1  **Trading Mental Effort for Confidence:**
2  **The Metacognitive Control of Value-Based Decision-Making**
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4  Douglas Lee[1,2]          Jean Daunizeau[1,3]
5
6  [1] Brain and Spine Institute (ICM), Paris, France
7  [2] Sorbonne University, Paris, France
8  [3] Translational Neuromodeling Unit (TNU), ETH, Zurich, Switzerland

21  Address for correspondence:
22  Jean Daunizeau
23  Motivation, Brain, and Behavior Group
24  Brain and Spine Institute
25  47, bvd de l'Hopital, 75013, Paris, France
26
27  Tel: +33 1 57 27 47 19
28  E-mail: jean.daunizeau@gmail.com

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**ABSTRACT**

Why do we sometimes opt for actions or items that we do not value the most? Under

current neurocomputational theories, such preference reversals are typically interpreted in

terms of errors that arise from the unreliable signaling of value to brain decision systems. But,

an alternative explanation is that people may change their mind because they are reassessing

the value of alternative options while pondering the decision. So, why do we carefully ponder

some decisions, but not others? In this work, we derive a computational model of the

metacognitive control of decisions or MCD. In brief, we assume that fast and automatic

processes first provide an initial (and largely uncertain) representation of options' values,

yielding prior estimates of decision difficulty. This uncertain value representation is then refined

by deploying cognitive (e.g., attentional, mnesic) resources, the allocation of which is controlled

by an effort-confidence tradeoff. Importantly, the anticipated benefit of allocating resources

varies in a decision-by-decision manner according to the prior estimate of decision difficulty.

The ensuing MCD model predicts choices, response time, subjective feeling of effort, choice

confidence, and choice-induced preference change. We test these predictions in a systematic

manner, using a dedicated behavioral paradigm. Our results provide a quantitative link between

mental effort, choice confidence, and preference reversals, which could inform interpretations

of related neuroimaging findings.

**INTRODUCTION**

Why do we carefully ponder some decisions, but not others? Decisions permeate every aspect of our lives—what to eat, where to live, whom to date, etc.—but the amount of effort that we put into different decisions varies tremendously. Rather than processing all decision-relevant information, we often rely on fast habitual and/or intuitive decision policies, which can lead to irrational biases and errors (Kahneman et al., 1982). For example, snap judgments about others are prone to unconscious stereotyping, which often has enduring and detrimental consequences (Greenwald and Banaji, 1995). Yet we don't always follow the fast but negligent lead of habits or intuitions. So, what determines how much time and effort we invest when making decisions?

Biased and/or inaccurate decisions can be triggered by psychobiological determinants such as stress (Porcelli and Delgado, 2009; Porcelli et al., 2012), emotions (Harlé and Sanfey, 2007; Martino et al., 2006; Sokol-Hessner et al., 2013), or fatigue (Blain et al., 2016). But, in fact, they also arise in the absence of such contextual factors. This is why they are sometimes viewed as the outcome of inherent neurocognitive constraints on the brain's decision processes, e.g., limited attentional and/or mnemonic capacity (Giguère and Love, 2013; Lim et al., 2011; Marois and Ivanoff, 2005) or unreliable neural representations of decision-relevant information (Drugowitsch et al., 2016; Wang and Busemeyer, 2016; Wyart and Koechlin, 2016). However, an alternative perspective is that the brain has a preference for efficiency over accuracy (Thorngate, 1980). For example, when making perceptual or motor decisions, people frequently trade accuracy for speed, even when time constraints are not tight (Heitz, 2014; Palmer et al., 2005). Related neural and behavioral data are best explained by "accumulation-to-bound"

process models, in which a decision is emitted when the accumulated perceptual evidence

reaches a bound (Gold and Shadlen, 2007; O'Connell et al., 2012; Ratcliff and McKoon, 2008;

Ratcliff et al., 2016). Further computational work demonstrated that some variants of these

models actually implement an optimal solution to speed-accuracy tradeoff problems (Ditterich,

2006; Drugowitsch et al., 2012). From a theoretical standpoint, this implies that accumulation-

to-bound policies can be viewed as an evolutionary adaptation, in response to selective

pressure that favors efficiency (Pirrone et al., 2014).

This line of reasoning, however, is not trivial to generalize to value-based decision

making, for which objective accuracy remains an elusive notion (Dutilh and Rieskamp, 2016;

Rangel et al., 2008). This is because, in contrast to evidence-based (e.g., perceptual) decisions,

there are no right or wrong value-based decisions. Nevertheless, people still make choices that

deviate from subjective reports of value, with a rate that decreases with value contrast. From

the perspective of accumulation-to-bound models, these preference reversals count as errors

and arise from the unreliable signaling of value to decision systems in the brain (Lim et al.,

2013). That value-based variants of accumulation-to-bound models are able to capture the

neural and behavioral effects of, e.g., overt attention (Krajbich et al., 2010; Lim et al., 2011),

external time pressure (Milosavljevic et al., 2010), confidence (De Martino et al., 2012) or

default preferences (Lopez-Persem et al., 2016), lends empirical support to this type of

interpretation. Further credit also comes from theoretical studies showing that these process

models, under some simplifying assumptions, optimally solve the problem of efficient value

comparison (Tajima et al., 2016, 2019). However, accumulation-to-bound models neglect the

possibility that people may reassess the value of alternative options during decisions (Slovic,

1995; Tversky and Thaler, 1990; Warren et al., 2011). For example, contemplating competing

possibilities during a choice may highlight features of alternative options that may not have

been considered thoroughly before (Sharot et al., 2010). Under this view, apparent preference

reversals are not errors: they are deliberate changes of mind. Here, we propose an alternative

computational model of value-based decision-making that resolves this concern.

We start with the premise that the brain generates representations of options' value in a

quick and automatic manner, even before attention is engaged for making a decision (Lebreton

et al., 2009). The brain also encodes the certainty of such value estimates (Lebreton et al.,

2015), from which *a priori* feelings of choice difficulty and confidence could, in principle, be

derived. Importantly, people are reluctant to make a choice that they are not confident about

(De Martino et al., 2013). Thus, when faced with a difficult decision, people should reassess

option values until they reach a satisfactory level of confidence about their preference. This

effortful mental deliberation would engage neurocognitive resources, such as attention and

memory, in order to process value-relevant information. In line with recent proposals regarding

the strategic deployment of cognitive control (Musslick et al., 2015; Shenhav et al., 2013), we

assume that the amount of allocated resources optimizes a tradeoff between expected effort

cost and confidence gain. Critically, we show how the system can anticipate the expected

benefit of allocating resources before having processed value-relevant information. The ensuing

*metacognitive control of decisions* or *MCD* thus adjusts mental effort on a decision-by-decision

basis, according to prior decision difficulty and importance (cf. Figure 1 below).

**Figure 1. The Metacognitive Control of Decisions**. First, automatic processes provide a "pre-effort" belief about option values. This belief is probabilistic, in the sense that it captures an uncertain prediction regarding the to-be-experienced value of a given option. This pre-effort belief serves to identify the anticipated impact of investing costly cognitive resources (i.e., effort) in the decision. In particular, investing effort is expected to increase decision confidence beyond its pre-effort level. But how much effort it should be worth investing depends upon the balance between expected confidence gain and effort costs. The system then allocates resources into value-relevant information processing up until the optimal effort investment is reached. At this point, a decision is triggered based upon the current post-effort belief about option values (in this example, the system has changed its mind, i.e. its preference has reversed). Note: we refer to the ensuing increase in the value difference between chosen and unchosen items as the "spreading of alternatives" (cf. Methods section).

As we will see, the MCD model makes clear quantitative predictions about several key

decision variables. We test these predictions by asking participants to report their judgments

about each item's value and their subjective certainty about their value judgements, both

before and after choosing between pairs of the items. Note that we also measure choice

confidence, response time, and subjective effort for each decision.

The objective of this work is to show how most non-trivial properties of value-based

decision making can be explained with a minimal (and mutually consistent) set of assumptions.

The MCD model predicts choice, response time, subjective effort, choice confidence, probability

of changing one's mind, and choice-induced preference change, out of two properties of pre-

choice value representations, namely: value ratings and value certainty ratings. Relevant details

regarding the model derivations, as well as the decision-making paradigm we designed to

evaluate those predictions, can be found in the Model and Methods sections below. In what

follows, we present our main dual computational/behavioral results.

**MODEL**

In what follows, we derive a computational model of the metacognitive control of

decisions or MCD. In brief, we assume that the amount of cognitive resources that is deployed

during a decision is controlled by an effort-confidence tradeoff. Critically, this tradeoff relies on

a prospective anticipation of how these resources will perturb the internal representations

of subjective values. As we will see, the computational properties of the MCD model are

critically different from accumulation-to-bound models of value-based decision-making, which

we briefly describe in the Supplementary Material.

**Deriving the expected value of decision control**

Let $z$ be the amount of cognitive (e.g., executive, mnemonic, or attentional) resources that serve to process value-relevant information. Allocating these resources will be associated with both a benefit $B(z)$, and a cost $C(z)$. As we will see, both are increasing functions of $z$: $B(z)$ derives from the refinement of internal representations of subjective values of alternative options or actions that compose the choice set, and $C(z)$ quantifies how aversive engaging cognitive resources is (mental effort). In line with the framework of expected value of control or EVC (Musslick et al., 2015; Shenhav et al., 2013), we assume that the brain chooses to allocate the amount of resources $\hat{z}$ that optimizes the following cost-benefit trade-off:

$$\hat{z} = \arg\max_{z} E\left[B(z) - C(z)\right]$$
(1)

where the expectation accounts for predictable stochastic influences that ensue from allocating resources (this will be more clear below). Note that the benefit term $B(z)$ is the (weighted) choice confidence $P_c(z)$:

$$B(z) = R \times P_c(z)$$
(2)

where the weight $R$ is analogous to a reward and quantifies the importance of making a confident decision (see below). As will be made more clear below, $P_c(z)$ plays a pivotal role in the model, in that it captures the efficacy of allocating resources for processing value-relevant information. So, how do we define choice confidence?

