partial physical non-dominance), thus illustrating how
preferences transcend physical bundle composition.

945   The preferences were graphically well represented by the asymmetric and nonlinear
ICs. Lack of overlap between ICs, validation by leave-one-out statistics and accuracy of
SVM and LDA decoding demonstrate proportionate contribution of individual bundles to
robust preference representations by ICs. The BDM bids corresponding to the ICs provided a
mechanism-independent validation of the revealed preferences. These empirically estimated
systematic ICs correspond well to, and thus implement, the abstract schematic graphs in
economic textbooks and decision research that are used for explaining human choices
(Laidler & Estrin 1989; Kreps 1990; Varian 1992; Mas-Colell et al. 1995; Perloff 2009). The
currently estimated ICs representing orderly revealed preferences allow the distinction from
irrational preferences not fully capturing the multi-component nature of choice options,
including lexicographic preferences restricted to single components. These properties make
the currently developed tests suitable for concept-driven studies of rational and irrational
behaviour in normal and brain-damaged humans.

958   The representation of vectorial, multi-component choice options by single-dimensional
neural signals is an open issue in neuroeconomics. Our behavioural tests provide a formal,
concept-driven foundation for investigating such signals in humans. These tests require
repeated trials rather than one-shot tests typical for experimental economics; the multi-trial
nature is well captured by stochastic choice theories that facilitate data interpretation due to
decades of decision research. The immediately consumed, small payouts facilitate
comparisons with animal studies and control for temporal discounting. The tangible payouts
after small sets of trials do not rely on language and should reduce mental ruminations about
future rewards and assure reliable cooperation by the participants, thus reducing interfering
and confounding brain activity. We were particularly interested in estimating whole maps of
well-ordered, fitted ICs derived from multiple IPs that conform to predictive mathematical
functions, rather than testing preferences for a few bundles with limited general validity. As a
result, neuroeconomic work using the demonstrated empirical tests could investigate decision
mechanisms not resolvable by purely behavioural examination. For example, it is unknown
whether the bundle components are combined into a common neural utility estimate or are
coded separately and integrated at a later stage. The ensuing neural decision process might
occur between whole bundles with all components integrated, or involve hierarchical or

parallel component-by-component comparisons. The systematic ICs may also allow to
investigate neural underpinnings of specific theories, such as the switching of attentional
processes between components conceptualised by multialternative decision field theory (Roe
et al. 2001). Finally, our detailed ICs would be helpful for investigating neural mechanisms
underlying inconsistent decision making when choice options are added (independence of
irrelevant alternatives, IIA), beyond the decoy and attraction effects already being addressed
(Chung et al. 2017; Gluth et al. 2017).
Our quantitative, psychophysically controlled preference assessment required repeated
choices (Green & Swets 1966). Such a multi-trial approach is dictated by statistical
requirements of neural studies, corresponds to the standard elicitation of choice functions in
reinforcement learning (Sutton & Barto 1998) and allows comparison with animal studies
requiring similar schedules (Kagel et al. 1975; Pastor-Bernier et al. 2017). In humans and
monkeys, these methods deliver systematically varying choice probabilities rather than
single, all-or-none selection. The visible trial-by-trial variations are assumed to reflect
underlying random processes that make the choice process stochastic, as captured by choice
probability and probabilistic choice functions in random-choice theories (Luce 1959;
McFadden & Richter 1990; McFadden 2004; Stott 2006). We appreciate that our multi-trial
schedule is at odds with the frequently employed, standard, single-shot, deterministic
assessment of ICs in experimental economics (Thurstone 1931; MacCrimmon & Toda 1969),
but we believe that the obtained consistent and robust ICs validate the approach.
Economic choice often involves substantial but imaginary sums of money and
consumer items, or random, singular payouts (Simonson 1989; Simonson, Tversky 1993;
Rieskamp et al. 2006). By contrast, our payout schedule was tailored to requirements of
neural studies and allowed the participants to actually consume the chosen rewards over tens
and hundreds of trials (while controlling for satiety effects), as is typically required for
animal experiments. These behavioural choices resembled small daily activities, such as
consuming snacks and drinks, and were met by the good motivation of our participants. In
this way, we obtained well-ordered ICs that provided valid representations of revealed
preferences, without requiring imagined items or unreasonably large sums of money.
Empirical investigations of economic choice of multi-component options have revealed
several inconsistencies, including reference biases, differences between willingness-to-pay
and willingness-to-sell, and violations of independence of irrelevant alternatives such as the
similarity, compromise, asymmetric dominance and attraction effects (Tversky 1972;
Knetsch & Sinden 1984; Simonson 1989; Tversky & Simonson 1993; Bateman et al. 1997;
Rieskamp et al. 2006). These phenomena may well be due to the intrinsically adaptive nature
of biological processes (Soltani et al. 2012; Li et al. 2018), rather than invalidating the
revealed preference concept itself. We aimed to avoid interference from adaptive processes
by designing stable and highly reproducible test conditions in a well-controlled laboratory
environment, non-reliance on verbal report, single, uninterrupted test sessions, singular
changes of bundle components, constant direction of testing (from top left to bottom right on
the revealed preference map), and preventing satiety by limiting total milkshake intake to 200
ml. As a result, our IPs remained stable over successive testing steps. We used the exact same
conditions for eliciting BDM bids, which may have facilitated their correspondence to
revealed preference ICs. With these testing conditions, we avoided some known
compromising factors, which might help to identify more basic factors underlying decision
irregularities.
The coefficients of hyperbolic fits to the ICs characterize numerically the representation
of revealed preferences. The slope coefficient indicates the relative weighting of the two
milkshakes. For example, the amount of equally revealed preferred single milkshakes
(graphed at the respective x-axis and y-axis intercepts of the two-dimensional map) was
lower for high-fat (component A) than high-sugar (component B) milkshakes in our
participants, which was represented by a IC slope steeper than -45 degrees; thus, participants
would have revealed preferred high-fat over high-sugar milkshakes if they came in same

