Trading Mental Effort for Confidence:
The Metacognitive Control of Value-Based Decision-Making

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ABSTRACT

Why do we sometimes opt for actions or items that we do not value the most? Under current neurocomputational theories, such preference reversals are typically interpreted in terms of errors that arise from the unreliable signaling of value to brain decision systems. But, an alternative explanation is that people may change their mind because they are reassessing the value of alternative options while pondering the decision. So, why do we carefully ponder some decisions, but not others? In this work, we derive a computational model of the metacognitive control of decisions or MCD. In brief, we assume that fast and automatic processes first provide an initial (and largely uncertain) representation of options' values, yielding prior estimates of decision difficulty. This uncertain value representation is then refined by deploying cognitive (e.g., attentional, mnesic) resources, the allocation of which is controlled by an effort-confidence tradeoff. Importantly, the anticipated benefit of allocating resources varies in a decision-by-decision manner according to the prior estimate of decision difficulty. The ensuing MCD model predicts choices, response time, subjective feeling of effort, choice confidence, and choice-induced preference change. We test these predictions in a systematic manner, using a dedicated behavioral paradigm. Our results provide a quantitative link between mental effort, choice confidence, and preference reversals, which could inform interpretations of related neuroimaging findings.

INTRODUCTION

Why do we carefully ponder some decisions, but not others? Decisions permeate every aspect of our lives—what to eat, where to live, whom to date, etc.—but the amount of effort that we put into different decisions varies tremendously. Rather than processing all decision-relevant information, we often rely on fast habitual and/or intuitive decision policies, which can lead to irrational biases and errors (Kahneman et al., 1982). For example, snap judgments about others are prone to unconscious stereotyping, which often has enduring and detrimental consequences (Greenwald and Banaji, 1995). Yet we don't always follow the fast but negligent lead of habits or intuitions. So, what determines how much time and effort we invest when making decisions?

Biased and/or inaccurate decisions can be triggered by psychobiological determinants such as stress (Porcelli and Delgado, 2009; Porcelli et al., 2012), emotions (Harlé and Sanfey, 2007; Martino et al., 2006; Sokol-Hessner et al., 2013), or fatigue (Blain et al., 2016). But, in fact, they also arise in the absence of such contextual factors. This is why they are sometimes viewed as the outcome of inherent neurocognitive constraints on the brain's decision processes, e.g., limited attentional and/or mnemonic capacity (Giguère and Love, 2013; Lim et al., 2011; Marois and Ivanoff, 2005) or unreliable neural representations of decision-relevant information (Drugowitsch et al., 2016; Wang and Busemeyer, 2016; Wyart and Koechlin, 2016). However, an alternative perspective is that the brain has a preference for efficiency over accuracy (Thorngate, 1980). For example, when making perceptual or motor decisions, people frequently trade accuracy for speed, even when time constraints are not tight (Heitz, 2014; Palmer et al., 2005). Related neural and behavioral data are best explained by "accumulation-to-bound" process models, in which a decision is emitted when the accumulated perceptual evidence reaches a bound (Gold and Shadlen, 2007; O'Connell et al.,

2012; Ratcliff and McKoon, 2008; Ratcliff et al., 2016). Further computational work

demonstrated that some variants of these models actually implement an optimal solution to

speed-accuracy tradeoff problems (Ditterich, 2006; Drugowitsch et al., 2012). From a

theoretical standpoint, this implies that accumulation-to-bound policies can be viewed as an

evolutionary adaptation, in response to selective pressure that favors efficiency (Pirrone et al.,

2014).

This line of reasoning, however, is not trivial to generalize to value-based decision

making, for which objective accuracy remains an elusive notion (Dutilh and Rieskamp, 2016;

Rangel et al., 2008). This is because, in contrast to evidence-based (e.g., perceptual) decisions,

there are no right or wrong value-based decisions. Nevertheless, people still make choices that

deviate from subjective reports of value, with a rate that decreases with value contrast. From

the perspective of accumulation-to-bound models, these preference reversals count as errors

and arise from the unreliable signaling of value to decision systems in the brain (Lim et al.,

2013). That value-based variants of accumulation-to-bound models are able to capture the

neural and behavioral effects of, e.g., overt attention (Krajbich et al., 2010; Lim et al., 2011),

external time pressure (Milosavljevic et al., 2010), confidence (De Martino et al., 2012a) or

default preferences (Lopez-Persem et al., 2016), lends empirical support to this type of

interpretation. Further credit also comes from theoretical studies showing that these process

models, under some simplifying assumptions, optimally solve the problem of efficient value

comparison (Tajima et al., 2016, 2019). However, accumulation-to-bound models neglect the

possibility that people may reassess the value of alternative options during decisions (Slovic,

1995; Tversky and Thaler, 1990; Warren et al., 2011). For example, contemplating competing

possibilities during a choice may highlight features of alternative options that may not have

been considered thoroughly before (Sharot et al., 2010). Under this view, apparent preference

reversals are not errors: they are deliberate changes of mind. Here, we propose an alternative

computational model of value-based decision-making that resolves this concern.

We start with the premise that the brain generates representations of options' value

in a quick and automatic manner, even before attention is engaged for making a decision

(Lebreton et al., 2009). The brain also encodes the certainty of such value estimates (Lebreton

et al., 2015), from which *a priori* feelings of choice difficulty and confidence could, in principle,

be derived. Importantly, people are reluctant to make a choice that they are not confident

about (De Martino et al., 2013). Thus, when faced with a difficult decision, people should

reassess option values until they reach a satisfactory level of confidence about their

preference. This effortful mental deliberation would engage neurocognitive resources, such

as attention and memory, in order to process value-relevant information. In line with recent

proposals regarding the strategic deployment of cognitive control (Musslick et al., 2015;

Shenhav et al., 2013), we assume that the amount of allocated resources optimizes a tradeoff

between expected effort cost and confidence gain. Critically, we show how the system can

anticipate the expected benefit of allocating resources before having processed value-relevant

information. The ensuing *metacognitive control of decisions* or *MCD* thus adjusts mental effort

on a decision-by-decision basis, according to prior decision difficulty and importance (cf. Figure

1 below).

Figure 1. The Metacognitive Control of Decisions. First, automatic processes provide a "pre-effort" belief about option values. This belief is probabilistic, in the sense that it captures an uncertain prediction regarding the to-be-experienced value of a given option. This pre-effort belief serves to identify the anticipated impact of investing costly cognitive resources (i.e., effort) in the decision. In particular, investing effort is expected to increase decision confidence beyond its pre-effort level. But how much effort it should be worth investing depends upon the balance between expected confidence gain and effort costs. The system then allocates resources into value-relevant information processing up until the optimal effort investment is reached. At this point, a decision is triggered based upon the current post-effort belief about option values (in this example, the system has changed its mind, i.e. its preference has reversed). Note: we refer to the ensuing increase in the value difference between chosen and unchosen items as the "spreading of alternatives" (cf. Methods section).

As we will see, the MCD model makes clear quantitative predictions about several key decision variables (cf. Model section below). We test these predictions by asking participants to report their judgments about each item's value and their subjective certainty about their

value judgements, both before and after choosing between pairs of the items. Note that we

also measure choice confidence, response time, and subjective effort for each decision.

The objective of this work is to show how most non-trivial properties of value-based

decision making can be explained with a minimal (and mutually consistent) set of assumptions.

The MCD model predicts choice, response time, subjective effort, choice confidence,

probability of changing one's mind, and choice-induced preference change, out of two

properties of pre-choice value representations, namely: value ratings and value certainty

ratings. Relevant details regarding the model derivations, as well as the decision-making

paradigm we designed to evaluate those predictions, can be found in the Model and Methods

sections below. In what follows, we present our main dual computational/behavioral results.

MODEL

In what follows, we derive a computational model of the metacognitive control of

decisions or MCD. In brief, we assume that the amount of cognitive resources that is deployed

during a decision is controlled by an effort-confidence tradeoff. Critically, this tradeoff relies

on a prospective anticipation of how these resources will perturb the internal

representations of subjective values. As we will see, the computational properties of the

MCD model are critically different from accumulation-to-bound models of value-based

decision-making, which we briefly describe in the Supplementary Material.

Deriving the expected value of decision control

Let z be the amount of cognitive (e.g., executive, mnemonic, or attentional)

resources that serve to process value-relevant information. Allocating these resources will

be associated with both a benefit $B(z)$, and a cost $C(z)$. As we will see, both are

increasing functions of z: $B(z)$ derives from the refinement of internal representations

of subjective values of alternative options or actions that compose the choice set, and

$C(z)$ quantifies how aversive engaging cognitive resources is (mental effort). In line with

the framework of expected value of control or EVC (Musslick et al., 2015; Shenhav et al.,

2013), we assume that the brain chooses to allocate the amount of resources $\hat{z}$ that

optimizes the following cost-benefit trade-off:

$$\hat{z} = \arg\max_{z} E\left[B(z) - C(z)\right] \qquad (1)$$

where the expectation accounts for predictable stochastic influences that ensue from

allocating resources (this will be more clear below). Note that the benefit term $B(z)$ is the

(weighted) choice confidence $P_c(z)$:

$$B(z) = R \times P_c(z) \qquad (2)$$

where the weight R is analogous to a reward and quantifies the importance of making a

confident decision (see below). As will be made more clear below, $P_c(z)$ plays a pivotal role

in the model, in that it captures the efficacy of allocating resources for processing value-

relevant information. So, how do we define choice confidence?

We assume that the decision maker may be unsure about how much he likes/wants

the alternative options that compose the choice set. In other words, the internal

representations of values V_i of alternative options are probabilistic. Such a probabilistic

representation of value can be understood in terms of, for example, an uncertain prediction

regarding the to-be-experienced value of a given option. Without loss of generality, the

probabilistic representation of option values take the form of Gaussian probability density

functions, as follows:

$$p(V_i) = N(\mu_i, \sigma_i) \qquad (3)$$

where μ_i and σ_i are the mode and the variance of the probabilistic value representations,

respectively (and i indexes alternative options in the choice set).