We assume that the decision maker may be unsure about how much he likes/wants the

alternative options that compose the choice set. In other words, the internal representations of

values $V_i$ of alternative options are probabilistic. Such a probabilistic representation of value

can be understood in terms of, for example, an uncertain prediction regarding the to-be-

experienced value of a given option. Without loss of generality, the probabilistic representation

of option values take the form of Gaussian probability density functions, as follows:

$$p(V_i) = N(\mu_i, \sigma_i)$$     (3)

where $\mu_i$ and $\sigma_i$ are the mode and the variance of the probabilistic value representations,

respectively (and $i$ indexes alternative options in the choice set).

This allows us to define choice confidence $P_c$ as the probability that the (predicted)

experienced value of the (to be) chosen item is higher than that of the (to be) unchosen item:

$$\begin{aligned} P_c &= \begin{cases} P(V_1 > V_2) & \text{if item \#1 is chosen} \\ P(V_2 > V_1) & \text{if item \#2 is chosen} \end{cases} \\ &= \begin{cases} P(V_1 > V_2) & \text{if } \Delta\mu > 0 \\ P(V_2 > V_1) & \text{if } \Delta\mu < 0 \end{cases} \\ &\approx s\left( \frac{\pi |\Delta\mu|}{\sqrt{3(\sigma_1 + \sigma_2)}} \right) \end{aligned}$$     (4)

where $s(x) = 1/1 + e^{-x}$ is the standard sigmoid mapping. Here the second line derives from

assuming that the choice follows the sign of the preference $\Delta\mu = \mu_1 - \mu_2$, and the last line

derives from a moment-matching approximation to the Gaussian cumulative density function

(Daunizeau, 2017).

As stated in the Introduction section, we assume that the brain valuation system

automatically generates uncertain estimates of options' value (Lebreton et al., 2009, 2015),

before cognitive effort is invested in decision making. In what follows, $\mu_i^0$ and $\sigma_i^0$ are the mode

and variance of the ensuing prior value representation (we treat them as inputs to the MCD

model). We also assume that this prior representation neglects existing value-relevant

information that would require cognitive effort to be retrieved and processed (Lopez-Persem et

al., 2016).

Now, how does the system anticipate the benefit of allocating resources to the decision

process? Recall that the purpose of allocating resources is to process (yet unavailable) value-

relevant information. The critical issue is thus to predict how both the uncertainty $\sigma_i$ and the

modes $\mu_i$ of value representations will change, before having allocated the resources (i.e.,

without having processed the information). In brief, allocating resources essentially has two

impacts: (i) it decreases the uncertainty $\sigma_i$, and (ii) it perturbs the modes $\mu_i$ in a stochastic

manner.

The former impact derives from assuming that the amount of information that will be

processed increases with the amount of allocated resources. Here, this implies that the variance

of a given probabilistic value representation decreases in proportion to the amount of allocated

effort, i.e.:

$$\sigma_i \triangleq \sigma_i(z) = \frac{1}{\dfrac{1}{\sigma_i^0} + \beta z} \tag{5}$$

where $\sigma_i^0$ is the prior variance of the representation (before any effort has been allocated), and

$\beta$ controls the efficacy with which resources increase the precision of value representations.

Formally speaking, Equation 5 has the form of a Bayesian update of the belief's precision in a

Gaussian-likelihood model, where the precision of the likelihood term is $\beta z$. More precisely, $\beta$

is the precision increase that follows from allocating a unitary amount of resources $z$. In what

follows, we will refer to $\beta$ as the "*type #1 effort efficacy*".

The latter impact follows from acknowledging the fact that the system cannot know how

processing more value-relevant information will affect its preference before having allocated

the corresponding resources. Let $\delta_i(z)$ be the change in the position of the mode of the $i^{\text{th}}$

value representation, having allocated allocating an amount $z$ of resources. The direction of the

mode's perturbation $\delta_i(z)$ cannot be predicted because it is tied to the information that would

be processed. However, a tenable assumption is to consider that the magnitude of the

perturbation increases with the amount of information that will be processed. This reduces to

stating that the variance of $\delta_i(z)$ increases in proportion to $z$, i.e.:

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$$
\begin{aligned}
\mu_i(z) &= \mu_i^0 + \delta_i \\
\delta_i &\sim N(0, \gamma z)
\end{aligned}
\tag{6}
$$

where $\mu_i^0$ is the mode of the value representation before any effort has been allocated, and $\gamma$

controls the relationship between the amount of allocated resources and the variance of the

perturbation term $\delta$. The higher $\gamma$, the greater the expected perturbation of the mode for a

given amount of allocated resources. In what follows, we will refer to $\gamma$ as the "*type #2 effort*

*efficacy*". Note that Equation 6 treats the impact of future information processing as a non-

specific random perturbation on the mode of the prior value representation. Our justification

for this assumption is twofold: (i) it is simple, and (ii) and it captures the idea that the MCD

controller does not know how the allocated resources will be used (here, by the value-based

decision system downstream). We will see that, in spite of this, the MCD controller can still

make quantitative predictions regarding the expected benefit of allocating resources.

Taken together, Equations 5 and 6 imply that predicting the net effect of allocating

resources onto choice confidence is not trivial. On the one hand, allocating effort will increase

the precision of value representations (cf. Equation 5), which mechanically increases choice

confidence, all other things being equal. On the other hand, allocating effort can either increase

or decrease the absolute difference $\left|\Delta\mu(z)\right|$ between the modes. This, in fact, depends upon the

sign of the perturbation terms $\delta$, which are not known in advance. Having said this, it is

possible to derive the *expected* absolute difference between the modes that would follow from

allocating an amount $z$ of resources:

$$E\left[\left|\Delta\mu\right| \middle| z\right] = 2\sqrt{\frac{\gamma z}{\pi}}\exp\left(-\frac{\left|\Delta\mu^0\right|^2}{4\gamma z}\right) + \Delta\mu^0\left(2\times s\left(\frac{\pi\,\Delta\mu^0}{\sqrt{6\gamma z}}\right)-1\right) \qquad (7)$$

where we have used the expression for the first-order moment of the so-called "folded normal

distribution", and the second term in the right-hand side of Equation 7 derives from the same

moment-matching approximation to the Gaussian cumulative density function as above. The

expected absolute means' difference $E\left[\left|\Delta\mu\right| \middle| z\right]$ depends upon both the absolute prior mean

difference $\left|\Delta\mu^0\right|$ and the amount of allocated resources $z$. This is depicted on Figure 2 below.

**Figure 2. The expected impact of allocated resources onto value representations.** Left: the expected absolute mean difference $E\left[\left|\Delta\mu\right|\middle| z\right]$ (y-axis) is plotted as a function of the absolute prior mean difference $\left|\Delta\mu^0\right|$ (x-axis) for different amounts $z$ of allocated resources (color code), having set type #2 effort efficacy to unity (i.e. $\gamma=1$). Right: Variance $V\left[\left|\Delta\mu\right|\middle| z\right]$ of the absolute mean difference ; same format.

One can see that $E\left[\left|\Delta\mu\right|\middle| z\right]-\left|\Delta\mu^0\right|$ is always greater than 0 and increases with $z$ (and if $z=0$, then $E\left[\left|\Delta\mu\right|\middle| z\right]=\left|\Delta\mu^0\right|$ ). In other words, allocating resources is expected to increase the value difference, despite the fact that the impact of the perturbation term can go either way. In addition, the expected gain in value difference afforded by allocating resources decreases with the absolute prior means' difference.

Similarly, the variance $V\left[\left|\Delta\mu\right|\middle| z\right]$ of the absolute means' difference is derived from the expression of the second-order moment of the corresponding folded normal distribution:

$$V\left[\left|\Delta\mu\right|\middle| z\right]=2\gamma z+\left|\Delta\mu^0\right|^2-E\left[\left|\Delta\mu\right|\middle| z\right]^2 \tag{8}$$

259     One can see on Figure 2 that $V\left[\left|\Delta\mu\right|\middle|z\right]$ increases with the amount $z$ of allocated

260     resources (but if $z=0$, then $V\left[\left|\Delta\mu\right|\middle|z\right]=0$).

261     Knowing the moments of the distribution of $\left|\Delta\mu\right|$ now enables us to derive the expected

262     confidence level $\bar{P}_c(z)$ that would result from allocating the amount of resource $z$ :

$$
\begin{aligned}
\bar{P}_c(z) &\triangleq E\left[P_c\middle|z\right] \\
&= E\left[s\left(\frac{\pi\left|\Delta\mu\right|}{\sqrt{6\sigma(z)}}\right)\middle|z\right] \\
&\approx s\left(\frac{\pi E\left[\left|\Delta\mu\right|\middle|z\right]}{\sqrt{6\left(\sigma(z)+\frac{1}{2}V\left[\left|\Delta\mu\right|\middle|z\right]\right)}}\right)
\end{aligned}
\tag{9}
$$

where we have assumed, for the sake of conciseness, that both prior value representations are

similarly uncertain (i.e., $\sigma_1^0 \approx \sigma_2^0 \triangleq \sigma^0$). It turns out that the expected choice confidence $\bar{P}_c(z)$

always increase with $z$, irrespective of the efficacy parameters $\beta$ and $\gamma$. These, however,

control the magnitude of the confidence gain that can be expected from allocating an amount $z$

of resources. Equation 9 is important, because it quantifies the expected benefit of resource

allocation, before having processed the ensuing value-relevant information. More details

regarding the accuracy of Equation 9 can be found in the Supplementary Material.

