# On the spillover effect and optimal size of marine reserves

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**Abstract**

Marine reserves are an essential component of model fisheries management. As implementing marine reserves induces an inherent tradeoff between the harvesting and conservation, to solidify the insight into fisheries management with marine reserves is fundamental for management success. Finding an optimal reserve size that improves the fishing yield is not only theoretical interest but also practically important to assess the underlying tradeoffs and to facilitate decision making. Also, since the species migration determines the degree of the spillover effect from a marine reserve, it is a key consideration to explore the performance of marine reserve. Here, we investigate an optimal reserve fraction and its management outcome under various spillover strength via a simple two-patch mathematical model, in which one patch is open to fishing, and the other is protected from fishing activities. The two-patch model is approximated by a single population dynamics when the migration rate is sufficiently larger than the growth rate of a target species. In this limit, it is shown that an optimal reserve size exists when the pre-reserve fishing is operated at the fishing mortality larger than the $f_{MSY}$, the fishing mortality at the maximum sustainable yield (MSY). Also, the fishing yield with the optimal reserve size becomes as large as MSY in the limit. Numerical simulations across various migration rates between two patches suggest that the maximum harvest under the management with a marine reserve is achieved in this limit, and this contrasts with the conservation benefit in which is maximized at the intermediate migration rate.

*Keywords: Fisheries; marine reserves; maximum sustainable yields*

# 1 Introduction

Marine reserves or no-take marine protected areas (MPAs) are a central tool in modern fisheries management to remedy harvesting and enhance ecosystem health [17, 25, 33]. As

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often an implementation of a marine reserve leads to a fishing closure, there is an inherent tradeoff between harvest and conservation [7, 20]. In addition, while one may postulate the positive influence of marine reserve introductions, previous studies have revealed unintended outcomes, including species loss due to an altered species interaction strength [32], intensified fishing activity close to the reserve boundary [10, 19], and escalating the competitions among fishermen that suppresses the reserve benefit [29]. Hence, to solidify our insight into the effect of the marine reserve is fundamental to facilitate the management decision making under the tradeoffs and robust design of a marine reserve establishment.

One key consideration for its effective implementation and for assessing the underlying tradeoffs is the condition for the marine reserve to improve the fishing yield [5, 15, 28]. In fact, marine reserves can merely reduce fishing yield when the stock is managed sustainably [15,22,35] or a target species has a high fecundity ability [28]. Previous studies suggested that the threshold for a marine reserve to increase the fishing yield is $f_{MSY}$, the fishing mortality at the maximum sustainable yield (MSY) [15, 21, 22]. One key aspect complementing to the previous findings, in regarding to the optimal reserve size, is the effect of the species migration between a marine reserve and fishing ground because no adult movement [15, 28] and well-mixed population [5] were previously investigated. Investigating its effect on the optimal reserve size with various migration rates will further strength current insight, since the spillover from the reserve is a central mechanism to enhance fishing outcomes outside the marine reserve [13, 14, 30].

Here, we investigate when the optimal reserve size, which improves the fishing yield, is achieved with the effect of the spillover due to the species migration. The model is a simple two patch model where one patch represents the fishing ground and the marine reserve for the other. Two patches are connected by the species migration at a species specific rate, and it affects the degree of the spillover from the marine reserve. The model also allows us to consider different migration modes, such as the positive/negative density-dependent as well as density-independent migrations. This simple approach yields analytical results by introducing an aggregated model when the migration rate is sufficiently larger than the growth rate of a target species [2, 5, 18, 29, 31]. The aggregated model allows us to derive the condition for an optimal reserve size to exist and the fishing yield and total population size under the management with the optimal reserve fraction. These suggest that an optimal reserve size exists when the fishing mortality is larger than the $f_{MSY}$, and the fishing yield and the total population size under the management correspond to MSY and $X_{MSY}$, the population size at MSY.

Numerical calculations across various migration rates suggest that the aggregated model gives the highest fishing yields among various species migration rates. That is, the fishing yield is maximized under the well-mixed population with a large migration rate. On the other hand, our model shows that a marine reserve provides a larger total population when the migration ability of a target species is low or moderate. These contrasting results emphasize the importance of the spillover effects in consideration of the tradeoffs under the management with marine reserves.