This allows us to define choice confidence P_c as the probability that the (predicted)

experienced value of the (to be) chosen item is higher than that of the (to be) unchosen item:

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$$P_c = \begin{cases} P(V_1 > V_2) & \text{if item \#1 is chosen} \\ P(V_2 > V_1) & \text{if item \#2 is chosen} \end{cases}$$
$$= \begin{cases} P(V_1 > V_2) & \text{if } \Delta\mu > 0 \\ P(V_2 > V_1) & \text{if } \Delta\mu < 0 \end{cases} \tag{4}$$
$$\approx s\left(\frac{\pi |\Delta\mu|}{\sqrt{3(\sigma_1 + \sigma_2)}} \right)$$

where $s(x) = 1/1 + e^{-x}$ is the standard sigmoid mapping. Here the second line derives from

assuming that the choice follows the sign of the preference $\Delta\mu = \mu_1 - \mu_2$, and the last line

derives from a moment-matching approximation to the Gaussian cumulative density function

(Daunizeau, 2017).

As stated in the Introduction section, we assume that the brain valuation system

automatically generates uncertain estimates of options' value (Lebreton et al., 2009, 2015),

before cognitive effort is invested in decision making. In what follows, μ_i^0 and σ_i^0 are the

mode and variance of the ensuing prior value representation (we treat them as inputs to the

MCD model). We also assume that this prior representation neglects existing value-relevant

information that would require cognitive effort to be retrieved and processed (Lopez-Persem

et al., 2016).

Now, how does the system anticipate the benefit of allocating resources to the

decision process? Recall that the purpose of allocating resources is to process (yet unavailable)

value-relevant information. The critical issue is thus to predict how both the uncertainty σ_i

and the modes μ_i of value representations will change, before having allocated the resources

(i.e., without having processed the information). In brief, allocating resources essentially has

two impacts: (i) it decreases the uncertainty σ_i, and (ii) it perturbs the modes μ_i in a stochastic

manner.

The former impact derives from assuming that the amount of information that will be

processed increases with the amount of allocated resources. Here, this implies that the

variance of a given probabilistic value representation decreases in proportion to the amount

of allocated effort, i.e.:

$$\sigma_i \triangleq \sigma_i(z) = \frac{1}{\frac{1}{\sigma_i^0} + \beta z} \qquad (5)$$

where σ_i^0 is the prior variance of the representation (before any effort has been allocated),

and β controls the efficacy with which resources increase the precision of value

representations. Formally speaking, Equation 5 has the form of a Bayesian update of the

belief's precision in a Gaussian-likelihood model, where the precision of the likelihood term is

βz. More precisely, β is the precision increase that follows from allocating a unitary amount

of resources z. In what follows, we will refer to β as the "*type #1 effort efficacy*".

The latter impact follows from acknowledging the fact that the system cannot know

how processing more value-relevant information will affect its preference before having

allocated the corresponding resources. Let $\delta_i(z)$ be the change in the position of the mode

of the ith value representation, having allocated allocating an amount z of resources. The

direction of the mode's perturbation $\delta_i(z)$ cannot be predicted because it is tied to the

information that would be processed. However, a tenable assumption is to consider that the

magnitude of the perturbation increases with the amount of information that will be

processed. This reduces to stating that the variance of $\delta_i(z)$ increases in proportion to z, i.e.:

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$$\mu_i(z) = \mu_i^0 + \delta_i$$
$$\delta_i \sim N(0, \gamma z)$$
(6)

where μ_i^0 is the mode of the value representation before any effort has been allocated, and

γ controls the relationship between the amount of allocated resources and the variance of

the perturbation term δ. The higher γ, the greater the expected perturbation of the mode

for a given amount of allocated resources. In what follows, we will refer to γ as the "*type #2*

effort efficacy". Note that Equation 6 treats the impact of future information processing as a

non-specific random perturbation on the mode of the prior value representation. Our

justification for this assumption is twofold: (i) it is simple, and (ii) and it captures the idea that

the MCD controller does not know how the allocated resources will be used (here, by the

value-based decision system downstream). We will see that, in spite of this, the MCD

controller can still make quantitative predictions regarding the expected benefit of allocating

resources.

Taken together, Equations 5 and 6 imply that predicting the net effect of allocating

resources onto choice confidence is not trivial. On the one hand, allocating effort will increase

the precision of value representations (cf. Equation 5), which mechanically increases choice

confidence, all other things being equal. On the other hand, allocating effort can either

increase or decrease the absolute difference $|\Delta\mu(z)|$ between the modes. This, in fact,

depends upon the sign of the perturbation terms δ, which are not known in advance. Having

said this, it is possible to derive the *expected* absolute difference between the modes that

would follow from allocating an amount z of resources:

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$$E\left[\left|\Delta\mu\right|\mid z\right]=2\sqrt{\frac{\gamma z}{\pi}}\exp\left(-\frac{\left|\Delta\mu^{0}\right|^{2}}{4\gamma z}\right)+\Delta\mu^{0}\left(2\times s\left(\frac{\pi\,\Delta\mu^{0}}{\sqrt{6\gamma z}}\right)-1\right)$$
(7)

where we have used the expression for the first-order moment of the so-called "folded normal

distribution", and the second term in the right-hand side of Equation 7 derives from the same

moment-matching approximation to the Gaussian cumulative density function as above. The

expected absolute means' difference $E\left[\left|\Delta\mu\right|\mid z\right]$ depends upon both the absolute prior mean

difference $\left|\Delta\mu^{0}\right|$ and the amount of allocated resources z. This is depicted on Figure 2 below.

Figure 2. The expected impact of allocated resources onto value representations. Left: the

expected absolute mean difference $E\left[\left|\Delta\mu\right|\mid z\right]$ (y-axis) is plotted as a function of the absolute

prior mean difference $\left|\Delta\mu^{0}\right|$ (x-axis) for different amounts z of allocated resources (color

code), having set type #2 effort efficacy to unity (i.e. $\gamma=1$). Right: Variance $V\left[\left|\Delta\mu\right|\mid z\right]$ of the

absolute mean difference ; same format.

One can see that $E\left[\left|\Delta\mu\right|\mid z\right]-\left|\Delta\mu^{0}\right|$ is always greater than 0 and increases with z (and

if $z=0$, then $E\left[\left|\Delta\mu\right|\mid z\right]=\left|\Delta\mu^{0}\right|$). In other words, allocating resources is expected to increase

the value difference, despite the fact that the impact of the perturbation term can go either

way. In addition, the expected gain in value difference afforded by allocating resources

decreases with the absolute prior means' difference.

Similarly, the variance $V\left[\left|\Delta\mu\right|\middle| z\right]$ of the absolute means' difference is derived from the

expression of the second-order moment of the corresponding folded normal distribution:

$V\left[\left|\Delta\mu\right|\middle| z\right] = 2\gamma z + \left|\Delta\mu^{0}\right|^{2} - E\left[\left|\Delta\mu\right|\middle| z\right]^{2}$ (8)

One can see on Figure 2 that $V\left[\left|\Delta\mu\right|\middle| z\right]$ increases with the amount z of allocated

resources (but if $z = 0$, then $V\left[\left|\Delta\mu\right|\middle| z\right] = 0$).

Knowing the moments of the distribution of $\left|\Delta\mu\right|$ now enables us to derive the

expected confidence level $\bar{P}_{c}(z)$ that would result from allocating the amount of resource z

:

$$
\begin{aligned}
\bar{P}_{c}(z) &\triangleq E\left[P_{c}\middle| z\right] \\
&= E\left[s\left(\frac{\pi\left|\Delta\mu\right|}{\sqrt{6\sigma(z)}}\right)\middle| z\right] \\
&\approx s\left(\frac{\pi E\left[\left|\Delta\mu\right|\middle| z\right]}{\sqrt{6\left(\sigma(z) + \frac{1}{2}V\left[\left|\Delta\mu\right|\middle| z\right]\right)}}\right)
\end{aligned}
$$

(9)

where we have assumed, for the sake of conciseness, that both prior value representations

are similarly uncertain (i.e., $\sigma_{1}^{0} \approx \sigma_{2}^{0} \triangleq \sigma^{0}$). It turns out that the expected choice confidence

$\bar{P}_{c}(z)$ always increase with z, irrespective of the efficacy parameters β and γ. These,

however, control the magnitude of the confidence gain that can be expected from allocating

an amount z of resources. Equation 9 is important, because it quantifies the expected benefit

of resource allocation, before having processed the ensuing value-relevant information. More

details regarding the accuracy of Equation 9 can be found in the Supplementary Material.

To complete the cost-benefit model, and without loss of generality, we will assume

that the cost of allocating resources to the decision process linearly scales with the amount of

resources, i.e.:

$C(z) = \alpha z$ (10)

where α determines the effort cost of allocating a unitary amount of resources z. In what

follows, we will refer to α as the "effort unitary cost". We note that weak nonlinearities in

the cost function (e.g., quadratic terms) would not qualitatively change the model predictions.

In brief, the MCD-optimal resource allocation $\hat{z} \triangleq \hat{z}(\alpha, \beta, \gamma)$ is simply given by:

$$\hat{z} = \arg \max_{z} \left[R \times \bar{P}_c(z) - \alpha z \right]$$

282   (11)

which does not have any closed-form analytic solution. Nevertheless, it can easily be identified

numerically, having replaced Equations 7-9 into Equation 11. We refer the readers interested

in the impact of model parameters $\{\alpha, \beta, \gamma\}$ on the MCD-optimal control to the

Supplementary Material.

At this point, we do not specify how Equation 11 is solved by neural networks in the

brain. Many alternatives are possible, from gradient ascent (Seung, 2003) to winner-take-all

competition of candidate solutions (Mao and Massaquoi, 2007). We will comment on the

specific issue of prospective (offline) versus reactive (online) MCD processes in the Discussion

section.