To complete the cost-benefit model, and without loss of generality, we will assume that

the cost of allocating resources to the decision process linearly scales with the amount of

resources, i.e.:

$C(z) = \alpha z$ (10)

where $\alpha$ determines the effort cost of allocating a unitary amount of resources $z$. In what

follows, we will refer to $\alpha$ as the "effort unitary cost". We note that weak nonlinearities in the

cost function (e.g., quadratic terms) would not qualitatively change the model predictions.

In brief, the MCD-optimal resource allocation $\hat{z} \triangleq \hat{z}(\alpha, \beta, \gamma)$ is simply given by:

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$$\hat{z} = \arg\max_{z}\left[ R \times \bar{P}_c(z) - \alpha z \right]$$
            (11)

which does not have any closed-form analytic solution. Nevertheless, it can easily be identified

numerically, having replaced Equations 7-9 into Equation 11. We refer the readers interested in

the impact of model parameters $\{\alpha, \beta, \gamma\}$ on the MCD-optimal control to the Supplementary

Material.

At this point, we do not specify how Equation 11 is solved by neural networks in the

brain. Many alternatives are possible, from gradient ascent (Seung, 2003) to winner-take-all

competition of candidate solutions (Mao and Massaquoi, 2007). We will comment on the

specific issue of prospective (offline) versus reactive (online) MCD processes in the Discussion

section.

<u>Note</u>: implicit in the above model derivation is the assumption that the allocation of

resources is similar for both alternative options in the choice set (i.e. $z_1 \approx z_2 \triangleq z$). This

simplifying assumption is justified by eye-tracking data (cf. Supplementary Material).

**Corollary predictions of the MCD model**

In the previous section, we derived the MCD-optimal resource allocation, which

effectively best balances the expected choice confidence with the expected effort costs, given

the predictable impact of stochastic perturbations that arise from processing value-relevant

information. This quantitative prediction is effectively shown on Figure 5 (and/or Figure S4 of

the Supplementary Material), as a function of (empirical proxies for) the prior absolute

difference between modes $\left|\Delta\mu^0\right|$ and the prior certainty $1/\sigma^0$ of value representations. But, this

mechanism has a few interesting corollary implications.

To begin with, note that knowing $\hat{z}$ enables us to predict what confidence level the

system should reach. In fact, one can define the MCD-optimal confidence level as the expected

confidence evaluated at the MCD-optimal amount of allocated resources, i.e., $\bar{P}_c(\hat{z})$. This is

important, because it implies that the model can predict both the effort the system invests and

its associated confidence, on a decision-by-decision basis. This quantitative prediction is shown

on Figure 6.

Similarly, one can predict the MCD-optimal probability of changing one's mind. Recall

that the probability $Q(z)$ of changing one's mind depends on the amount of allocated

resources $z$, i.e.:

$$
\begin{aligned}
Q(z) &\triangleq P\left(sign(\Delta\mu) \neq sign(\Delta\mu^0)\middle|z\right) \\
309 \quad &= \begin{cases} P(\Delta\mu > 0|z) & \text{if } \Delta\mu^0 < 0 \\ P(\Delta\mu < 0|z) & \text{if } \Delta\mu^0 > 0 \end{cases} \\
&\approx s\left(-\frac{\pi\left|\Delta\mu^0\right|}{\sqrt{6\gamma z}}\right)
\end{aligned}
\tag{12}
$$

One can see that the MCD-optimal probability of changing one's mind $Q(\hat{z})$ is a simple

monotonic function of the allocated effort $\hat{z}$. Note that, by definition, choice accuracy (i.e.,

congruence of choice and prior preference $\Delta\mu^0$) is but $1-Q(z)$, which is shown on Figure 4

below.

Lastly, we can predict choice-induced preference change, i.e., how value representations

are supposed to spread apart during the decision. Such an effect is typically measured in terms

of the so-called "spreading of alternatives" or SoA, which is defined as follows:

$$
\begin{aligned}
SOA &= \left( \mu_{chosen}^{(post-choice)} - \mu_{unchosen}^{(post-choice)} \right) - \left( \mu_{chosen}^{(pre-choice)} - \mu_{unchosen}^{(pre-choice)} \right) \\
&= \begin{cases} \Delta\mu(z) - \Delta\mu^0 & \text{if } \Delta\mu(z) > 0 \\ \Delta\mu^0 - \Delta\mu(z) & \text{if } \Delta\mu(z) < 0 \end{cases} \\
&= \begin{cases} \Delta\delta(z) & \text{if } \Delta\delta(z) > -\Delta\mu^0 \\ -\Delta\delta(z) & \text{if } \Delta\delta(z) < -\Delta\mu^0 \end{cases}
\end{aligned}
\tag{13}
$$

318    where $\Delta\delta(z) \sim N(0, 2\gamma z)$ is the cumulative perturbation term of the modes' difference. Taking

319    the expectation of the right-hand term of Equation 13 under the distribution of $\Delta\delta(z)$ and

320    evaluating it at $z = \hat{z}$ now yields the MCD-optimal spreading of alternatives $\overline{SOA}(\hat{z})$:

$$
\begin{aligned}
\overline{SOA}(\hat{z}) &= E\left[ SOA | \hat{z} \right] \\
&= E\left[ \Delta\delta(\hat{z}) | \Delta\delta(\hat{z}) > -\Delta\mu^0 \right] P\left( \Delta\delta(\hat{z}) > -\Delta\mu^0 \right) \\
&\quad - E\left[ \Delta\delta(\hat{z}) | \Delta\delta(\hat{z}) < -\Delta\mu^0 \right] P\left( \Delta\delta(\hat{z}) < -\Delta\mu^0 \right) \\
&= 2\sqrt{\frac{\gamma\hat{z}}{\pi}} \exp\left( -\frac{|\Delta\mu^0|^2}{4\gamma\hat{z}} \right)
\end{aligned}
\tag{14}
$$

where the last line derives from the expression of the first-order moment of the truncated

Gaussian distribution. Note that the expected preference change also increases monotonically

with the allocated effort $\hat{z}$.

**Correspondence between model variables and empirical measures**

In summary, the MCD model predicts choice accuracy, choice confidence, choice-

induced preference change, response time and/or subjective feelings of effort, given the prior

absolute difference between modes $|\Delta\mu^0|$ and the prior certainty $1/\sigma^0$ of value

representations. The inputs to the model are the prior moments of value representations,

whose trial-by-trial variations determine variations in model predictions. Here, we simply

assume that pre-choice value and value certainty ratings provide us with an approximation of

these prior moments. More precisely, we use $\Delta VR^0$ and $CVR^0$ as empirical proxies for $\Delta\mu^0$ and

$1/\sigma^0$, respectively. Accordingly, we consider post-choice value and value certainty ratings as

empirical proxies for the posterior mean $\mu(\hat{z})$ and precision $1/\sigma(\hat{z})$ of value representations,

at the time when the decision was triggered (i.e., after having invested the effort $\hat{z}$). Similarly,

we match expected choice confidence $\bar{P}_c(z)$ (i.e., after having invested the effort $\hat{z}$) with

empirical choice confidence.

Note that the MCD model does not specify what the allocated resources are. In principle,

both mnesic and attentional resources may be engaged when processing value-relevant

information. At this point, what really matters is assessing the magnitude $z$ of decision-related

effort. We think of $z$ as the cumulative engagement of neurocognitive resources, which varies

both in terms of duration and intensity. Empirically, we relate $\hat{z}$ to two different measures,

namely: subjective feeling of effort and response time. The former relies on the subjective cost

incurred when deploying neurocognitive resources, which would be signaled by experiencing

mental effort. The latter makes sense if one thinks of response time in terms of effort duration.

Although it is a more objective measurement than subjective rating of effort, response time

only approximates $\hat{z}$ if effort intensity shows relatively small variations. We will comment on

this in the Discussion section.

Finally, the MCD model is also agnostic about the definition of "decision importance", i.e.

the weight $R$ in Equation 2. In this work, we simply investigate the effect of decision

importance by comparing subjective effort and response time in "neutral" versus

"consequential" decisions (cf. Methods section). We will also comment on this in the Discussion

section.

**METHODS**

Participants for our study were recruited from the RISC (*Relais d'Information sur les*

*Sciences de la Cognition*) subject pool through the ICM (*Institut du Cerveau et de la Moelle*

*épinière*). All participants were native French speakers. All participants were from the non-

patient population with no reported history of psychiatric or neurological illness.

Written instructions provided detailed information about the sequence of tasks within

the experiment, the mechanics of how participants would perform the tasks, and images

illustrating what a typical screen within each task section would look like. The experiment was

developed using Matlab and PsychToolbox. The experiment was conducted entirely in French.

Eye gaze position and pupil size were continuously recorded throughout the duration of

the experiment using The Eye Tribe eye tracking devices. Participants' head positions were fixed

using stationary chinrests. In case of incidental movements, we corrected the pupil size data for

distance to screen, separately for each eye.

**Participants**

A total of 41 people (28 female; age: mean=28, stdev=5, min=20, max=40) participated in

this study. The experiment lasted approximately 2 hours, and each participant was paid a flat

rate of 20€ as compensation for his time plus a bonus, which was given to participants to

compensate for potential financial losses in the "penalized" trials (see below). More precisely, in

"penalized" trials, participants lose 0.20€ (out of a 5€ bonus) for each second that they took to

make their choice. This resulted in an average 4€ bonus (across participants). One group of 11

participants was excluded from the cross-condition analysis only (see below), due to technical

issues.