amounts. Such asymmetric IC slopes are also seen with various bundles in primates (Pastor-
Bernier et al. 2017). A key reason for the asymmetry may be the use of a physical amount
scale; in addition, the revealed preference among the milkshake bundles depended also on
their fat and sugar content, for which there is no simple common physical scale. Thus, the
scaling of milkshakes in units of physical amount would explain the IC asymmetry. Further,
the slope for identical bundle compositions varied between our participants, which
demonstrates an additional subjective component in revealed preferences. Despite these
scaling and subjectivity issues, our estimated ICs had well-ordered slopes and failed to
overlap.
Further, the convex IC curvature in most of our participants indicated that
unproportionately smaller amounts of combined sugar-fat milkshakes (at IC center) were
equally revealed preferred as larger amounts of milkshakes containing primarily sugar or fat
(positioned closer to IC boundaries near x- and y-intercepts). The BDM isolines confirmed
the convex curvature; unproportionately smaller amounts of combined sugar-fat milkshakes
(at isoline center) elicited the same bids as larger amounts of milkshakes containing primarily
sugar or fat (positioned closer to isoline x- and y- intercepts). A previous study on food
snacks found also higher BDM bids for sugar-fat combinations than for fat or sugar alone,
despite similar calories (DiFeliceantonio et al. 2018). However, not all ICs must be convex;
bundles with one unattractive component showed concave ICs in primates (Pastor-Bernier et
al. 2017). In addition, IC convexity may be ascribed to concave utility of each bundle
component (Perloff 2009); more of the same component (closer to IC x- or y-intercepts) has
lower marginal gain, and therefore the decision-maker is willing to trade in more of a
component of which she has much for one unit of the other component of which she has little.
This may be the reason why most participants showed convex ICs. Thus, the systematic
estimates of the defining slope and curvature coefficients suggest that the ICs reliably
represented well-ordered preferences for the milkshake bundles.
The estimating mechanism for BDM bids differs substantially from that underlying
revealed preference ICs. Both the auction-like bidding and the mechanics of cursor
movement differed from the choice between two simultaneously presented options. BDM
bidding is incentive compatible, such that erroneous or deceiving bids lead to suboptimal
outcome, as conceptualised by the expected cost of misbehaviour (Lusk et al. 2007); bidders
should state exactly their subjective value to avoid paying an exaggerated price or foregoing a
desired object. With these properties, BDM bidding constituted a well-controlled,
authoritative test for eliciting true subjective values, thus providing a useful validation
mechanism for preferences that were revealed by binary choice. Indeed, the obtained SVM-
and LDA-consistent BDM bids followed the preference scheme of ICs, namely higher bids
for revealed preferred bundles and similar bids with choice indifference despite varying
bundle composition. Most strikingly, hyperbolically fitted BDM isolines closely resembled
the hyperbolically fitted revealed preference ICs, both in terms of graphics and numeric
coefficients. The only notable difference was higher BDM bid variability. Our data align well
with, and extend, the previously noted correspondence between binary choices and BDM bids
for single-component options and bundle choices on paper or via verbal communication
(Roosen et al. 2017). Thus, the BDM estimating mechanism resulted in bids that
corresponded well to the graphic scheme of ICs conceptualised by Revealed Preference
Theory and thus validated our empirical IC assessments.
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1076 **Table 1.** Decoding accuracy, as assessed with support vector machine.
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Participant C – convex IC L – linear IC	Bundles			BDM bids		
	Lev1 vs. Lev2	Lev1 vs. Lev3	Lev2 vs. Lev3	Lev1 vs. Lev2	Lev1 vs. Lev3	Lev2 vs. Lev3
C1	83.9	100.0	68.8	70.6	81.0	54.4
C2	86.8	99.8	68.7	44.0	51.0	47.4
C3	72.7	96.0	58.7	56.6	71.3	59.3
C4	88.0	99.5	67.6	67.7	76.3	62.5
C5	99.1	100.0	81.4	64.8	69.9	49.8
C6	93.7	99.5	61.9	53.5	65.9	54.0
C7	66.2	75.5	49.5	70.0	72.2	43.6
C8	72.9	81.8	52.6	49.8	67.2	62.6
C9	80.0	100.0	80.0	68.4	83.8	62.1
C10	99.6	100.0	69.8	47.0	46.6	43.7
C11	90.2	99.5	55.4	68.9	75.7	53.1
C12	96.8	100.0	71.2	47.3	57.9	47.1
C12	86.1	87.7	56.7	58.7	60.9	48.2
C14	80.0	100.0	80.0	63.2	74.3	56.8
C15	99.8	100.0	85.5	67.8	78.6	54.8
C16	100.0	100.0	90.0	57.8	62.5	50.4
C17	100.0	100.0	100.0	44.8	70.9	68.4
C18	99.0	99.9	55.3	68.1	75.3	51.7
L19	100.0	100.0	88.6	43.5	65.8	59.6
L20	100.0	100.0	95.9	69.2	84.9	70.1
L21	100.0	100.0	100.0	62.2	66.0	44.4
L22	100.0	100.0	83.6	54.8	52.5	45.9
L23	100.0	100.0	100.0	63.2	58.4	47.8
L24	100.0	100.0	80.0	61.4	68.5	57.7