<u>Note</u>: implicit in the above model derivation is the assumption that the allocation of

resources is similar for both alternative options in the choice set (i.e. $z_1 \approx z_2 \triangleq z$). This

simplifying assumption is justified by eye-tracking data (cf. Supplementary Material).

Corollary predictions of the MCD model

In the previous section, we derived the MCD-optimal resource allocation, which

effectively best balances the expected choice confidence with the expected effort costs, given

the predictable impact of stochastic perturbations that arise from processing value-relevant

information. This quantitative prediction is effectively shown on Figure 5 (and/or Figure S4 of

the Supplementary Material), as a function of (empirical proxies for) the prior absolute

difference between modes $\left|\Delta\mu^0\right|$ and the prior certainty $1/\sigma^0$ of value representations. But,

this mechanism has a few interesting corollary implications.

To begin with, note that knowing $\hat{z}$ enables us to predict what confidence level the

system should reach. In fact, one can define the MCD-optimal confidence level as the expected

confidence evaluated at the MCD-optimal amount of allocated resources, i.e., $\overline{P}_c\left(\hat{z}\right)$. This is

important, because it implies that the model can predict both the effort the system invests

and its associated confidence, on a decision-by-decision basis. This quantitative prediction is

shown on Figure 6.

Similarly, one can predict the MCD-optimal probability of changing one's mind. Recall

that the probability $Q\left(z\right)$ of changing one's mind depends on the amount of allocated

resources z, i.e.:

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$$Q\left(z\right) \triangleq P\left(sign\left(\Delta\mu\right) \neq sign\left(\Delta\mu^0\right)\middle|z\right)$$
$$= \begin{cases} P\left(\Delta\mu > 0\middle|z\right) & \text{if } \Delta\mu^0 < 0 \\ P\left(\Delta\mu < 0\middle|z\right) & \text{if } \Delta\mu^0 > 0 \end{cases} \qquad (12)$$
$$\approx s\left(-\frac{\pi\left|\Delta\mu^0\right|}{\sqrt{6\gamma z}}\right)$$

One can see that the MCD-optimal probability of changing one's mind $Q\left(\hat{z}\right)$ is a simple

monotonic function of the allocated effort $\hat{z}$. Note that, by definition, choice accuracy (i.e.,

congruence of choice and prior preference $\Delta\mu^0$) is but $1-Q(z)$, which is shown on Figure 4

below.

Lastly, we can predict choice-induced preference change, i.e., how value

representations are supposed to spread apart during the decision. Such an effect is typically

measured in terms of the so-called "spreading of alternatives" or SoA, which is defined as

follows:

$$
\begin{aligned}
SOA &= \left(\mu_{chosen}^{(post-choice)} - \mu_{unchosen}^{(post-choice)}\right) - \left(\mu_{chosen}^{(pre-choice)} - \mu_{unchosen}^{(pre-choice)}\right) \\
&= \begin{cases} \Delta\mu(z) - \Delta\mu^0 & \text{if } \Delta\mu(z) > 0 \\ \Delta\mu^0 - \Delta\mu(z) & \text{if } \Delta\mu(z) < 0 \end{cases} \\
&= \begin{cases} \Delta\delta(z) & \text{if } \Delta\delta(z) > -\Delta\mu^0 \\ -\Delta\delta(z) & \text{if } \Delta\delta(z) < -\Delta\mu^0 \end{cases}
\end{aligned}
\tag{13}
$$

322 where $\Delta\delta(z) \sim N(0, 2\gamma z)$ is the cumulative perturbation term of the modes' difference.

323 Taking the expectation of the right-hand term of Equation 13 under the distribution of $\Delta\delta(z)$

324 and evaluating it at $z = \hat{z}$ now yields the MCD-optimal spreading of alternatives $\overline{SOA}(\hat{z})$:

$$
\begin{aligned}
\overline{SOA}(\hat{z}) &= E\left[SOA|\hat{z}\right] \\
&= E\left[\Delta\delta(\hat{z})\,\middle|\,\Delta\delta(\hat{z}) > -\Delta\mu^0\right] P\left(\Delta\delta(\hat{z}) > -\Delta\mu^0\right) \\
&\quad - E\left[\Delta\delta(\hat{z})\,\middle|\,\Delta\delta(\hat{z}) < -\Delta\mu^0\right] P\left(\Delta\delta(\hat{z}) < -\Delta\mu^0\right) \\
&= 2\sqrt{\frac{\gamma\hat{z}}{\pi}} \exp\left(-\frac{\left|\Delta\mu^0\right|^2}{4\gamma\hat{z}}\right)
\end{aligned}
\tag{14}
$$

326 where the last line derives from the expression of the first-order moment of the truncated

327 Gaussian distribution. Note that the expected preference change also increases monotonically

328 with the allocated effort $\hat{z}$.

Correspondence between model variables and empirical measures

In summary, the MCD model predicts choice accuracy, choice confidence, choice-

induced preference change, response time and/or subjective feelings of effort, given the prior

absolute difference between modes $\left|\Delta\mu^0\right|$ and the prior certainty $1/\sigma^0$ of value

representations. The inputs to the model are the prior moments of value representations,

whose trial-by-trial variations determine variations in model predictions. Here, we simply

assume that pre-choice value and value certainty ratings provide us with an approximation of

these prior moments. More precisely, we use ΔVR^0 and CVR^0 as empirical proxies for $\Delta\mu^0$ and

$1/\sigma^0$, respectively. Accordingly, we consider post-choice value and value certainty ratings as

empirical proxies for the posterior mean $\mu(\hat{z})$ and precision $1/\sigma(\hat{z})$ of value representations,

at the time when the decision was triggered (i.e., after having invested the effort $\hat{z}$). Similarly,

we match expected choice confidence $\overline{P}_c(z)$ (i.e., after having invested the effort $\hat{z}$) with

empirical choice confidence.

Note that the MCD model does not specify what the allocated resources are. In

principle, both mnesic and attentional resources may be engaged when processing value-

relevant information. At this point, what really matters is assessing the magnitude z of

decision-related effort. We think of z as the cumulative engagement of neurocognitive

resources, which varies both in terms of duration and intensity. Empirically, we relate $\hat{z}$ to

two different measures, namely: subjective feeling of effort and response time. The former

relies on the subjective cost incurred when deploying neurocognitive resources, which would

be signaled by experiencing mental effort. The latter makes sense if one thinks of response

time in terms of effort duration. Although it is a more objective measurement than

subjective rating of effort, response time only approximates $\hat{z}$ if effort intensity shows

relatively small variations. We will comment on this in the Discussion section.

Finally, the MCD model is also agnostic about the definition of "decision importance",

i.e. the weight R in Equation 2. In this work, we simply investigate the effect of decision

importance by comparing subjective effort and response time in "neutral" versus

"consequential" decisions (cf. Methods section). We will also comment on this in the

Discussion section.

METHODS

Participants for our study were recruited from the RISC (*Relais d'Information sur les*

Sciences de la Cognition) subject pool through the ICM (*Institut du Cerveau et de la Moelle*

épinière). All participants were native French speakers. All participants were from the non-

patient population with no reported history of psychiatric or neurological illness.

Written instructions provided detailed information about the sequence of tasks within

the experiment, the mechanics of how participants would perform the tasks, and images

illustrating what a typical screen within each task section would look like. The experiment was

developed using Matlab and PsychToolbox. The experiment was conducted entirely in French.

Eye gaze position and pupil size were continuously recorded throughout the duration

of the experiment using The Eye Tribe eye tracking devices. Participants' head positions were

fixed using stationary chinrests. In case of incidental movements, we corrected the pupil size

data for distance to screen, separately for each eye.

Participants

A total of 41 people (28 female; age: mean=28, stdev=5, min=20, max=40) participated

in this study. The experiment lasted approximately 2 hours, and each participant was paid a

flat rate of 20€ as compensation for his time plus a bonus, which was given to participants to

compensate for potential financial losses in the "penalized" trials (see below). More precisely,

in "penalized" trials, participants lose 0.20€ (out of a 5€ bonus) for each second that they took

to make their choice. This resulted in an average 4€ bonus (across participants). One group of

11 participants was excluded from the cross-condition analysis only (see below), due to

technical issues.

Materials

The stimuli for this experiment were 148 digital images, each representing a distinct

food item (50 fruits, 50 vegetables, 48 various snack items including nuts, meats, and cheeses).

Food items were selected such that most items would be well known to most participants.

Procedure

Prior to commencing the testing session of the experiment, participants underwent a

brief training session. The training tasks were identical to the experimental tasks, although

different stimuli were used (beverages). The experiment itself began with an initial section

where all individual items were displayed in a random sequence for 1.5 seconds each, in order

to familiarize the participants with the set of options they would later be considering and form

an impression of the range of subjective value for the set. The main experiment was divided

into three sections, following the classic Free-Choice Paradigm protocol (e.g., Izuma and

Murayama, 2013): pre-choice item ratings, choice, and post-choice item ratings. There was no

time limit for the overall experiment, nor for the different sections, nor for the individual trials.

Item rating and choice sessions are described below (see Figure 3).

Figure 3. Experimental design. Left: pre-choice item rating session: participants are asked to rate how much they like each food item and how certain they are about it (value certainty rating). Center: choice session: participants are asked to choose between two food items, to rate how confident they are about their choice, and to report the feeling of effort associated with the decision. Right: post-choice item rating session (same as pre-choice item rating session).

Item rating (same for pre-choice and post-choice sessions): Participants were asked to rate the entire set of items in terms of how much they liked each item. The items were presented one at a time in a random sequence (pseudo-randomized across participants). At the onset of each trial, a fixation cross appeared at the center of the screen for 750ms. Next, a solitary image of a food item appeared at the center of the screen. Participants had to respond to the question, "How much do you like this item?" using a horizontal slider scale (from "I hate it!" to "I love it!") to indicate their value rating for the item. The middle of the scale was the point of neutrality ("I don't care about it."). Hereafter, we refer to the reported value as the "pre-choice value rating". Participants then had to respond to the question, "How certain are you about the item's value?" by expanding a solid bar symmetrically around the cursor of the value slider scale to indicate the range of possible value ratings that would be compatible with their subjective feeling. We measured participants' certainty about value rating in terms of the

percentage of the value scale that is not occupied by the reported range of compatible value ratings. We refer to this as the "pre-choice value certainty rating". At that time, the next trial began.