**Materials**

The stimuli for this experiment were 148 digital images, each representing a distinct food

item (50 fruits, 50 vegetables, 48 various snack items including nuts, meats, and cheeses). Food

items were selected such that most items would be well known to most participants.

**Procedure**

Prior to commencing the testing session of the experiment, participants underwent a

brief training session. The training tasks were identical to the experimental tasks, although

different stimuli were used (beverages). The experiment itself began with an initial section

where all individual items were displayed in a random sequence for 1.5 seconds each, in order

to familiarize the participants with the set of options they would later be considering and form

an impression of the range of subjective value for the set. The main experiment was divided into

three sections, following the classic Free-Choice Paradigm protocol (e.g., Izuma and Murayama,

2013): pre-choice item ratings, choice, and post-choice item ratings. There was no time limit for

the overall experiment, nor for the different sections, nor for the individual trials. Item rating

and choice sessions are described below (see Figure 3).

**Figure 3. Experimental design**. Left: pre-choice item rating session: participants are asked to rate how much they like each food item and how certain they are about it (value certainty rating). Center: choice session: participants are asked to choose between two food items, to rate how confident they are about their choice, and to report the feeling of effort associated with the decision. Right: post-choice item rating session (same as pre-choice item rating session).

Item rating (same for pre-choice and post-choice sessions): Participants were asked to rate the entire set of items in terms of how much they liked each item. The items were presented one at a time in a random sequence (pseudo-randomized across participants). At the onset of each trial, a fixation cross appeared at the center of the screen for 750ms. Next, a solitary image of a food item appeared at the center of the screen. Participants had to respond to the question, "How much do you like this item?" using a horizontal slider scale (from "I hate it!" to "I love it!") to indicate their value rating for the item. The middle of the scale was the point of neutrality ("I don't care about it."). Hereafter, we refer to the reported value as the "pre-choice value rating". Participants then had to respond to the question, "How certain are you about the item's value?" by expanding a solid bar symmetrically around the cursor of the value slider scale to indicate the

range of possible value ratings that would be compatible with their subjective feeling. We

measured participants' certainty about value rating in terms of the percentage of the value scale

that is not occupied by the reported range of compatible value ratings. We refer to this as the

"pre-choice value certainty rating". At that time, the next trial began.

Choice: Participants were asked to choose between pairs of items in terms of which item they

preferred. The entire set of items was presented one pair at a time in a random sequence. Each

item appeared in only one pair. At the onset of each trial, a fixation cross appeared at the center

of the screen for 750ms. Next, two images of snack items appeared on the screen: one towards

the left and one towards the right. Participants had to respond to the question, "Which do you

prefer?" using the left or right arrow key. We measured response time in terms of the delay

between the stimulus onset and the response. Participants then had to respond to the question,

"Are you sure about your choice?" using a vertical slider scale (from "Not at all!" to

"Absolutely!"). We refer to this as the report of choice confidence. Finally, participants had to

respond to the question, "To what extent did you think about this choice?" using a horizontal

slider scale (from "Not at all!" to "Really a lot!"). We refer to this as the report of subjective

effort. At that time, the next trial began.

Note: In the Results section, we refer to $\Delta VR^0$ as the difference between pre-choice value

ratings of items composing a choice set. Similarly, $CVR^0$ is the average pre-choice value certainty

ratings across items composing a choice set.

**Conditions**

The choice section of the experiment included trials of three different conditions:

"neutral" (60 trials), "consequential" (7 trials), and "penalized" (7 trials), which were randomly

intermixed. Immediately prior to each "consequential" trial, participants were instructed that

they would be required to eat, at the end of the experiment, a portion of the item that they

were about to choose. Immediately prior to each "penalized" trial, participants were instructed

that they would lose 0.20€ (out of a 5€ bonus) for each second that they would take to make

their choice.

**RESULTS**

Here, we test the predictions that the MCD model makes. We note that basic descriptive

statistics of our data, including measures of test-retest reliability and replications of previously

reported effects on confidence in value-based choices (De Martino et al., 2013), are appended

in the Supplementary Materials.

**Evaluating response time and choice confidence**

In what follows, we inspect the three-way relationships between pre-choice value and

value certainty ratings and each choice feature (namely: prediction accuracy, response time, and

confidence). At this point, suffice it to say that, under the MCD model, value uncertainty ratings

capture the precision of subjective value representations before the choice. Unless stated

otherwise, we will focus on both the absolute difference between pre-choice value ratings

(hereafter: $|\Delta VR^0|$) and the mean pre-choice value certainty rating across paired choice items

(hereafter: $VCR^0$). In each case, we will summarize the empirical data and the corresponding

MCD model prediction.

First, we checked how choice prediction accuracy relates to $|\Delta VR^0|$ and $VCR^0$. Here, we

measure choice accuracy in terms of the rate of choices that are congruent with preferences

derived from pre-choice value ratings $\Delta VR^0$. Under the MCD model, choice accuracy should

increase with both $|\Delta VR^0|$ and $VCR^0$. This is because increasing $|\Delta VR^0|$ and/or $VCR^0$ will

decrease the demand for effort, which implies that the probability of changing one's mind will

be smaller. Figure 4 below shows the quantitative model predictions and summarizes the

corresponding empirical data.

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**Figure 4. Three-way relationship between choice accuracy, value, and value certainty**. Top row:

prediction of the MCD model: left: choice accuracy is shown as a function of $|\Delta VR^0|$ (x-axis) and

$VCR^0$ (color code); right: GLM regression beta weights for $|\Delta VR^0|$ and $VCR^0$ on choice accuracy.

Bottom row: empirical data (cross-participant means): same format. (Error bars represent s.e.m.,

$|\Delta VR^0|$ p<0.001, $VCR^0$ p=0.046.)

One can see that the data seem to conform to the model's predictions. To confirm this,

we ran, for each participant, a multiple logistic regression of choice accuracy against $|\Delta VR^0|$ and

$VCR^0$. A random effect analysis shows that both have a significant positive effect at the group

level ($|\Delta VR^0|$: mean GLM beta=0.17, s.e.m.=0.02, p<0.001; $VCR^0$: mean GLM beta=0.07,

s.e.m.=0.03, p=0.004). Note that people make "inaccurate" choices either because they make

mistakes or because they change their mind during the decision. In principle, we can

discriminate between these two explanations because we can check whether "inaccurate"

choices are congruent with post-choice value ratings (change of mind) or not (error). This is

important, because accumulation-to-bound models do not allow for the possibility that value

representations change during decisions (hence all "inaccurate" choices would be deemed

"errors"). It turns out that, among "inaccurate" choices, mind changes are more frequent than

errors (mean rate difference=2.3%, s.e.m.=0.01, p=0.032). Note that analyses of mind changes

yield qualitatively identical results as choice accuracy (we refer the interested reader to

Supplementary Material).

Second, we checked how response time relates to pre-choice value and value certainty

ratings. Under the MCD model, response time can be thought of as a proxy for effort duration.

Here, increasing $|\Delta VR^0|$ and/or $VCR^0$ will decrease the demand for effort, which will result in

smaller expected response time. Figure 5 below shows the quantitative model predictions and

summarizes the corresponding empirical data.

**Figure 5. Three-way relationship between response time, value, and value certainty**. Top row: prediction of the MCD model: left: log(RT) is shown as a function of $|\Delta VR^0|$ (x-axis) and $VCR^0$ (color code); right: GLM regression beta weights for $|\Delta VR^0|$ and $VCR^0$ on log(RT). Bottom row: empirical data (cross-participant means): same format. (Error bars represent s.e.m., $|\Delta VR^0|$ p<0.001, $VCR^0$ p<0.001.)

One can see that the response time data behave as predicted by the MCD model. Here, we also ran, for each participant, a multiple linear regression of response times against $|\Delta VR^0|$ and $VCR^0$. A random effect analysis shows that both have a significant and negative effect at the group level ($|\Delta VR^0|$: mean GLM beta=-0.13, s.e.m.=0.02, p<0.001; $CR^0$: mean GLM beta=-0.06, s.e.m.=0.02, p=0.005).

Third, we checked how choice confidence relates to $|\Delta VR^0|$ and $VCR^0$. Under the MCD

model, confidence reflects the discriminability of value representations after optimal resource

allocation. Critically, although more resources are allocated to the decision when either $|\Delta VR^0|$

or $VCR^0$ decrease, this does not overcompensate for decision difficulty, and thus choice

confidence decreases. Figure 6 below shows the quantitative model predictions and summarizes

the corresponding empirical data.

**Figure 6. Three-way relationship between choice confidence, value, and value certainty**. Top

row: prediction of the MCD model: left: choice confidence is shown as a function of $|\Delta VR^0|$ (x-

axis) and $VCR^0$ (color code); right: GLM regression beta weights for $|\Delta VR^0|$ and $VCR^0$ on choice

confidence. Bottom row: empirical data (cross-participant means): same format. (Error bars

represent s.e.m., $|\Delta VR^0|$ p<0.001, $VCR^0$ p<0.001.)

One can see that the choice confidence follows the MCD model predictions. Again, we

ran, for each participant, a multiple linear regression of confidence against $|\Delta VR^0|$ and $VCR^0$. A

random effect analysis shows that both have a significant and positive effect at the group level

($|\Delta VR^0|$: mean GLM beta=0.23, s.e.m.=0.02, p<0.001; $VCR^0$: mean GLM beta=0.15, s.e.m.=0.03,

p<0.001). Note that this is unlikely to be a trivial consequence of peoples' response time

readout, since confidence is only mildly correlated with response time (mean correlation=-0.32,

s.e.m.=0.03, p<0.001).