# 2 Model

## 2.1 Basic model

Our starting point is the commonly used Schaefer model [11, 27] where the population dynamics of the target species $x$ is described with the growth rate $r$, carrying capacity $K$, and fishing mortality rate $f$ as follows:

$$\frac{dx}{dt} = rx\left(1 - \frac{x}{K}\right) - fx. \tag{1}$$

In this model, MSY, the fishing mortality rate at MSY, $f_{MSY}$, and the population size at MSY, $X_{MSY}$, are described as

$$\text{MSY} = \frac{rK}{4},\ f_{MSY} = \frac{r}{2},\ \text{and } X_{MSY} = \frac{K}{2}. \tag{2}$$

To investigate the effect of marine reserve, we employ a common two-patch model (e.g., [9, 30, 32]) where one patch ($i = 1$) is open to fishing and another patch ($i = 2$) is protected from fishing activity (i.e., $f = 0$), and the species migration at the rate $m$ connects the two patches (Fig. 1). Then, Eq. (1) becomes

$$\frac{dx_1}{dt} = rx_1\left(1 - \frac{x_1}{(1-\alpha)K}\right) - fx_1 + M(x_1, x_2), \tag{3a}$$

$$\frac{dx_2}{dt} = rx_2\left(1 - \frac{x_2}{\alpha K}\right) - M(x_1, x_2), \tag{3b}$$

where, $x_1$ and $x_2$ are the population of fishing ground and marine reserve, respectively. The last terms describe the species migration and has the following form:

$$M(x_1, x_2) = m\left\{\left(\frac{x_2}{\alpha K}\right)^s (1-\alpha)x_2 - \left(\frac{x_1}{(1-\alpha)K}\right)^s \alpha x_1\right\}, \tag{4}$$

where, $s$ is the parameter controlling the migration mode [1]: negative density-dependent migration when $-1 < s < 0$; random migration when $s = 0$; and density-dependent migration when $s > 0$.

We use the notations to describe the fishing yield and the total population size under the management with a marine reserve as

$$Y_{res} = fx_1,\ \text{and } X_{res} = x_1 + x_2. \tag{5}$$

where, these quantities are used in the following to compare the management without marine reserve.

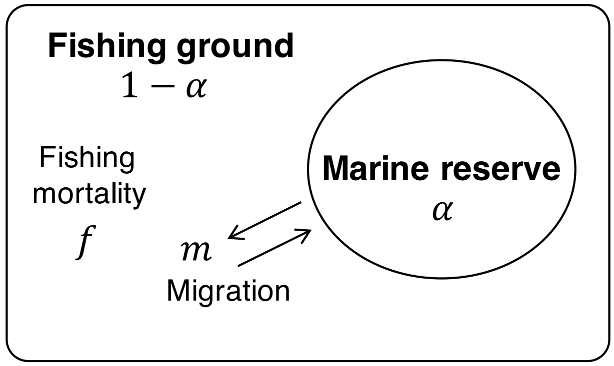


Figure 1: Schematic description of the model. The space is divided into the fishing ground (fraction $1-\alpha$) with fishing mortality, $f$, and the marine reserve (fraction $\alpha$) without fishing activity. The species migration at a rate $m$ connects the dynamics of two patches. Species migration is either positively/negatively density-dependent or density-independent.

### 2.2 Model aggregation

When the migration rate is sufficiently larger than vital rate ($m \gg r$), there are fast and slow dynamics operating at different time scales [2, 18]. Then, the migration term has a negligible effect on the total population $X = x_1 + x_2$ operated at the time scale of fast parameter $\tau = mt$. Thus, Eq. (3) can be approximated by a single aggregated model [31]

$$\frac{dX}{d\tau} = rX\left(1 - \frac{X}{K}\right) - f(1-\alpha)X. \tag{6}$$

In the following, we discuss the analytical aspect of the aggregated model (6), and perform numerical calculations across the species migration rate $m$ including the situation where the model aggregation is not valid.

## 3 Results

### 3.1 Analysis of the aggregated model Eq. (6)

The aggregated model allows us to obtain explicit form of the equilibrium as follows:

$$X^{AG} = K\left(1 - \frac{f(1-\alpha)}{r}\right), \tag{7}$$

and fishing yield is

$$Y^{AG} = f(1-\alpha)K\left(1 - \frac{f(1-\alpha)}{r}\right), \tag{8}$$

where, the superscript $AG$ indicates the equilibrium of the aggregated model. By solving $dY^{AG}/df = 0$ about $f$ and $\alpha$, respectively, we obtain the optimal fishing effort and reserve size:

$$f^{AG*} = \frac{r}{2(1-\alpha)}, \quad (9)$$

$$\alpha^{AG*} = 1 - \frac{r}{2f}. \quad (10)$$

Eq. (9) suggests that one needs to increase the fishing mortality with a rate inversely proportional to the fraction of fishing ground $1-\alpha$ after an establishment of a marine reserve, and it becomes infinitely large when the fraction of the marine reserve approaches to unity. On the other hand, Eq. (10) represents that for an (positive) optimal reserve size to exist the fishing yield should be larger than MSY level:

$$f > \frac{r}{2} = f_{MSY}. \quad (11)$$

In other words, if the fishing effort is smaller than the MSY level, there is no optimal MPA size to improve fishing yield. Also, the optimal size approaches 1 (i.g., complete fishing ban) as the fishing mortality becomes large. These explanations are visualized in Fig. 2.