Choice: Participants were asked to choose between pairs of items in terms of which item they preferred. The entire set of items was presented one pair at a time in a random sequence. Each item appeared in only one pair. At the onset of each trial, a fixation cross appeared at the center of the screen for 750ms. Next, two images of snack items appeared on the screen: one towards the left and one towards the right. Participants had to respond to the question, "Which do you prefer?" using the left or right arrow key. We measured response time in terms of the delay between the stimulus onset and the response. Participants then had to respond to the question, "Are you sure about your choice?" using a vertical slider scale (from "Not at all!" to "Absolutely!"). We refer to this as the report of choice confidence. Finally, participants had to respond to the question, "To what extent did you think about this choice?" using a horizontal slider scale (from "Not at all!" to "Really a lot!"). We refer to this as the report of subjective effort. At that time, the next trial began.

Note: In the Results section, we refer to ΔVR^0 as the difference between pre-choice value ratings of items composing a choice set. Similarly, CVR^0 is the average pre-choice value certainty ratings across items composing a choice set.

Conditions

The choice section of the experiment included trials of three different conditions: "neutral" (60 trials), "consequential" (7 trials), and "penalized" (7 trials), which were randomly intermixed. Immediately prior to each "consequential" trial, participants were instructed that they would be required to eat, at the end of the experiment, a portion of the item that they were about to choose. Immediately prior to each "penalized" trial, participants were

439   instructed that they would lose 0.20€ (out of a 5€ bonus) for each second that they would take

440   to make their choice.

Probabilistic model fit

Recall that the quantitative relationship between the MCD trial-by-trial inputs (namely:

value ratings' difference ΔVR^0 and mean value certainty rating VCR^0) and its outputs (namely:

the probability of changing one's mind, choice confidence, spreading of alternatives, response

time, and subjective effort ratings) is controlled by three parameters: the *effort unitary cost*

α , *type #1 effort efficacy* β and *type #2 effort efficacy* γ . In principle, these parameters may

vary across people, thereby capturing idiosyncrasies in peoples' (meta-)cognitive apparatus.

However, in addition to estimating these three parameters, fitting the MCD model to each

participant's data also requires a rescaling of the model's input and output variables. This is

because the empirical measure of these variables has no reason to match their theoretical

scale. We thus inserted two additional nuisance parameters per input and output MCD

variable, which operate a linear rescaling (affine transformation). More precisely, each

input/output MCD variable y_t (where t is the trial index) was passed through an affine

transformation $y_t \rightarrow a \times y_t + b$, where the slope parameter a was constrained to be positive

to prevent arbitrary and/or counter-intuitive model predictions. Importantly, these nuisance

parameters cannot induce any trial-by-trial variation in the MCD input/output variables. In

particular, they cannot yield differences in MCD outputs across MCD inputs. In other terms,

the MCD really has only three degrees of freedom. All five MCD dependent variables were

then fitted concurrently with a single set of subject-specific parameters (to ensure a balanced

fit, we z-scored MCD outputs across trials). Probabilistic parameter estimation was performed

using the variational Laplace approach (Daunizeau, 2018; Friston et al., 2007), which is made

available from the VBA toolbox (Daunizeau et al., 2014). In what follows, we report summary

statistics of the MCD fits at the group level, in terms of measured and fitted MCD dependent

variables when binned according to ΔVR^0 and VCR^0. We refer the reader interested in within-

subject MCD fit accuracy measures to the appended Supplementary Material.

RESULTS

Here, we test the predictions of the MCD model. We note that basic descriptive

statistics of our data, including measures of test-retest reliability and replications of previously

reported effects on confidence in value-based choices (De Martino et al., 2013), are appended

in the Supplementary Materials.

Evaluating response time and choice confidence

In what follows, we inspect the three-way relationships between pre-choice value and

value certainty ratings and each choice feature (namely: prediction accuracy, response time,

and confidence). At this point, suffice it to say that, under the MCD model, value uncertainty

ratings capture the precision of subjective value representations before the choice. Unless

stated otherwise, we will focus on both the absolute difference between pre-choice value

ratings (hereafter: $|\Delta VR^0|$) and the mean pre-choice value certainty rating across paired

choice items (hereafter: VCR^0). In each case, we will summarize the empirical data and the

corresponding MCD model prediction.

First, we checked how choice prediction accuracy relates to $|\Delta VR^0|$ and VCR^0. Here,

we measure choice accuracy in terms of the rate of choices that are congruent with

preferences derived from pre-choice value ratings ΔVR^0. Under the MCD model, choice

accuracy should increase with both $|\Delta VR^0|$ and VCR^0. This is because increasing $|\Delta VR^0|$ and/or

VCR^0 will decrease the demand for effort, which implies that the probability of changing one's

mind will be smaller. Figure 4 below shows the quantitative model predictions and summarizes

the corresponding empirical data.

Figure 4. Three-way relationship between choice accuracy, value, and value certainty. Left:

choice accuracy (color code) is shown as a function of $|\Delta VR^0|$ (x-axis) and VCR^0 (y-axis). Center:

choice accuracy is shown as a function of $|\Delta VR^0|$ (x-axis) and VCR^0 (color code); solid lines

indicate empirical data, dashed lines represent MCD model predictions. Right: GLM regression

beta weights for $|\Delta VR^0|$ and VCR^0 on choice accuracy. (Error bars represent s.e.m., $|\Delta VR^0|$

p<0.001, VCR^0 p=0.046.)

One can see that the data seem to conform to the model predictions. To confirm this,

we ran, for each participant, a multiple logistic regression of choice accuracy against $|\Delta VR^0|$

and VCR^0. A random effect analysis shows that both have a significant positive effect at the

group level ($|\Delta VR^0|$: mean GLM beta=0.17, s.e.m.=0.02, p<0.001; VCR^0: mean GLM beta=0.07,

s.e.m.=0.03, p=0.004). Note that people make "inaccurate" choices either because they make

mistakes or because they change their mind during the decision. In principle, we can

discriminate between these two explanations because we can check whether "inaccurate"

choices are congruent with post-choice value ratings (change of mind) or not (error). This is

important, because accumulation-to-bound models do not allow for the possibility that value

representations change during decisions (hence all "inaccurate" choices would be deemed

"errors"). It turns out that, among "inaccurate" choices, mind changes are more frequent than

errors (mean rate difference=2.3%, s.e.m.=0.01, p=0.032). Note that analyses of mind changes

yield qualitatively identical results as choice accuracy (we refer the interested reader to

Supplementary Material).

Second, we checked how response time relates to pre-choice value and value certainty

ratings. Under the MCD model, response time can be thought of as a proxy for effort duration.

Here, increasing $|\Delta VR^0|$ and/or VCR^0 will decrease the demand for effort, which will result in

smaller expected response time. Figure 5 below shows the quantitative model predictions and

summarizes the corresponding empirical data.

Figure 5. Three-way relationship between response time, value, and value certainty. Left:

response time (RT; color code) is shown as a function of $|\Delta VR^0|$ (x-axis) and VCR^0 (y-axis).

Center: RT is shown as a function of $|\Delta VR^0|$ (x-axis) and VCR^0 (color code); solid lines indicate

empirical data, dashed lines represent MCD model predictions. Right: GLM regression beta

weights for $|\Delta VR^0|$ and VCR^0 on RT. (Error bars represent s.e.m., $|\Delta VR^0|$ p<0.001, VCR^0

p<0.001.)

One can see that the response time data behave as predicted by the MCD model. Here,

we also ran, for each participant, a multiple linear regression of response times against $|\Delta VR^0|$

and VCR^0. A random effect analysis shows that both have a significant and negative effect at

the group level ($|\Delta VR^0|$: mean GLM beta=-0.13, s.e.m.=0.02, p<0.001; CR^0: mean GLM beta=-

0.06, s.e.m.=0.02, p=0.005).

Third, we checked how choice confidence relates to $|\Delta VR^0|$ and VCR^0. Under the MCD

model, confidence reflects the discriminability of value representations after optimal resource

allocation. Critically, although more resources are allocated to the decision when either

$|\Delta VR^0|$ or VCR^0 decrease, this does not overcompensate for decision difficulty, and thus choice

confidence decreases. Figure 6 below shows the quantitative model predictions and

summarizes the corresponding empirical data.

Figure 6. Three-way relationship between choice confidence, value, and value certainty. Left: choice confidence (color code) is shown as a function of $|\Delta VR^0|$ (x-axis) and VCR^0 (y-axis). Center: choice confidence is shown as a function of $|\Delta VR^0|$ (x-axis) and VCR^0 (color code); solid lines indicate empirical data, dashed lines represent MCD model predictions. Right: GLM regression beta weights for $|\Delta VR^0|$ and VCR^0 on choice confidence. (Error bars represent s.e.m., $|\Delta VR^0|$ $p<0.001$, VCR^0 $p<0.001$.)

One can see that the choice confidence follows the MCD model predictions. Again, we

ran, for each participant, a multiple linear regression of confidence against $|\Delta VR^0|$ and VCR^0.

A random effect analysis shows that both have a significant and positive effect at the group

level ($|\Delta VR^0|$: mean GLM beta=0.23, s.e.m.=0.02, $p<0.001$; VCR^0: mean GLM beta=0.15,

s.e.m.=0.03, $p<0.001$). Note that this is unlikely to be a trivial consequence of peoples'

response time readout, since confidence is only mildly correlated with response time (mean

correlation=-0.32, s.e.m.=0.03, $p<0.001$).