**Subjective feeling of effort, choice-induced preference change, decision importance, and cost**

**of time**

So far, we have provided evidence that choice confidence and response time can be

explained by the MCD model. In what follows, we will evaluate some additional novel

quantitative predictions that are specific to the MCD model. The derivation of each of these

predictions is detailed in the Model section above.

First, recall that MCD really is about the allocation of costly cognitive resources, i.e.

mental effort, into the decision process. One may thus ask whether the subjective feeling of

effort *per se* follows the MCD predictions. Recall that increasing $|\Delta VR^0|$ and/or $VCR^0$ will

decrease the demand for mental resources, which will result in the decision being associated

with a lower feeling of effort. To check this, we thus performed a multiple linear regression of

subjective effort ratings against $|\Delta VR^0|$ and $VCR^0$. A random effect analysis shows that both

have a significant and negative effect at the group level ($|\Delta VR^0|$: mean GLM beta=-0.20,

s.e.m.=0.03, p<0.001; $CR^0$: mean GLM beta=-0.05, s.e.m.=0.02, p=0.025). A graphical summary

of the data can be seen in the Supplementary Material.

Second, the MCD model predicts how value representations will be modified during the

decision process. In particular, choice-induced preference change should globally follow the

optimal effort allocation. More precisely, the reported value of alternative options should

spread apart, and the expected spreading of alternatives (i.e. $[VR_{chosen} - VR_{unchosen}]^{post\text{-}choice}$ -

$[VR_{chosen} - VR_{unchosen}]^{pre\text{-}choice}$) should be decreasing with $|\Delta VR^0|$ and $VCR^0$. Figure 7 below shows

the model predictions and summarizes the corresponding empirical data.

**Figure 7. Three-way relationship between choice-induced preference change, value, and value

certainty**. Top row: prediction of the MCD model: left: SoA is shown as a function of $|\Delta VR^0|$ (x-

axis) and $VCR^0$ (color code); right: GLM regression beta weights for $|\Delta VR^0|$ and $VCR^0$ on SoA.

Bottom row: empirical data (cross-participant means): same format. (Error bars represent s.e.m.,

$|\Delta VR^0|$ p<0.001, $VCR^0$ p=0.079.)

One can see that the spreading of alternatives follows the MCD model predictions. A

random effect analysis confirms this, showing that both $|\Delta VR^0|$ and $CR^0$ have a significant

negative effect at the group level ($|\Delta VR^0|$: mean GLM beta=-0.09, s.e.m.=0.03, p<0.001; $CR^0$:

mean GLM beta=-0.04, s.e.m.=0.02, p=0.027). Note that this replicates our previous findings on

choice-induced preference change (Lee and Daunizeau, 2020; Lee and Coricelli, 2020). In

addition to expected changes in value ratings, the MCD model predicts that the precision of

value representations should increase after the decision has been made (cf. "β-effect" in the

Supplementary Material). Indeed, post-choice value certainty ratings are significantly higher

than pre-choice value certainty ratings (mean difference=1.34, s.e.m.=0.51, p=0.006).

Importantly, under the MCD model, post-choice ratings are simply reports of modified value

representations at the time when the choice is triggered. Therefore, choice and its associated

confidence level should be better predicted with post-choice ratings than with pre-choice

ratings. Indeed, we found that the predictive power of post-choice ratings is significantly higher

than that of pre-choice ratings, both for choice (mean prediction accuracy difference=7%,

s.e.m.=0.01, p<0.001) and choice confidence (mean prediction accuracy difference=3%,

s.e.m.=0.01, p=0.004). Details regarding this analysis can be found in the Supplementary

Material.

Third, the MCD model predicts that, all else being equal, effort increases with decision

importance and decreases with costs. We checked the former prediction by asking participants

to make a few decisions where they knew that the choice would be real, i.e. they would actually

have to eat the chosen food item. We refer to these trials as "consequential" decisions. To check

the latter prediction, we imposed a financial penalty that increases with response time. These

experimental manipulations are described in the Methods section. Figure 8 below shows

subjective effort ratings and response times for "neutral", "consequential" and "penalized"

decisions, when controlling for $|\Delta VR^0|$ and $VCR^0$ (see the Supplementary Material for more

details).

**Figure 8. Comparison of "neutral", "consequential", and "penalized" decisions.** Left: Mean (+/-

s.e.m.) response time are shown for "neutral" (blue), "consequential" (red) and "penalized"

(yellow) decisions. Right: Mean (+/- s.e.m.) effort rating (same format). Both datasets were

corrected for $|\Delta VR^0|$ and $VCR^0$.

One can see that subjective effort reports and response times follow the MCD model

predictions. More precisely, both subjective effort reports and response times were significantly

higher for "consequential" decisions than for "neutral" decisions (mean effort difference=0.39,

s.e.m.=0.12, p=0.001; mean response time difference=0.43, s.e.m.=0.19, p=0.017). In addition,

response times are significantly faster for "penalized" than for "neutral" decisions (mean

response time difference=-0.51, s.e.m.=0.08, p<0.001). Note that although the difference in

reported effort between "neutral" and "penalized" decisions does not reach statistical

significance (mean effort difference=-0.13, s.e.m.=0.12, p=0.147), it goes in the right direction.

**DISCUSSION**

In this work, we have presented a novel computational model of decision-making which explains the intricate relationships between choice accuracy, response time, subjective effort, choice confidence, and choice-induced preference change. This model assumes that deciding between alternative options whose values are uncertain induces a demand for allocating cognitive resources to processing value-relevant information. Cognitive resource allocation then optimally trades effort for confidence, given the discriminability of prior value representations. Such metacognitive control of decisions or MCD makes novel predictions that differ from standard accumulation-to-bound models of decision-making, including a drift-diffusion model that was proposed as an optimal policy for value-based decision making (Tajima et al., 2016). But, how can these two frameworks both be optimal? The answer lies in the distinct computational problems that they solve. The MCD solves the problem of finding the optimal amount of effort to invest under the possibility that yet-unprocessed value-relevant information might change the decider's mind. In fact, this resource allocation problem would be vacuous, would it not be possible to reassess preferences during the decision process. In contrast, the DDM provides an optimal solution to the problem of efficiently comparing option values, which may be unreliably signaled, but remain stationary nonetheless. Of course, the DDM decision variable (i.e., the "evidence" for a given choice option) may fluctuate, e.g. it may first drift towards the upper bound, and then eventually reach the lower bound (Kiani et al., 2014; Resulaj et al., 2009). But, critically, these fluctuations are not caused by changes in the underlying value signal. This is why the DDM cannot predict choice-induced preference changes, i.e. changes in

options' values. This critical distinction extends to other types of accumulation-to-bound

models, including race models (De Martino et al, 2013; Tajima et al, 2019). We note that both

these models could be equipped with pre-choice value priors, and then driven with "true"

values derived from post-choice ratings. Pre-choice to post-choice rating change would then

map to an update from prior to posterior value estimates. But then, simulating these models

would require both pre-choice and post-choice ratings, which implies that choice-induced value

changes cannot be *predicted* from pre-choice ratings.

As a side note, we acknowledge that variants of these accumulation-to-bound models

could potentially use pre-choice value and value certainty ratings differently from how current

models use them (Lee and Usher, 2020). However, distorting existing models may be

problematic because some computational variables may lose their very meaning. For example,

in the context of Tajima's DDM, if the drift rate was set to some form of certainty-weighted

value difference, then the ensuing DDM's stochastic state could not be mapped to the posterior

mean of a bayesian belief on value. In turn, the model predictions would not be optimally

solving the value-comparison problem.

Now, let us highlight that the MCD model offers a plausible alternative interpretation for

the two main reported neuroimaging findings regarding confidence in value-based choices (De

Martino et al., 2013). First, the ventromedial prefrontal cortex or vmPFC was found to respond

positively to both value difference (i.e., $\Delta VR^0$) and choice confidence. Second, the right

rostrolateral prefrontal cortex or rRLPFC was more active during low-confidence versus high-

confidence choices. These findings were originally interpreted through the framework of the

race model that we compared to the MCD model. In brief, rRLPFC was thought to perform a

readout of choice confidence (for the purpose of subjective metacognitive report) from the

racing value accumulators hosted in the vmPFC. Under the MCD framework, the contribution of

the vmPFC to value-based choices might rather be to anticipate and monitor the benefit of

effort investment (i.e., confidence). This would be consistent with recent fMRI studies

suggesting that vmPFC confidence computations signal the attainment of task goals (Hebscher

and Gilboa, 2016; Lebreton et al., 2015). Now, recall that the MCD model predicts that

confidence and effort should be anti-correlated. Thus, the puzzling negative correlation

between choice confidence and rRLPFC activity could be simply explained under the assumption

that rRLPFC provides the neurocognitive resources that are instrumental for processing value-

relevant information during decisions. This resonates with the known involvement of rRLPFC in

reasoning (Desrochers et al., 2015; Dumontheil, 2014) or memory retrieval (Benoit et al., 2012;

Westphal et al., 2019).

At this point, we would like to discuss a few features of the MCD model. First, we did not

specify what determines the reward component, which quantifies decision importance and acts

as an effective weight for confidence against effort costs (cf. $R$ in Equation 2 of the Model

section). We know, from the comparison of "consequential" and "neutral" choices that

increasing decision importance eventually increases effort, as predicted by the MCD model.