It is obvious from Eq. (6) that the maximum fishing yields coincides to MSY of Eq. (1). In fact, substituting either Eq. (9) or (10) into Eq. (8) recovers Eq. (2). We denote this by

$$Y^{AG*}|_{f^*} = Y^{AG*}|_{\alpha^*} = \text{MSY}. \quad (12)$$

We often use notation $Y^{AG*}$ when the substitution is obvious. Similarly, we regard $X^{AG*}$ as the population size when fishing yield is given by Eq. (12).

## 3.2 Numerical investigation for general situation

When the migration rate is not large enough, then the aggregated model is not valid. Yet, we will show that the the analytical results above provides a maximum value of fishing yield, and it becomes a benchmark to discuss the performance of an introduced marine reserve.

Here, we numerically perform Eq. (3) to gain the fishing yield and the optimal reserve size $\alpha^*$, as well as total population size under various reserve sizes and migration rates. To compare these quantities with those of the management without marine reserve, we introduce the following two normalized quantities: the fishing yield normalized by the MSY, $Y_{res}/\text{MSY}$, and the total population size normalized by $X_{MSY}$, $X_{res}/X_{MSY}$. Note from the analysis above, the aggregated model under the optimal reserve size, $\alpha^{AG*}$, gives the values of normalized fishing yield and population size $Y^{AG*}/\text{MSY} = X^{AG*}/X_{MSY} = 1$.

The top three panels in Figure 3 show the normalize fishing yield under the density-independent migration ($s = 0$). As expected, the optimal reserve size $\alpha^*$, if any, approaches that of the aggregated model $\alpha^{AG*}$ as the migration rate $m$ becomes large. Also, the normalized fishing yield of the aggregated model gives the upper bound: ($Y_{res}/\text{MSY} \leq Y^{AG*}/\text{MSY}$). The numerical calculations also suggest that the condition for the positive optimal reserve size to exists (Eq. 11) still holds for the various migration rate. Therefore, the improvement

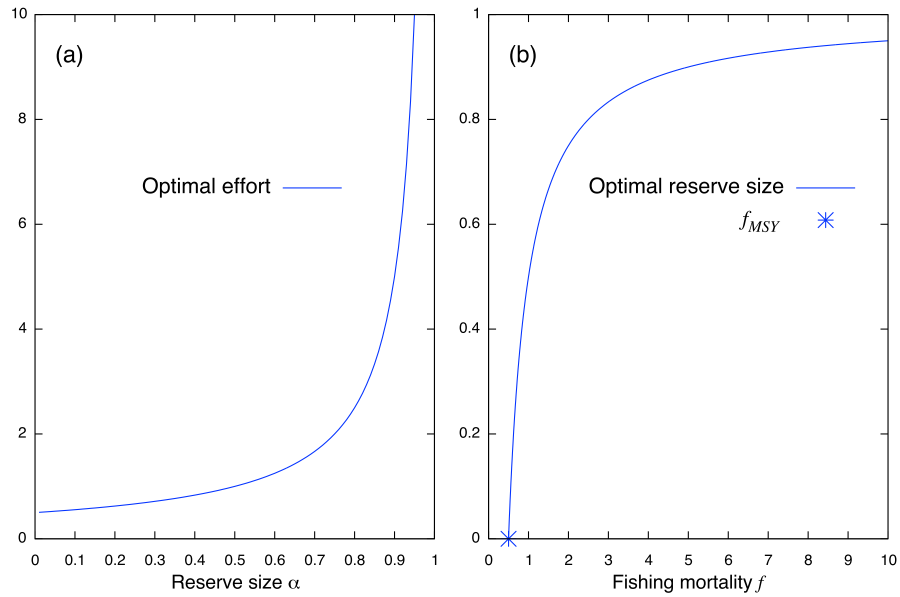


Figure 2: (a) Optimal fishing effort of the aggregated model (Eq. (9)) and (b) optimal reserve size (Eq. (10)). Positive reserve size exists only when fishing mortality is larger than $f_{MSY}$.

of fishing yield by introducing marine reserve occurs only when the initial fishing mortality exceeds the MSY level. Although this condition may be necessary for a marine reserve to increase the harvesting when the migration rate is not high enough, it does not guarantee the existence of an optimal reserve size. For example, when the fishing mortality rate is moderately high ($f = 0.75$; Figure 3b), the spillover effect from the marine reserve is necessary for an optimal reserve size to exist.