Subjective feeling of effort, choice-induced preference change, decision importance, and

cost of time

So far, we have provided evidence that choice confidence and response time can be

explained by the MCD model. In what follows, we will evaluate some additional novel

quantitative predictions that are specific to the MCD model. The derivation of each of these

predictions is detailed in the Model section above.

First, recall that MCD really is about the allocation of costly cognitive resources, i.e.

mental effort, into the decision process. One may thus ask whether the subjective feeling of

effort *per se* follows the MCD predictions. Recall that increasing $|\Delta VR^0|$ and/or VCR^0 will

decrease the demand for mental resources, which will result in the decision being associated

with a lower feeling of effort. To check this, we thus performed a multiple linear regression of

subjective effort ratings against $|\Delta VR^0|$ and VCR^0. A random effect analysis shows that both

have a significant and negative effect at the group level ($|\Delta VR^0|$: mean GLM beta=-0.20,

s.e.m.=0.03, p<0.001; CR^0: mean GLM beta=-0.05, s.e.m.=0.02, p=0.025). A graphical summary

of the data can be seen in the Supplementary Material.

Second, the MCD model predicts how value representations will be modified during

the decision process. In particular, choice-induced preference change should globally follow

the optimal effort allocation. More precisely, the reported value of alternative options should

spread apart, and the expected spreading of alternatives (i.e. $[VR_{chosen} - VR_{unchosen}]^{post-choice}$ -

$[VR_{chosen} - VR_{unchosen}]^{pre-choice}$) should be decreasing with $|\Delta VR^0|$ and VCR^0. Figure 7 below shows

the model predictions and summarizes the corresponding empirical data.

Figure 7. Three-way relationship between spreading of alternatives, value, and value certainty. Left: spreading of alternatives (SoA; color code) is shown as a function of $|\Delta VR^0|$ (x-axis) and VCR^0 (y-axis). Center: SoA is shown as a function of $|\Delta VR^0|$ (x-axis) and VCR^0 (color code); solid lines indicate empirical data, dashed lines represent MCD model predictions. Right: GLM regression beta weights for $|\Delta VR^0|$ and VCR^0 on SoA. (Error bars represent s.e.m., $|\Delta VR^0|$ p<0.001, VCR^0 p=0.079.)

One can see that the spreading of alternatives follows the MCD model predictions. A random effect analysis confirms this, showing that both $|\Delta VR^0|$ and CR^0 have a significant negative effect at the group level ($|\Delta VR^0|$: mean GLM beta=-0.09, s.e.m.=0.03, p<0.001; CR^0: mean GLM beta=-0.04, s.e.m.=0.02, p=0.027). Note that this replicates our previous findings on choice-induced preference change (Lee and Daunizeau, 2020; Lee and Coricelli, 2020). In addition to expected changes in value ratings, the MCD model predicts that the precision of value representations should increase after the decision has been made (cf. "β-effect" in the Supplementary Material). Indeed, post-choice value certainty ratings are significantly higher than pre-choice value certainty ratings (mean difference=1.34, s.e.m.=0.51, p=0.006). Importantly, under the MCD model, post-choice ratings are simply reports of modified value representations at the time when the choice is triggered. Therefore, choice and its associated confidence level should be better predicted with post-choice ratings than with pre-choice ratings. Indeed, we found that the predictive power of post-choice ratings is significantly higher than that of pre-choice ratings, both for choice (mean prediction accuracy difference=7%, s.e.m.=0.01, p<0.001) and choice confidence (mean prediction accuracy difference=3%, s.e.m.=0.01, p=0.004). Details regarding this analysis can be found in the Supplementary Material.

Third, the MCD model predicts that, all else being equal, effort increases with decision importance and decreases with costs. We checked the former prediction by asking

participants to make a few decisions where they knew that the choice would be real, i.e. they

would actually have to eat the chosen food item. We refer to these trials as "consequential"

decisions. To check the latter prediction, we imposed a financial penalty that increases with

response time. These experimental manipulations are described in the Methods section.

Figure 8 below shows subjective effort ratings and response times for "neutral",

"consequential" and "penalized" decisions, when controlling for $|\Delta VR^0|$ and VCR^0 (see the

Supplementary Material for more details).

Figure 8. Comparison of "neutral", "consequential", and "penalized" decisions. Left: Mean

(+/- s.e.m.) response time is shown for "neutral" (blue), "consequential" (green) and

"penalized" (red) decisions (significance stars: *p<0.05, ***p<0.001). Right: Mean (+/- s.e.m.)

subjective effort (same format). Both datasets were corrected for $|\Delta VR^0|$ and VCR^0.

605   One can see that subjective effort reports and response times follow the MCD model

predictions. More precisely, both subjective effort reports and response times were

significantly higher for "consequential" decisions than for "neutral" decisions (mean effort

difference=0.39, s.e.m.=0.12, p=0.001; mean response time difference=0.43, s.e.m.=0.19,

p=0.017). In addition, response times are significantly faster for "penalized" than for "neutral"

decisions (mean response time difference=-0.51, s.e.m.=0.08, p<0.001). Note that although

the difference in reported effort between "neutral" and "penalized" decisions does not reach

statistical significance (mean effort difference=-0.13, s.e.m.=0.12, p=0.147), it goes in the right

direction.

DISCUSSION

In this work, we have presented a novel computational model of decision-making

which explains the intricate relationships between choice accuracy, response time, subjective

effort, choice confidence, and choice-induced preference change. This model assumes that

deciding between alternative options whose values are uncertain induces a demand for

allocating cognitive resources to processing value-relevant information. Cognitive resource

allocation then optimally trades effort for confidence, given the discriminability of prior value

representations. Such metacognitive control of decisions or MCD makes novel predictions that

differ from standard accumulation-to-bound models of decision-making, including a drift-

diffusion model that was proposed as an optimal policy for value-based decision making

(Tajima et al., 2016). But, how can these two frameworks both be optimal? The answer lies in

the distinct computational problems that they solve. The MCD solves the problem of finding

the optimal amount of effort to invest under the possibility that yet-unprocessed value-

relevant information might change the decider's mind. In fact, this resource allocation

problem would be vacuous, would it not be possible to reassess preferences during the

decision process. In contrast, the DDM provides an optimal solution to the problem of

efficiently comparing option values, which may be unreliably signaled, but remain stationary

nonetheless. Of course, the DDM decision variable (i.e., the "evidence" for a given choice

option) may fluctuate, e.g. it may first drift towards the upper bound, and then eventually

reach the lower bound (Kiani et al., 2014; Resulaj et al., 2009). But, critically, these fluctuations

are not caused by changes in the underlying value signal. This is why the DDM cannot predict

choice-induced preference changes, i.e. changes in options' values. This critical distinction

extends to other types of accumulation-to-bound models, including race models (De Martino

et al, 2013; Tajima et al, 2019). We note that both these models could be equipped with pre-

choice value priors, and then driven with "true" values derived from post-choice ratings. Pre-

choice to post-choice rating change would then map to an update from prior to posterior value

estimates. But then, simulating these models would require both pre-choice and post-choice

ratings, which implies that choice-induced value changes cannot be *predicted* from pre-choice

ratings.

As a side note, we acknowledge that variants of these accumulation-to-bound models

could potentially use pre-choice value and value certainty ratings differently from how current

models use them (Lee and Usher, 2020). However, distorting existing models may be

problematic because some computational variables may lose their very meaning. For example,

in the context of Tajima's DDM, if the drift rate was set to some form of certainty-weighted

value difference, then the ensuing DDM's stochastic state could not be mapped to the

posterior mean of a Bayesian belief on value. In turn, the model predictions would not be

optimally solving the value-comparison problem.

Now, let us highlight that the MCD model offers a plausible alternative interpretation

for the two main reported neuroimaging findings regarding confidence in value-based choices

(De Martino et al., 2013). First, the ventromedial prefrontal cortex or vmPFC was found to

respond positively to both value difference (i.e., ΔVR^0) and choice confidence. Second, the

right rostrolateral prefrontal cortex or rRLPFC was more active during low-confidence versus

high-confidence choices. These findings were originally interpreted through the framework of

the race model that we compared to the MCD model. In brief, rRLPFC was thought to perform

a readout of choice confidence (for the purpose of subjective metacognitive report) from the

racing value accumulators hosted in the vmPFC. Under the MCD framework, the contribution

of the vmPFC to value-based choices might rather be to anticipate and monitor the benefit of

effort investment (i.e., confidence). This would be consistent with recent fMRI studies

suggesting that vmPFC confidence computations signal the attainment of task goals (Hebscher

and Gilboa, 2016; Lebreton et al., 2015). Now, recall that the MCD model predicts that

confidence and effort should be anti-correlated. Thus, the puzzling negative correlation

between choice confidence and rRLPFC activity could be simply explained under the

assumption that rRLPFC provides the neurocognitive resources that are instrumental for

processing value-relevant information during decisions. This resonates with the known

involvement of rRLPFC in reasoning (Desrochers et al., 2015; Dumontheil, 2014) or memory

retrieval (Benoit et al., 2012; Westphal et al., 2019).

At this point, we would like to discuss a few features of the MCD model. First, we did

not specify what determines the reward component, which quantifies decision importance

and acts as an effective weight for confidence against effort costs (cf. R in Equation 2 of the

Model section). We know, from the comparison of "consequential" and "neutral" choices that

increasing decision importance eventually increases effort, as predicted by the MCD model.

However, decision importance may have many determinants, such as, for example, the

commitment time of the decision (cf. partner choices), the breadth of its repercussions (cf.

political decisions), or its instrumentality with respect to the achievement of superordinate

goals (cf. moral decisions). How these determinants are combined and/or moderated by the

decision context is virtually unknown (Locke and Latham, 2002, 2006). In addition, decision

importance might also be influenced by the prior (intuitive/emotional/habitual) appraisal of

option values. For example, we found that, all else equal, people spent much more time and

effort deciding between two disliked items than between two liked items (results not shown).

This reproduces recent results regarding the evaluation of choice sets (Shenhav and

Karmarkar, 2019). Probing this type of influence will be the focus of forthcoming publications.