However, decision importance may have many determinants, such as, for example, the

commitment time of the decision (cf. partner choices), the breadth of its repercussions (cf.

political decisions), or its instrumentality with respect to the achievement of superordinate

goals (cf. moral decisions). How these determinants are combined and/or moderated by the

decision context is virtually unknown (Locke and Latham, 2002, 2006). In addition, decision

importance might also be influenced by the prior (intuitive/emotional/habitual) appraisal of

option values. For example, we found that, all else equal, people spent much more time and

effort deciding between two disliked items than between two liked items (results not shown).

This reproduces recent results regarding the evaluation of choice sets (Shenhav and Karmarkar,

2019). Probing this type of influence will be the focus of forthcoming publications.

Second, our current version of the MCD model relies upon a simple variant of resource

costs. We note that rendering the cost term nonlinear (e.g., quadratic) does not change the

qualitative nature of the MCD model predictions. More problematic, perhaps, is the fact that we

did not consider distinct types of effort, which could, in principle, be associated with different

costs. For example, the cost of allocating attention to a given option may depend upon whether

this option would be *a priori* chosen or not. This might eventually explain systematic decision

biases and differences in response times between default and non-default choices (Lopez-

Persem et al., 2016). Another possibility is that effort might be optimized along two canonical

dimensions, namely: duration and intensity. The former dimension essentially justifies the fact

that we used response time as a proxy for the amount of allocated resources. This is because, if

effort intensity stays constant, then longer response times essentially signal a greater resource

expenditure. In fact, as is evident from the comparison between "penalized" and "neutral"

choices, imposing an external penalty cost on response time reduces, as expected, the ensuing

subjective effort. More generally, however, the dual optimization of effort dimensions might

render the relationship between effort and response time more complex. For example, beyond

memory span or attentional load, effort intensity could be related to processing speed. This

would explain why, although "penalized" choices are made much faster than "neutral" choices,

the associated feeling of effort is not as strongly impacted (cf. Figure 8). In any case, the

relationship between effort and response time might depend upon the relative costs of effort

duration and intensity, which might itself be partially driven by external availability constraints

(cf. time pressure or multitasking). We note that the essential nature of the cost of mental effort

in cognitive tasks (e.g., neurophysiological cost, interferences cost, opportunity cost) is still a

matter of intense debate (Kurzban et al., 2013; Musslick et al., 2015; Ozcimder et al., 2017).

Progress towards addressing this issue will be highly relevant for future extensions of the MCD

model.

Third, we did not consider the issue of identifying plausible neuro-computational

implementations of MCD. This issue is tightly linked to the previous one, in that distinct cost

types would likely impose different constraints on candidate neural network architectures (Feng

et al., 2014; Petri et al., 2017). For example, underlying brain circuits are likely to operate MCD

in a more reactive manner, eventually adjusting resource allocation from the continuous

monitoring of relevant decision variables (e.g., experienced costs and benefits). Such a reactive

process contrasts with our current, prospective-only, variant of MCD, which sets resource

allocation based on anticipated costs and benefits. We already checked that simple reactive

scenarios, where the decision is triggered whenever the online monitoring of effort or

confidence reaches the optimal threshold, make predictions qualitatively similar to those we

have presented here. We tend to think however, that such reactive processes should be based

upon a dynamic programming perspective on MCD, as was already done for the problem of

optimal efficient value comparison (Tajima et al., 2016, 2019). We will pursue this and related

neuro-computational issues in subsequent publications.

**DATA AVAILABILITY**

The data that support the findings of this study are available from the corresponding

author upon reasonable request.

**CODE AVAILABILITY**

The computer code and algorithms that support the findings of this study are available

from the corresponding author upon reasonable request.

**ETHICAL COMPLIANCE**

This study complies with all relevant ethical regulations and conforms to the guidelines

established by our local ethics committee. In accordance with the Helsinki declaration, all

participants gave informed consent prior to commencing the experiment.

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 **Trading mental effort for confidence: supplementary material**

**1. Data descriptive statistics and sanity checks**

Recall that we collect value ratings and value certainty ratings both before and after the

choice session. We did this for the purpose of validating specific predictions of the MCD model

(in particular: choice-induced preference changes: see Figure 7 of the main text). It turns out

this also enables us to assess the test-retest reliability of both value and value certainty ratings.

We found that both ratings were significantly reproducible (value: mean correlation=0.88,

s.e.m.=0.01, p <0.001, value certainty: mean correlation=0.37, s.e.m.=0.04, p <0.001).

We also checked whether choices were consistent with pre-choice ratings. For each

participant, we thus preformed a logistic regression of choices against the difference in value

ratings. We found that the balanced prediction accuracy was beyond chance level (mean

accuracy=0.68, s.e.m.=0.01, p<0.001).

**2. Does choice confidence moderate the relationship between choice and pre-choice**

**value ratings?**

Previous studies regarding confidence in value-base choices showed that choice

confidence moderates choice prediction accuracy (De Martino et al., 2012). We thus splat our

logistic regression of choices into high- and low-confidence trials, and tested whether higher

confidence was consistently associated with increased choice accuracy. A random effect analysis

showed that the regression slopes were significantly higher for high- than for low-confidence

trials (mean slope difference=0.14, s.e.m.=0.03, p<0.001). For the sake of completeness, the

 impact of choice confidence on the slope of the logistic regression (of choice onto the difference

in pre-choice value ratings) is shown on Figure S1 below.

**Figure S1. Relationship between choices, pre-choice value ratings and choice confidence.** Left:

the probability of choosing the item on the right (y-axis) is shown as a function of the pre-choice

value difference (x-axis), for high- (blue) versus low- (red) confidence trials. The plain lines show

the logistic prediction that would follow from group-averages of the corresponding slope

estimates. Right: the corresponding logistic regression slope (y-axis) is shown for both high-

(blue) and low- (red) confidence trials (group means +/- s.e.m.).

These results clearly replicate the findings of De Martino and colleagues (2012), which

were interpreted with a race model variant of the accumulation-to-bound principle. We note,

however, that this effect is also predicted by the MCD model. Here, variations in both (i) the

prediction accuracy of choice from pre-choice value ratings, and (ii) choice confidence, are

driven by variations in resource allocation. In brief, the expected magnitude of the perturbation

of value representations increases with the amount of allocated resources. This eventually

degrades the prediction accuracy of choice from pre-choice value ratings (which have been

changed during the decision process). However, although more resources are allocated to the

decision, this does not overcompensate for decision difficulty, and thus choice confidence

decreases. Thus, low-confidence choices will be those choices that cannot be well predicted

with pre-choice value ratings. We note that the anti-correlation between choice confidence and

choice accuracy can be seen by comparing Figures 4 and 6 of the main text.

**3. How do choice confidence, difference in pre-choice value ratings, and decision time**

**relate to each other?**

In the main text, we show that trial-by-trial variations in choice confidence is

concurrently explained by both pre-choice value and value certainty ratings. Here, we reproduce

previous findings relating choice confidence to both absolute value difference $\Delta VR^0$ and decision

time (De Martino et al., 2012). First, we regressed, for each participant, decision time

concurrently against both $|\Delta VR^0|$ and choice confidence. A random effect showed that both

have a significant main effect on decision time ($\Delta VR^0$: mean GLM beta=-0.016, s.em.=0.003,

p<0.001; choice confidence: mean GLM beta=-0.014, s.em.=0.002; p<0.001), without any two-

way interaction (p=0.133). This analysis is summarized in Figure S2 below, together with the full

three-way relationship between $|\Delta VR^0|$, confidence and decision time.

**Figure S2. Relationship between pre-choice value ratings, choice confidence and decision times.** Left: decision times (y-axis) are plotted as a function of low- and high- $|\Delta VR^0|$ (x-axis) for both low- (red) and high- (blue) confidence trials. Errorbar represent s.e.m. Right: A heatmap of mean z-scored confidence is shown as a function of both decision time (x-axis) and $|\Delta VR^0|$ (y-axis).

In brief, confidence increases with the absolute value difference and decreases with decision time. This effect is also predicted by the MCD model, for reasons identical to the explanation of the relationship between confidence and choice accuracy (see above). Recall that, overall, an increase in choice difficulty is expected to yield an increase in decision time and a decrease in choice confidence. This would produce the same data pattern as Figure S2, although the causal relationships implicit in this data representation is partially incongruent with the computational mechanisms underlying MCD.

## 4. Analysis of changes of mind

In the main manuscript, we show that choice accuracy increases with pre-choice value difference $\Delta VR^0$ and pre-choice value certainty $VCR^0$. Recall that choice accuracy was defined in terms of the rate of choices that are congruent with preferences derived from pre-choice value

ratings. Now people make "inaccurate" choices either because they make mistakes or because

they change their mind during the decision. In principle, we can discriminate between these two

explanations because we can check whether "inaccurate" choices are congruent with post-

choice value ratings (change of mind) or not (error). This is important, because accumulation-to-

bound models do not allow for the possibility that value representations change during

decisions. Hence all "inaccurate" choices would be deemed "errors", which are driven by

stochastic noise in the evidence accumulation process. It turns out that most choices are

"accurate" (mean choice accuracy =73.3%, s.e.m.=1%), and less than half of the "inaccurate"

choices are classified as "errors" (mean error rate=12%, s.e.m.=0.01), which is significantly less

than "mind changes" (mean rate difference=2%, s.e.m.=0.01, p=0.032). In addition, choice

confidence and (post- versus pre-choice) value certainty gain were significantly higher for

"changes of mind" than for "errors" (choice confidence: mean difference=13.7, s.e.m.=2.1,

p<0.001; value certainty gain: mean difference=2.6, s.e.m.=1.4, p=0.035).