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Accuracy is shown in % of numbers of bundles and BDM bids correctly assigned to one of
the two tested revealed preference levels (averages from 150 iterations of classification of 10
pseudorandomly selected bundles / BDM bids from two tested revealed preference levels).
Bundles were tested at psychophysically assessed choice indifference points (IP) on one of
three revealed preference levels (corresponding to the three indifference curves, IC, fitted to
the IPs). The tested bundles had been pseudorandomly selected from 5 unique estimated IPs
on each of two revealed preference levels under study. BDM: Becker-DeGroot-Marschak
auction-like bidding mechanism. Lev1, Lev2, Lev3: revealed preference levels, numbered
according to distance from origin. The 24 participants were labelled as C or L according to
their IPs being fitted best to convex or linear ICs.
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Author contributions

K. V., A. P.-B., F. G. and W.S. designed the research, K. V. performed the experiments, K. V., A. P.-B. and A.S. analysed the data, K. V. and W.S. wrote the manuscript.

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Figure S1. Satiety control.
(a) Choice function obtained by sigmoid fit with the probit model. From seven randomly
selected fixed amounts of component B in the Variable Bundle, the two colored dots show
two amounts associated with choice probabilities closest to the indifference point (IP).
(b) Lack of satiety across typical test duration. Choice probabilities varied insignificantly
above (blue) and below (orange) the IP across six repeated choice steps, using 42 trials at
each step. Data were averaged from all 24 participants. The total duration of the six steps was
20 ± 1.38 minutes (mean $\pm$ SEM).
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