Second, our current version of the MCD model relies upon a simple variant of resource

costs. We note that rendering the cost term nonlinear (e.g., quadratic) does not change the

qualitative nature of the MCD model predictions. More problematic, perhaps, is the fact that

we did not consider distinct types of effort, which could, in principle, be associated with

different costs. For example, the cost of allocating attention to a given option may depend

upon whether this option would be *a priori* chosen or not. This might eventually explain

systematic decision biases and differences in response times between default and non-default

choices (Lopez-Persem et al., 2016). Another possibility is that effort might be optimized along

two canonical dimensions, namely: duration and intensity. The former dimension essentially

justifies the fact that we used response time as a proxy for the amount of allocated resources.

This is because, if effort intensity stays constant, then longer response times essentially signal

a greater resource expenditure. In fact, as is evident from the comparison between

"penalized" and "neutral" choices, imposing an external penalty cost on response time

reduces, as expected, the ensuing subjective effort. More generally, however, the dual

optimization of effort dimensions might render the relationship between effort and response

time more complex. For example, beyond memory span or attentional load, effort intensity

could be related to processing speed. This would explain why, although "penalized" choices

are made much faster than "neutral" choices, the associated feeling of effort is not as strongly

impacted (cf. Figure 8). In any case, the relationship between effort and response time might

depend upon the relative costs of effort duration and intensity, which might itself be partially

driven by external availability constraints (cf. time pressure or multitasking). We note that the

essential nature of the cost of mental effort in cognitive tasks (e.g., neurophysiological cost,

interferences cost, opportunity cost) is still a matter of intense debate (Kurzban et al., 2013;

Musslick et al., 2015; Ozcimder et al., 2017). Progress towards addressing this issue will be

highly relevant for future extensions of the MCD model.

Third, we did not consider the issue of identifying plausible neuro-computational

implementations of MCD. This issue is tightly linked to the previous one, in that distinct cost

types would likely impose different constraints on candidate neural network architectures

(Feng et al., 2014; Petri et al., 2017). For example, underlying brain circuits are likely to operate

MCD in a more reactive manner, eventually adjusting resource allocation from the continuous

monitoring of relevant decision variables (e.g., experienced costs and benefits). Such a

reactive process contrasts with our current, prospective-only, variant of MCD, which sets

resource allocation based on anticipated costs and benefits. We already checked that simple

reactive scenarios, where the decision is triggered whenever the online monitoring of effort

or confidence reaches the optimal threshold, make predictions qualitatively similar to those

we have presented here. We tend to think however, that such reactive processes should be

based upon a dynamic programming perspective on MCD, as was already done for the

problem of optimal efficient value comparison (Tajima et al., 2016, 2019). We will pursue this

and related neuro-computational issues in subsequent publications.

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

CODE AVAILABILITY

The computer code and algorithms that support the findings of this study are available from the corresponding author upon reasonable request.

ETHICAL COMPLIANCE

This study complies with all relevant ethical regulations and conforms to the guidelines established by our local ethics committee. In accordance with the Helsinki declaration, all participants gave informed consent prior to commencing the experiment.

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Trading mental effort for confidence: supplementary material

1. Data descriptive statistics and sanity checks

Recall that we collect value ratings and value certainty ratings both before and after the choice session. We did this for the purpose of validating specific predictions of the MCD model (in particular: choice-induced preference changes: see Figure 7 of the main text). It turns out this also enables us to assess the test-retest reliability of both value and value certainty ratings. We found that both ratings were significantly reproducible (value: mean correlation=0.88, s.e.m.=0.01, p <0.001, value certainty: mean correlation=0.37, s.e.m.=0.04, p <0.001).

We also checked whether choices were consistent with pre-choice ratings. For each participant, we thus preformed a logistic regression of choices against the difference in value ratings. We found that the balanced prediction accuracy was beyond chance level (mean accuracy=0.68, s.e.m.=0.01, p<0.001).

2. Does choice confidence moderate the relationship between choice and pre-choice value ratings?

Previous studies regarding confidence in value-base choices showed that choice confidence moderates choice prediction accuracy (De Martino et al., 2012b). We thus splat our logistic regression of choices into high- and low-confidence trials, and tested whether higher confidence was consistently associated with increased choice accuracy. A random effect analysis showed that the regression slopes were significantly higher for high- than for low-confidence trials (mean slope difference=0.14, s.e.m.=0.03, p<0.001). For the sake of completeness, the impact of choice confidence on the slope of the logistic regression (of choice onto the difference in pre-choice value ratings) is shown on Figure S1 below.

Figure S1. Relationship between choices, pre-choice value ratings and choice confidence.

Left: the probability of choosing the item on the right (y-axis) is shown as a function of the pre-choice value difference (x-axis), for high- (blue) versus low- (red) confidence trials. The plain lines show the logistic prediction that would follow from group-averages of the corresponding slope estimates. Right: the corresponding logistic regression slope (y-axis) is shown for both high- (blue) and low- (red) confidence trials (group means +/- s.e.m.).

These results clearly replicate the findings of De Martino and colleagues (2012), which were interpreted with a race model variant of the accumulation-to-bound principle. We note, however, that this effect is also predicted by the MCD model. Here, variations in both (i) the prediction accuracy of choice from pre-choice value ratings, and (ii) choice confidence, are driven by variations in resource allocation. In brief, the expected magnitude of the perturbation of value representations increases with the amount of allocated resources. This eventually degrades the prediction accuracy of choice from pre-choice value ratings (which have been changed during the decision process). However, although more resources are allocated to the decision, this does not overcompensate for decision difficulty, and thus choice confidence decreases. Thus, low-confidence choices will be those choices that cannot be well predicted with pre-choice value ratings. We note that the anti-correlation between choice confidence and choice accuracy can be seen by comparing Figures 4 and 6 of the main text.

3. How do choice confidence, difference in pre-choice value ratings, and decision time

relate to each other?

In the main text, we show that trial-by-trial variations in choice confidence is

concurrently explained by both pre-choice value and value certainty ratings. Here, we

reproduce previous findings relating choice confidence to both absolute value difference ΔVR^0

and decision time (De Martino et al., 2012b). First, we regressed, for each participant, decision

time concurrently against both $|\Delta VR^0|$ and choice confidence. A random effect showed that

both have a significant main effect on decision time (ΔVR^0: mean GLM beta=-0.016,

s.em.=0.003, p<0.001; choice confidence: mean GLM beta=-0.014, s.em.=0.002; p<0.001),

without any two-way interaction (p=0.133). This analysis is summarized in Figure S2 below,

together with the full three-way relationship between $|\Delta VR^0|$, confidence and decision time.

Figure S2. Relationship between pre-choice value ratings, choice confidence and decision

times. Left: decision times (y-axis) are plotted as a function of low- and high- $|\Delta VR^0|$ (x-axis)

for both low- (red) and high- (blue) confidence trials. Errorbar represent s.e.m. Right: A

heatmap of mean z-scored confidence is shown as a function of both decision time (x-axis) and

$|\Delta VR^0|$ (y-axis).

In brief, confidence increases with the absolute value difference and decreases with

decision time. This effect is also predicted by the MCD model, for reasons identical to the

explanation of the relationship between confidence and choice accuracy (see above). Recall

that, overall, an increase in choice difficulty is expected to yield an increase in decision time

and a decrease in choice confidence. This would produce the same data pattern as Figure S2,

although the causal relationships implicit in this data representation is partially incongruent

with the computational mechanisms underlying MCD.

4. MCD fit accuracy

To capture idiosyncrasies in participants' metacognitive control of decisions, the MCD

model was fitted to trial-by-trial data, where all five MCD outputs (namely: the probability of

changing one's mind, choice confidence, spreading of alternatives, response time, and

subjective effort ratings) were considered together. In the main manuscript, we present

summary statistics at the group level, which validate the qualitative predictions that can be

derived from the MCD model. But can the MCD model predict within-subject data beyond

chance level? To address this question, we performed Monte-Carlo simulations under the null,

i.e. we fitted the MCD model to dummy (random) data. Note that we used the exact same

procedure as the one we use for experimental data (same model parameterization, same

number of trials, etc). For each Monte-Carlo simulation, we computed the percentage of

explained variance for each of the five dummy dependent variables. We set chance level as

the average percentage of explained variance across Monte-Carlo simulations. This serves as

a reference point for evaluating the predictive power of the MCD model, which is shown in

Figure S3 below.

Figure S3. Within-subject explanatory power of the MCD model. The mean percentage of explained trial-by-trial variance (y-axis) is shown for each MCD output (x-axis: Pc= chcoice confidence, SoA= spreading of alternatives, RT= response time, Qcom = probability of changing one's mind, Eff= subjective effort rating). Errorbars depict standard error around the mean. Horizontal red lines depict chance level (i.e. average percentage of explained variance under the null).

One can see that the predictive power of the MCD model is higher than chance level for

each dependent variable. Also, the MCD model explains very accurately trial-by-trial variations

in choice confidence, but has less explanatory power for, e.g., spreading of alternative. In

addition, subjective effort ratings seem to be a more reliable proxy for the amount of allocated

resources than response time. Recall that trial-by-trial variations in all of these dependent

variables had to be explained with only three parameters, namely: the *effort unitary cost* α,

type #1 effort efficacy β and *type #2 effort efficacy* γ. The predictive power of the MCD is

thus remarkable, given that the MCD model possesses only three degrees of freedom.