Thus, one may wonder what the impacts of both pre-choice value difference $\Delta VR^0$ and

pre-choice value certainty $VCR^0$ on choice accuracy would be, if one were to remove "errors"

from "inaccurate" choices. Figure S3 below shows both the predicted and measured three-way

relationship between the probability of changing one's mind, $\Delta VR^0$ and $VCR^0$.

**Figure S3. Relationship between the probability of changing one's mind, value ratings, and certainty ratings.** Top row: prediction of the MCD model: left: changing of mind frequency is shown as a function of $|\Delta VR^0|$ (x-axis) and $VCR^0$ (color code); right: GLM regression beta weights for $|\Delta VR^0|$ and $VCR^0$ on changing of mind frequency. Bottom row: empirical data (cross-participant means): same format. (Error bars represent s.e.m., $|\Delta VR^0|$ p<0.001, $VCR^0$ p=0.006.)

Recall that, under the MCD model, the probability of changing one's mind increases with the resource demand, which decreases when either $|\Delta VR^0|$ or $VCR^0$ increase. One can see that the data seem to conform to this prediction. To check this, we ran, for each participant, a multiple logistic regression of change of mind against $|\Delta VR^0|$ and $VCR^0$. A random effect analysis shows that both have a significant and negative effect at the group level ($\Delta VR^0$: mean GLM beta=-0.16, s.e.m.=-0.02, p<0.001; $VCR^0$: mean GLM beta=-0.08, s.e.m.=0.02, p<0.001).

These results are qualitatively similar to the analysis of choice accuracy (cf. Figure 4 in the main

text).

**5.  Analysis of the subjective feeling of effort**

In the main manuscript, we show that decision time decreases with pre-choice value

difference $\Delta VR^0$ and pre-choice value certainty $VCR^0$. The focus on decision time was motivated

by the fact that all models could make quantitative -and thus comparable- predictions. In brief,

we found that the effect of $VCR^0$ on decision time was consistent with the MCD model, but not

with accumulation-to-bound models. Now, under the MCD model, decision time is but a proxy

for effort duration. Here, we ask whether the subjective feeling of effort *per se* follows the MCD

model predictions. This is possible because we asked participants to rate how effortful each

decision felt. Figure S4 below shows both the predicted and the measured three-way

relationship between effort, $|\Delta VR^0|$ and $VCR^0$.

**Figure S4. Relationship between subjective effort, value ratings, and certainty ratings.** Top row: prediction of the MCD model: left: subjective effort is shown as a function of $|\Delta VR^0|$ (x-axis) and $VCR^0$ (color code); right: GLM regression beta weights for $|\Delta VR^0|$ and $VCR^0$ on subjective effort. Bottom row: empirical data (cross-participant means): same format. (Error bars represent s.e.m., $|\Delta VR^0|$ $p<0.001$, $VCR^0$ $p=0.021$.)

One can see that the reported subjective feeling of effort closely matches model predictions. One may ask whether people's effort reports may be a trivial post-choice read-outs of decision time and/or choice confidence. This, however, is unlikely, given that people's subjective effort is reducible neither to decision time (mean correlation=0.39, s.e.m.=0.04), nor to choice confidence (mean correlation=-0.48, s.e.m.=, p=0.05).

## 6. Do post-choice ratings better predict choice and choice confidence than pre-choice ratings?

The MCD model assumes that value representations are modified during the decision process, until the MCD-optimal amount of resources is met. This eventually triggers the decision, whose properties (i.e. which alternative option is eventually preferred, and with which confidence level) then reflects the modified value representations. If post-choice ratings are reports of modified value representations at the time when the choice is triggered, then choice and its associated confidence level should be better predicted with post-choice ratings than with pre-choice ratings. In what follows, we test this prediction.

In the first section of this Supplementary Material, we report the result of a logistic regression of choice against pre-choice value ratings (see also Figure S1). We performed the same regression analysis, but this time against post-choice value ratings. Figure S5 below shows the ensuing predictive power (here, in terms of balanced accuracy or BA) for both pre-choice and post-choice ratings. The main text also features the result of a multiple linear regression of choice confidence ratings onto $|\Delta VR^0|$ and $VCR^0$ (cf. Figure 6). Again, we performed the same regression, this time against post-choice ratings. Figure S5 below shows the ensuing predictive power (here, in terms of percentage of explained variance or $R^2$) for both pre-choice and post-choice ratings.

**Figure S5. Comparison of the predictive power of pre-choice versus post-choice ratings.** Left: Mean (+/- s.e.m.) BA of logistic regressions of choice against pre-choice (left) and post-choice (right) value ratings. Right: Mean (+/- s.e.m.) $R^2$ of multiple linear regressions of choice confidence against pre-choice (left) and post-choice (right) ratings.

A simple random effect analysis shows that the predictive power of post-choice ratings is significantly higher than that of pre-choice ratings, both for choice (mean difference in BA=7%, s.e.m.=0.01, $p<0.001$) and choice confidence (mean difference in $R^2$=3%, s.e.m.=0.01, p=0.004).

## 7. Cross-condition analysis: decision importance and cost of decision time

As featured in the main manuscript, we intermixed "neutral" trials with two specific sets of trials, in which we either manipulated decision importance (cf. "consequential" decisions) or the cost of decision time (cf. "penalized" decisions). Figure S6 below shows the mean subjective effort ratings and decision times for "neutral", "consequential" and "penalized" decisions.

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**Figure S6. Comparison of "neutral", "consequential" and "penalized" decisions.** Left: Mean (+/- s.e.m.) response time are shown for "neutral" (blue), "consequential" (red) and "penalized" (yellow) decisions. Right: Mean (+/- s.e.m.) effort rating (same format).

Overall, the data partially follows the model predictions. In particular, subjective effort and decision time are both significantly higher for "consequential" than for "neutral" decisions (effort: mean difference=9.0, s.e.m.=2.2, p<0.001; DT: mean difference=0.56, s.e.m.=0.32, p=0.043). In addition, decision time is significantly lower for "penalized" than for "neutral" decision (mean DT difference=-0.46, s.e.m.=0.08, p<0.001). However, there is no noticeable difference between reported efforts in "neutral" and "penalized" decisions (mean effort difference=0.6, s.e.m.=2.1, p=0.604).

This comparison however, may be confounded by between-condition differences in $\Delta VR^0$ or $VCR^0$. For each participant, we thus performed a multiple linear regression of effort and DT onto $|\Delta VR^0|$ and $VCR^0$, including all types of trials. Corrected effort and DT can now be compared, after having removed the effects of $|\Delta VR^0|$ and $VCR^0$. This is what Figure 8 of the main text shows. As one can see, the overall pattern is similar to Figure S6. As before, subjective effort and decision time are both significantly higher for "consequential" than for "neutral"

1032  decisions (effort: mean GLM beta difference=0.39, s.e.m.=0.12, p=0.001; DT: mean GLM beta

1033  difference=0.43, s.e.m.=0.19, p=0.017), and decision time is significantly lower for "penalized"

1034  than for "neutral" decisions (mean DT GLM beta difference=-0.51, s.e.m.=0.08, p<0.001). Finally,

1035  the difference between reported efforts in "neutral" and "penalized" decisions is now almost

1036  significant (mean effort GLM beta difference=-0.13, s.e.m.=0.12, p=0.147).

1037

**8. Analysis of eye-tracking data**

We first checked whether pupil dilation positively correlates with participants' reports of

subjective effort. We epoched the pupil size data into trial-by-trial time series, and temporally

co-registered the epochs either at stimulus onset (starting 1.5 seconds before the stimulus

onset and lasting 5 seconds) or at choice response (starting 3.5 seconds before the choice

response and lasting 5 seconds). Data was baseline-corrected at stimulus onset. For each

participant, we then regressed, at each time point during the decision, pupil size onto effort

ratings (across trials). Time series of regression coefficients were then reported at the group

level, and tested for statistical significance (correction for multiple comparison was performed

using random field theory 1D-RFT). Figure S7 below summarizes this analysis, in terms of the

baseline-corrected time series of regression coefficients.

**Figure S7. Correlation between pupil size and reports of subjective effort during decision time.**
Left: Mean (+/- s.e.m.) correlation between pupil size and subjective effort (y-axis) is plotted as a function of peristimulus time (x-axis). Here, epochs are co-registered w.r.t. stimulus onset (the green line indicates stimulus onset and the red dotted line indicates the average choice response). Right: Same, but for epochs co-registered w.r.t. choice response (the green line indicates choice response and the red dotted line indicates the average stimulus onset).

We found that the correlation between effort and pupil dilation was becoming significant from 500ms after stimulus onset onwards. Note that, using the same approach, we found a negative correlation between pupil dilation and pre-choice absolute value difference $|\Delta VR^0|$. However, this relationship disappeared when we entered both $|\Delta VR^0|$ and effort into the same regression model.

Our eye-tracking data also allowed us to ascertain which item was being gazed at for each point in peristimulus time (during decisions). Using the choice responses, we classified each time point as a gaze at the (to be) chosen item or at the (to be) rejected item. We then derived, for each decision, the ratio of time spent gazing at chosen/unchosen items versus the total duration of the decision (between stimulus onset and choice response). The difference between these two gaze ratios measures the overt attentional bias towards the chosen item.

1067 We refer to this as the gaze bias. Consistent with previous studies, we found that chosen items

1068 were gazed at more than rejected items (mean gaze bias=0.02, s.e.m.=0.01, p=0.067).  However,

1069 we also found that this effect was in fact limited to low effort choices. Figure S8 below shows

1070 the gaze bias for low and high effort trials, based upon a median-split of subjective effort.

1071

1072

1073 **Figure S8. Gaze bias for low and high effort trials.** Mean (+/- s.e.m.) gaze bias is plotted for both
1074 low (left) and high (right) effort trials.