5. Analysis of changes of mind

In the main manuscript, we show that choice accuracy increases with pre-choice value difference ΔVR^0 and pre-choice value certainty VCR^0. Recall that choice accuracy was defined in terms of the rate of choices that are congruent with preferences derived from pre-choice value ratings. Now people make "inaccurate" choices either because they make mistakes or because they change their mind during the decision. In principle, we can discriminate between these two explanations because we can check whether "inaccurate" choices are congruent with post-choice value ratings (change of mind) or not (error). This is important, because accumulation-to-bound models do not allow for the possibility that value representations change during decisions. Hence all "inaccurate" choices would be deemed "errors", which are driven by stochastic noise in the evidence accumulation process. It turns out that most choices are "accurate" (mean choice accuracy =73.3%, s.e.m.=1%), and less than half of the "inaccurate" choices are classified as "errors" (mean error rate=12%, s.e.m.=0.01), which is significantly less than "mind changes" (mean rate difference=2%, s.e.m.=0.01, p=0.032). In addition, choice confidence and (post- versus pre-choice) value certainty gain were significantly higher for "changes of mind" than for "errors" (choice confidence: mean difference=13.7, s.e.m.=2.1, p<0.001; value certainty gain: mean difference=2.6, s.e.m.=1.4, p=0.035).

Thus, one may wonder what the impacts of both pre-choice value difference ΔVR^0 and pre-choice value certainty VCR^0 on choice accuracy would be, if one were to remove "errors" from "inaccurate" choices. Figure S4 below shows both the predicted and measured three-way relationship between the probability of changing one's mind, ΔVR^0 and VCR^0.

Figure S4. Relationship between change of mind frequency, value ratings, and certainty

ratings. Left: change of mind frequency (CoM; color code) is shown as a function of $|\Delta VR^0|$ (x-axis)

and VCR^0 (y-axis). Center: CoM is shown as a function of $|\Delta VR^0|$ (x-axis) and VCR^0 (color code); solid

lines indicate empirical data, dashed lines represent MCD model predictions. Right: GLM regression

beta weights for $|\Delta VR^0|$ and VCR^0 on CoM. (Error bars represent s.e.m., $|\Delta VR^0|$ p<0.001, VCR^0

p=0.006.)

Recall that, under the MCD model, the probability of changing one's mind increases

with the resource demand, which decreases when either $|\Delta VR^0|$ or VCR^0 increase. One can

see that the data seem to conform to this prediction. To check this, we ran, for each

participant, a multiple logistic regression of change of mind against $|\Delta VR^0|$ and VCR^0. A

random effect analysis shows that both have a significant and negative effect at the group

level (ΔVR^0: mean GLM beta=-0.16, s.e.m.=-0.02, p<0.001; VCR^0: mean GLM beta=-0.08,

s.e.m.=0.02, p<0.001). These results are qualitatively similar to the analysis of choice accuracy

(cf. Figure 4 in the main text).

6. Analysis of the subjective feeling of effort

In the main manuscript, we show that decision time decreases with pre-choice value

difference ΔVR^0 and pre-choice value certainty VCR^0. The focus on decision time was

motivated by the fact that all models could make quantitative -and thus comparable-

predictions. In brief, we found that the effect of VCR^0 on decision time was consistent with the

1011 MCD model, but not with accumulation-to-bound models. Now, under the MCD model,

1012 decision time is but a proxy for effort duration. Here, we ask whether the subjective feeling of

1013 effort *per se* follows the MCD model predictions. This is possible because we asked

1014 participants to rate how effortful each decision felt. Figure S5 below shows both the predicted

1015 and the measured three-way relationship between effort, $|\Delta VR^0|$ and VCR^0.

1016

Figure S5. Relationship between subjective effort, value ratings, and certainty ratings. Left:

subjective effort (color code) is shown as a function of $|\Delta VR^0|$ (x-axis) and VCR^0 (y-axis). Center:

subjective effort is shown as a function of $|\Delta VR^0|$ (x-axis) and VCR^0 (color code); solid lines indicate

empirical data, dashed lines represent MCD model predictions. Right: GLM regression beta weights for

$|\Delta VR^0|$ and VCR^0 on subjective effort. (Error bars represent s.e.m., $|\Delta VR^0|$ $p<0.001$, VCR^0 $p=0.021$.)

One can see that the reported subjective feeling of effort closely matches model

predictions. One may ask whether people's effort reports may be a trivial post-choice read-

outs of decision time and/or choice confidence. This, however, is unlikely, given that people's

subjective effort is reducible neither to decision time (mean correlation=0.39, s.e.m.=0.04),

nor to choice confidence (mean correlation=-0.48, s.e.m.=, p=0.05).

7. Do post-choice ratings better predict choice and choice confidence than pre-choice

ratings?

The MCD model assumes that value representations are modified during the decision

process, until the MCD-optimal amount of resources is met. This eventually triggers the

decision, whose properties (i.e. which alternative option is eventually preferred, and with

which confidence level) then reflects the modified value representations. If post-choice ratings

are reports of modified value representations at the time when the choice is triggered, then

choice and its associated confidence level should be better predicted with post-choice ratings

than with pre-choice ratings. In what follows, we test this prediction.

In the first section of this Supplementary Material, we report the result of a logistic

regression of choice against pre-choice value ratings (see also Figure S1). We performed the

same regression analysis, but this time against post-choice value ratings. Figure S5 below

shows the ensuing predictive power (here, in terms of balanced accuracy or BA) for both pre-

choice and post-choice ratings. The main text also features the result of a multiple linear

regression of choice confidence ratings onto $|\Delta VR^0|$ and VCR^0 (cf. Figure 6). Again, we

performed the same regression, this time against post-choice ratings. Figure S6 below shows

the ensuing predictive power (here, in terms of percentage of explained variance or R^2) for

both pre-choice and post-choice ratings.

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Figure S6. Comparison of the predictive power of pre-choice versus post-choice ratings. Left:

Mean (+/- s.e.m.) BA of logistic regressions of choice against pre-choice (left) and post-choice

(right) value ratings. Right: Mean (+/- s.e.m.) R^2 of multiple linear regressions of choice

confidence against pre-choice (left) and post-choice (right) ratings.

A simple random effect analysis shows that the predictive power of post-choice ratings

is significantly higher than that of pre-choice ratings, both for choice (mean difference in

BA=7%, s.e.m.=0.01, p<0.001) and choice confidence (mean difference in R^2=3%, s.e.m.=0.01,

p=0.004).

8. Cross-condition analysis: decision importance and cost of decision time

As featured in the main manuscript, we intermixed "neutral" trials with two specific

sets of trials, in which we either manipulated decision importance (cf. "consequential"

decisions) or the cost of decision time (cf. "penalized" decisions). Figure S7 below shows the

mean subjective effort ratings and decision times for "neutral", "consequential" and

"penalized" decisions.

1060

Figure S7. Comparison of "neutral", "consequential" and "penalized" decisions. Left: Mean

(+/- s.e.m.) response time is shown for "neutral" (blue), "consequential" (green) and

"penalized" (red) decisions (significance stars: *p<0.05, ***p<0.001). Right: Mean (+/- s.e.m.)

subjective effort (same format).

Overall, the data partially follows the model predictions. In particular, subjective effort

and decision time are both significantly higher for "consequential" than for "neutral" decisions

(effort: mean difference=9.0, s.e.m.=2.2, p<0.001; DT: mean difference=0.56, s.e.m.=0.32,

p=0.043). In addition, decision time is significantly lower for "penalized" than for "neutral"

decision (mean DT difference=-0.46, s.e.m.=0.08, p<0.001). However, there is no noticeable

difference between reported efforts in "neutral" and "penalized" decisions (mean effort

difference=0.6, s.e.m.=2.1, p=0.604).

This comparison however, may be confounded by between-condition differences in

ΔVR^0 or VCR^0. For each participant, we thus performed a multiple linear regression of effort

and DT onto $|\Delta VR^0|$ and VCR^0, including all types of trials. Corrected effort and DT can now be

compared, after having removed the effects of $|\Delta VR^0|$ and VCR^0. This is what Figure 8 of the

main text shows. As one can see, the overall pattern is similar to Figure S6. As before,

subjective effort and decision time are both significantly higher for "consequential" than for

"neutral" decisions (effort: mean GLM beta difference=0.39, s.e.m.=0.12, p=0.001; DT: mean

GLM beta difference=0.43, s.e.m.=0.19, p=0.017), and decision time is significantly lower for

"penalized" than for "neutral" decisions (mean DT GLM beta difference=-0.51, s.e.m.=0.08,

p<0.001). Finally, the difference between reported efforts in "neutral" and "penalized"

decisions is now almost significant (mean effort GLM beta difference=-0.13, s.e.m.=0.12,

p=0.147).

9. Analysis of eye-tracking data

We first checked whether pupil dilation positively correlates with participants' reports

of subjective effort. We epoched the pupil size data into trial-by-trial time series, and

temporally co-registered the epochs either at stimulus onset (starting 1.5 seconds before the

stimulus onset and lasting 5 seconds) or at choice response (starting 3.5 seconds before the

choice response and lasting 5 seconds). Data was baseline-corrected at stimulus onset. For

each participant, we then regressed, at each time point during the decision, pupil size onto

effort ratings (across trials). Time series of regression coefficients were then reported at the

group level, and tested for statistical significance (correction for multiple comparison was

performed using random field theory 1D-RFT). Figure S8 below summarizes this analysis, in

terms of the baseline-corrected time series of regression coefficients.

Figure S8. Correlation between pupil size and reports of subjective effort during decision

time. Left: Mean (+/- s.e.m.) correlation between pupil size and subjective effort (y-axis) is

plotted as a function of peristimulus time (x-axis). Here, epochs are co-registered w.r.t.

stimulus onset (the green line indicates stimulus onset and the red dotted line indicates the

average choice response). Right: Same, but for epochs co-registered w.r.t. choice response

(the green line indicates choice response and the red dotted line indicates the average

stimulus onset).

We found that the correlation between effort and pupil dilation was becoming

significant from 500ms after stimulus onset onwards. Note that, using the same approach, we

found a negative correlation between pupil dilation and pre-choice absolute value difference

$|\Delta VR^0|$. However, this relationship disappeared when we entered both $|\Delta VR^0|$ and effort into

the same regression model.

Our eye-tracking data also allowed us to ascertain which item was being gazed at for

each point in peristimulus time (during decisions). Using the choice responses, we classified

each time point as a gaze at the (to be) chosen item or at the (to be) rejected item. We then

derived, for each decision, the ratio of time spent gazing at chosen/unchosen items versus the

total duration of the decision (between stimulus onset and choice response). The difference

between these two gaze ratios measures the overt attentional bias towards the chosen item.

We refer to this as the gaze bias. Consistent with previous studies, we found that chosen items

were gazed at more than rejected items (mean gaze bias=0.02, s.e.m.=0.01, p=0.067).

However, we also found that this effect was in fact limited to low effort choices. Figure S9

below shows the gaze bias for low and high effort trials, based upon a median-split of

subjective effort.

1118

Figure S9. Gaze bias for low and high effort trials. Mean (+/- s.e.m.) gaze bias is plotted for

both low (left) and high (right) effort trials.

We found that there was a significant gaze bias for low effort choices (mean gaze ratio

difference=0.033, s.e.m.=0.013, p=0.009), but not for high effort choices (mean gaze ratio

difference=0.002, s.e.m.=0.014, p=0.453). A potential trivial explanation for the fact that the

gaze bias is large for low effort trials is that these are the trials where participants immediately

recognize their favorite option, which attracts their attention. More interesting is the fact that

the gaze bias is null for high effort trials. This may be taken as evidence for the fact that, on

average, people allocate the same amount of (attentional) resources on both items. This is

important, because we use this simplifying assumption in our MCD model derivations.

10. On the accuracy of the predicted confidence gain

The MCD model relies on the system's ability to anticipate the benefit of allocating

resources to the decision process. Given the mathematical expression of choice confidence

(cf. Equation 4 in the main text), this reduces to finding an analytical approximation to the

following expression:

$$\overline{P} = E\left[s\left(\lambda |x| \right) \right] \tag{S1}$$

where $x \rightarrow s(x) = 1/1 + e^{-x}$ is the sigmoid mapping, λ is an arbitrary constant, and the

expectation is taken under the gaussian distribution of $x \sim N\left(\mu, \sigma^2 \right)$, whose mean and

variance are μ and σ^2, respectively.

Note that the absolute value mapping $x \rightarrow |x|$ follows a folded normal distribution,

whose first two moments $E\left[|x| \right]$ and $V\left[|x| \right]$ have known expressions:

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$$\begin{cases} E\left[|x| \right] = \sigma \sqrt{\dfrac{2}{\pi}} \exp\left(-\dfrac{|\mu|^2}{2\sigma^2} \right) + \mu\left(2 \times s\left(\dfrac{\pi \mu}{\sigma\sqrt{3}} \right) - 1 \right) \\ V\left[|x| \right] = \mu^2 + \sigma^2 - E\left[|x| \right]^2 \end{cases} \tag{S2}$$

where the first line relies on a moment-matching approximation to the cumulative normal

distribution function (Daunizeau, 2017). This allows us to derive the following analytical

approximation to Equation S1:

$$\overline{P} \approx s\left(\dfrac{E\left[|x| \right]}{\sqrt{\dfrac{1}{\lambda^2} + aV\left[|x| \right]}} \right) \tag{S3}$$

where setting $a \approx 3/\pi^2$ makes this approximation tight (Daunizeau, 2017).

The quality of this approximation can be evaluated by drawing samples of

$x \sim N(\mu, \sigma^2)$, and comparing the Monte-Carlo average of $s(\lambda|x|)$ with the expression given

in Equation S3. This is summarized in Figure S10 below, where the range of variation for the

moments of x were set as follows: $\mu \in [-4, 4]$ and $\sigma^2 \in [0, 4]$.

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Figure S10: Quality of the analytical approximation to $\bar{P}$. Upper left panel: the Monte-Carlo

estimate of $\bar{P}$ (colour-coded) is shown as a function of both the mean $\mu \in [-4, 4]$ (y-axis) and

the variance $\sigma^2 \in [0, 4]$ (x-axis) of the parent process $x \sim N(\mu, \sigma^2)$. Upper right panel:

analytic approximation to $\bar{P}$ as given by Equation S3 (same format). Lower left panel: the

error, i.e. the difference between the Mon-Carlo and the analytic approximation (same

format). Lower right panel: the analytic approximation (y-axis) is plotted as a function of the

Monte-Carlo estimate (x-axis) for each pair of moments $\{\mu, \sigma^2\}$ of the parent distribution.

One can see that the error rarely exceeds 5%, across the whole range of moments

$\{\mu, \sigma^2\}$ of the parent distribution. This is how tight the semi-analytic approximation of the

expected confidence gain (Equation 9 in the main text) is.

11. On the impact of model parameters for the MCD model

First, note that the properties of the metacognitive control of decisions (in terms

of effort allocation and/or confidence) actually depends upon the demand for resources,

which is itself determined by prior value representations. Now the way the MCD-optimal

control responds to the resource demand (which is fully specified by the prior uncertainty

σ^0 and the absolute means' difference $\left|\Delta\mu^0\right|$) is determined by effort efficacy and unitary cost

parameters.

Let us first ask what would be the MCD-optimal effort $\hat{z}$ and confidence $\overline{P}_c(\hat{z})$

when $\gamma = 0$, i.e. if the only effect of allocating resources is to increase the precision of

value representations. We call this the "β-effect". It is depicted on Figure S11 below.

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Figure S11. The β-effect: MCD-optimal effort and confidence when effort has no impact on

the value difference. MCD-optimal effort (left) and confidence (right) are shown as a function

of the absolute prior mean difference $\left|\Delta\mu^0\right|$ (x-axis) and prior variance σ^0 (y-axis).

One can see that, overall, increasing the prior variance σ^0 increases the resource

demand, which eventually increases the MCD-optimal allocated effort $\hat{z}$. This, however, does

not overcompensate for the loss of confidence incurred when increasing the prior variance.

This is why the MCD-optimal confidence $\overline{P}_c\left(\hat{z}\right)$ always decreases with the prior variance σ^0.

Note that, for the same reason, the MCD-optimal confidence always increases with the

absolute prior means' difference $\left|\Delta\mu^0\right|$.

Now the impact of the absolute prior means' difference $\left|\Delta\mu^0\right|$ on $\hat{z}$ is less trivial. In

brief, when $\left|\Delta\mu^0\right|$ is high, the MCD-optimal allocated effort $\hat{z}$ decreases with $\left|\Delta\mu^0\right|$. This is

due to the fact that the resource demand decreases with $\left|\Delta\mu^0\right|$. However, if $\left|\Delta\mu^0\right|$ decreases

even more, it eventually reaches a critical point, below which the MCD-optimal allocated

effort $\hat{z}$ *increases* with $\left|\Delta\mu^0\right|$. This is because, although the resource demand still decreases

with $\left|\Delta\mu^0\right|$, the cost of allocating resources overcompensates the gain in confidence. For such

difficult decisions, the system does not follow the demand anymore, and progressively de-

motivates the allocation of resources as $\left|\Delta\mu^0\right|$ continues to decrease. In brief, the amount $\hat{z}$

of allocated resources decreases away from a "sweet spot", which is the absolute prior means'

difference that yields the maximal confidence gain per effort unit. Critically, the position of

this sweet spot decreases with β and increases with α. This is because the confidence

gain increases, by definition, with effort efficacy, whereas it becomes more costly when α

increases.

Let us now ask what would be the MCD-optimal effort $\hat{z}$ and confidence $\bar{P}_c(\hat{z})$

when $\beta = 0$, i.e. if the only effect of allocating resources is to perturb the value difference.

The ensuing "γ -effect" is depicted on Figure S12 below.

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1198   Figure S12. The γ-effect: MCD-optimal effort and confidence when effort has no impact on

1199   value precision. Same format as Fig S10.

In brief, the overall picture is reversed, with a few minor differences. One can see that

increasing the absolute prior means' difference $\left|\Delta\mu^0\right|$ decreases the resource demand, which

eventually decreases the MCD-optimal allocated effort $\hat{z}$. This does decrease confidence,

because the γ-effect of allocated effort overcompensates the effect of variations in $\left|\Delta\mu^0\right|$.

When no effort is allocated however, confidence is driven by $\left|\Delta\mu^0\right|$, i.e. it becomes an

increasing function of $\left|\Delta\mu^0\right|$. In contrast, variations in the prior variance σ^0 always

overcompensate the ensuing changes in effort, which is why confidence always decreases with

σ^0. In addition, the amount $\hat{z}$ of allocated resources decreases away from a sweet prior

variance spot, which is the prior variance σ^0 that yields the maximal confidence gain per effort

unit. Critically, the position of this sweet spot increases with γ and decreases with α, for

reasons similar to the β-effect.

Now one can ask what happens in the presence of both the β-effect and the γ-effect.

If the effort unitary cost α is high enough, the MCD-optimal effort allocation is essentially

the superposition of both effects. This means that there are two "sweet spots": one

around some value of $\left|\Delta\mu^0\right|$ at high σ^0 (β-effect) and one around some value of σ^0 at high

$\left|\Delta\mu^0\right|$ (γ-effect). If the effort unitary cost α decreases, then the position of the β-sweet

spot increases and that of the β-sweet spot decreases, until they effectively merge

together. This is exemplified on Figure S13 below.

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Figure S13. MCD-optimal effort and confidence when both types of effort efficacy are

operant. Same format as Fig S10.

One can see that, somewhat paradoxically, the effort response is now much simpler.

In brief, the MCD-optimal effort allocation $\hat{z}$ increases with the prior variance σ^0 and

decreases with the absolute prior means' difference $\left|\Delta\mu^0\right|$. The landscape of the ensuing MCD-

optimal confidence level $\bar{P}_c\left(\hat{z}\right)$ is slightly less trivial, but globally, it can be thought of as

increasing with $\left|\Delta\mu^0\right|$ and decreasing with σ^0. Here again, this is because variations in $\left|\Delta\mu^0\right|$

and/or σ^0 almost always overcompensate the ensuing changes in allocated effort.

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