1075

We found that there was a significant gaze bias for low effort choices (mean gaze ratio

difference=0.033, s.e.m.=0.013, p=0.009), but not for high effort choices (mean gaze ratio

difference=0.002, s.e.m.=0.014, p=0.453). A potential trivial explanation for the fact that the

gaze bias is large for low effort trials is that these are the trials where participants immediately

recognize their favorite option, which attracts their attention. More interesting is the fact that

the gaze bias is null for high effort trials. This may be taken as evidence for the fact that, on

average, people allocate the same amount of (attentional) resources on both items. This is

important, because we use this simplifying assumption in our MCD model derivations.

1084

## 9. On the accuracy of the predicted confidence gain

1085

The MCD model relies on the system's ability to anticipate the benefit of allocating

resources to the decision process. Given the mathematical expression of choice confidence (cf.

Equation 4 in the main text), this reduces to finding an analytical approximation to the following

expression:

$$\bar{P} = E\left[s\left(\lambda|x|\right)\right] \tag{S1}$$

where $x \to s(x) = 1/1 + e^{-x}$ is the sigmoid mapping, $\lambda$ is an arbitrary constant, and the

expectation is taken under the gaussian distribution of $x \sim N\left(\mu, \sigma^2\right)$, whose mean and variance

are $\mu$ and $\sigma^2$, respectively.

Note that the absolute value mapping $x \to |x|$ follows a folded normal distribution,

whose first two moments $E\left[|x|\right]$ and $V\left[|x|\right]$ have known expressions:

1096
$$\begin{cases} E\left[|x|\right] = \sigma\sqrt{\dfrac{2}{\pi}}\exp\left(-\dfrac{|\mu|^2}{2\sigma^2}\right) + \mu\left(2 \times s\left(\dfrac{\pi\mu}{\sigma\sqrt{3}}\right) - 1\right) \\ V\left[|x|\right] = \mu^2 + \sigma^2 - E\left[|x|\right]^2 \end{cases} \tag{S2}$$

1097 where the first line relies on a moment-matching approximation to the cumulative normal

1098 distribution function (Daunizeau, 2017). This allows us to derive the following analytical

1099 approximation to Equation S1:

1100
$$\bar{P} \approx s\left(\dfrac{E\left[|x|\right]}{\sqrt{\dfrac{1}{\lambda^2} + aV\left[|x|\right]}}\right) \tag{S3}$$

1101 where setting $a \approx 3/\pi^2$ makes this approximation tight (Daunizeau, 2017).

1102 The quality of this approximation can be evaluated by drawing samples of $x \sim N\left(\mu, \sigma^2\right)$,

1103 and comparing the Monte-Carlo average of $s\left(\lambda|x|\right)$ with the expression given in Equation S3.

1104 This is summarized in Figure S9 below, where the range of variation for the moments of $x$ were

1105 set as follows: $\mu \in \left[-4, 4\right]$ and $\sigma^2 \in \left[0, 4\right]$.

1106

1107

**Figure S9: Quality of the analytical approximation to $\bar{P}$.** Upper left panel: the Monte-Carlo

estimate of $\bar{P}$ (colour-coded) is shown as a function of both the mean $\mu \in \left[-4, 4\right]$ (y-axis) and

the variance $\sigma^2 \in \left[0, 4\right]$ (x-axis) of the parent process $x \sim N\left(\mu, \sigma^2\right)$. Upper right panel: analytic

approximation to $\bar{P}$ as given by Equation S3 (same format). Lower left panel: the error, i.e. the

difference between the Mon-Carlo and the analytic approximation (same format). Lower right

panel: the analytic approximation (y-axis) is plotted as a function of the Monte-Carlo estimate

(x-axis) for each pair of moments $\{\mu, \sigma^2\}$ of the parent distribution.

1115

1116    One can see that the error rarely exceeds 5%, across the whole range of moments

1117    $\{\mu, \sigma^2\}$ of the parent distribution. This is how tight the semi-analytic approximation of the

1118    expected confidence gain (Equation 9 in the main text) is.

1119

**10. On the impact of model parameters for the MCD model**

First, note that the properties of the metacognitive control of decisions (in terms of

effort allocation and/or confidence) actually depends upon the demand for resources,

which is itself determined by prior value representations. Now the way the MCD-optimal

control responds to the resource demand (which is fully specified by the prior uncertainty

$\sigma^0$ and the absolute means' difference $\left|\Delta\mu^0\right|$) is determined by effort efficacy and unitary cost

parameters.

Let us first ask what would be the MCD-optimal effort $\hat{z}$ and confidence $\overline{P}_c(\hat{z})$ when

$\gamma = 0$, i.e. if the only effect of allocating resources is to increase the precision of value

representations. We call this the "β-effect". It is depicted on Figure S10 below.

1130

**Figure S10. The β-effect: MCD-optimal effort and confidence when effort has no impact on the value difference.** MCD-optimal effort (left) and confidence (right) are shown as a function of the absolute prior mean difference $\left|\Delta\mu^0\right|$ (x-axis) and prior variance $\sigma^0$ (y-axis).

One can see that, overall, increasing the prior variance $\sigma^0$ increases the resource demand, which eventually increases the MCD-optimal allocated effort $\hat{z}$. This, however, does not overcompensate for the loss of confidence incurred when increasing the prior variance. This is why the MCD-optimal confidence $\overline{P}_c(\hat{z})$ always decreases with the prior variance $\sigma^0$. Note that, for the same reason, the MCD-optimal confidence always increases with the absolute prior means' difference $\left|\Delta\mu^0\right|$.

Now the impact of the absolute prior means' difference $\left|\Delta\mu^0\right|$ on $\hat{z}$ is less trivial. In brief, when $\left|\Delta\mu^0\right|$ is high, the MCD-optimal allocated effort $\hat{z}$ decreases with $\left|\Delta\mu^0\right|$. This is due to the fact that the resource demand decreases with $\left|\Delta\mu^0\right|$. However, if $\left|\Delta\mu^0\right|$ decreases even more, it eventually reaches a critical point, below which the MCD-optimal allocated effort $\hat{z}$ *increases* with $\left|\Delta\mu^0\right|$. This is because, although the resource demand still decreases with $\left|\Delta\mu^0\right|$, the cost

of allocating resources overcompensates the gain in confidence. For such difficult decisions, the

system does not follow the demand anymore, and progressively de-motivates the allocation of

resources as $\left|\Delta\mu^0\right|$ continues to decrease. In brief, the amount $\hat{z}$ of allocated resources

decreases away from a "sweet spot", which is the absolute prior means' difference that yields

the maximal confidence gain per effort unit. Critically, the position of this sweet spot decreases

with $\beta$ and increases with $\alpha$. This is because the confidence gain increases, by definition,

with effort efficacy, whereas it becomes more costly when $\alpha$ increases.

Let us now ask what would be the MCD-optimal effort $\hat{z}$ and confidence $\overline{P}_c\left(\hat{z}\right)$ when

$\beta=0$, i.e. if the only effect of allocating resources is to perturb the value difference. The

ensuing "γ -effect" is depicted on Figure S11 below.

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1159    **Figure S11. The γ-effect: MCD-optimal effort and confidence when effort has no impact on**

1160    **value precision.** Same format as Fig S10.

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In brief, the overall picture is reversed, with a few minor differences. One can see that

increasing the absolute prior means' difference $\left|\Delta\mu^0\right|$ decreases the resource demand, which

eventually decreases the MCD-optimal allocated effort $\hat{z}$. This does decrease confidence,

because the γ-effect  of allocated effort overcompensates the effect of variations in $\left|\Delta\mu^0\right|$.

When no effort is allocated however, confidence is driven by $\left|\Delta\mu^0\right|$, i.e. it becomes an increasing

function of $\left|\Delta\mu^0\right|$. In contrast, variations in the prior variance $\sigma^0$ always overcompensate the

ensuing changes in effort, which is why confidence always decreases with $\sigma^0$. In addition, the

amount $\hat{z}$ of allocated resources decreases away from a sweet prior variance spot, which is the

prior variance $\sigma^0$ that yields the maximal confidence gain per effort unit. Critically, the position

of this sweet spot increases with $\gamma$ and decreases with $\alpha$, for reasons similar to the β-effect.

Now one can ask what happens in the presence of both the β-effect and the γ-effect. If

the effort unitary cost $\alpha$ is high enough, the MCD-optimal effort allocation is essentially the

superposition of both effects. This means that there are two "sweet spots": one around

some value of $\left|\Delta\mu^0\right|$ at high $\sigma^0$ (β-effect) and one around some value of $\sigma^0$ at high $\left|\Delta\mu^0\right|$ (γ-

effect). If the effort unitary cost $\alpha$ decreases, then the position of the β-sweet spot

increases and that of the β-sweet spot decreases, until they effectively merge together. This

is exemplified on Figure S12 below.

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1181 **Figure S12. MCD-optimal effort and confidence when both types of effort efficacy are operant.**

1182 Same format as Fig S10.

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One can see that, somewhat paradoxically, the effort response is now much simpler. In

brief, the MCD-optimal effort allocation $\hat{z}$ increases with the prior variance $\sigma^0$ and decreases

with the absolute prior means' difference $\left|\Delta\mu^0\right|$. The landscape of the ensuing MCD-optimal

confidence level $\overline{P}_c\left(\hat{z}\right)$ is slightly less trivial, but globally, it can be thought of as increasing with

$\left|\Delta\mu^0\right|$ and decreasing with $\sigma^0$. Here again, this is because variations in $\left|\Delta\mu^0\right|$ and/or $\sigma^0$ almost

always overcompensate the ensuing changes in allocated effort.

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