On the other hand, the bottom three panels of Figure 3 show the normalized total population size. The management with an optimal size of the marine reserve tends to give a smaller population size than $X_{MSY}$ (i.e., $X_{res}/X_{MSY} < 1$) except for the migration rate becomes sufficiently large. Also, these show that an increasing the reserve size provides a higher normalized population size, and the population size becomes larger at low to moderate migration rates ($m$ is about 1 in Figure 3) at a given reserve size. These suggest a mismatch between an optimal harvesting and conservation benefit.

The qualitatively similar trends are obtained in two alternative migration modes: density-dependent ($s = 1$) and negatively density-dependent ($s = -0.5$) migrations between patches (see Appendix figures).

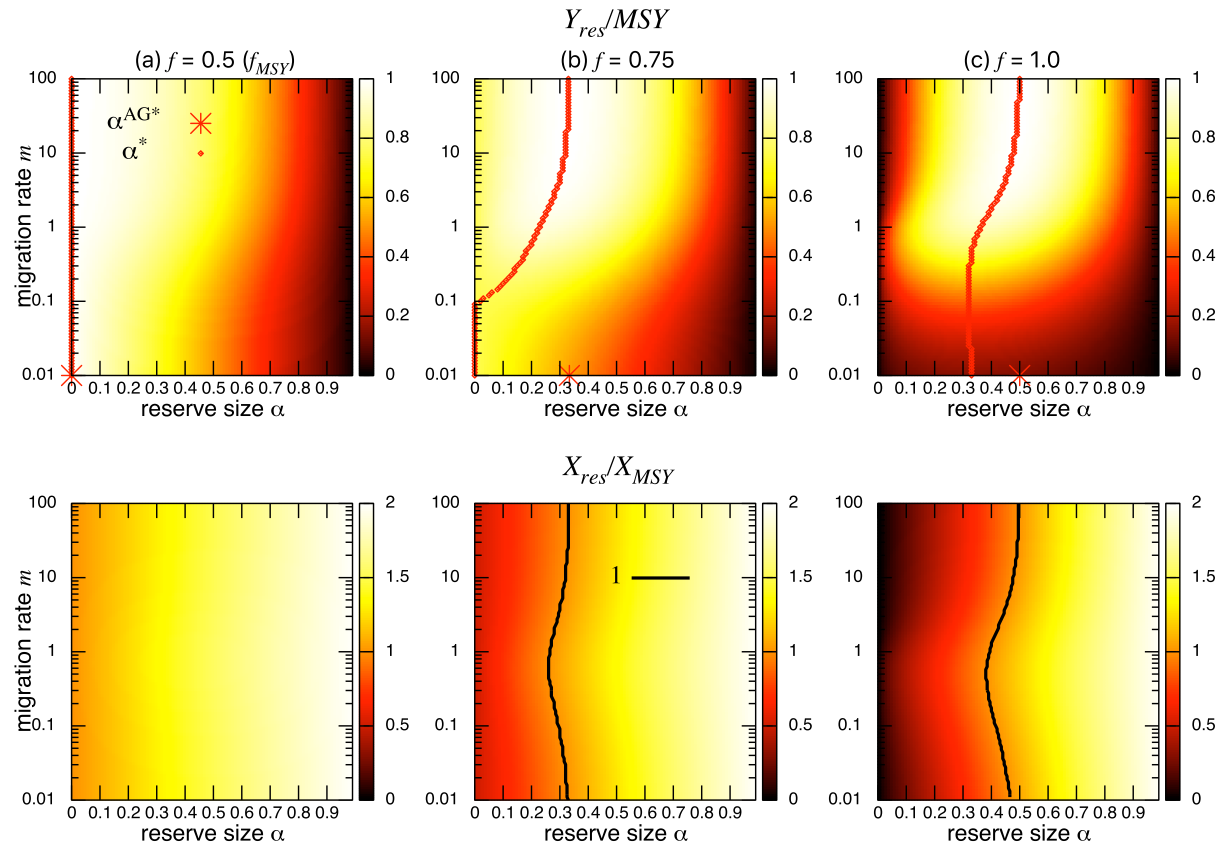


Figure 3: (Top) the normalized fishing yield ($Y_{res}$/MSY) and optimal reserve size (diamond); and (bottom) the normalized population size ($X_{ref}/X_{MSY}$) when the species migration is density independent. Optimal reserve size predicted by the aggregated model Eq. (6) is also shown on $x$-axis (star). The parameter values used are $r = 1$, $K = 10$, and $s = 0$.

# 4 Discussion

The effect of a marine reserve on harvesting is a crucial consideration as creating a marine reserve in the existing fishing ground can reduce fishing opportunities. By taking advantage of the simple mathematical model investigated, here we demonstrated some important theoretical predictions to stipulate an optimal size of the marine reserve along with the fishing yield and total population.

The analysis of the aggregated model, describing the situation where the species migration rate is sufficiently large, suggests that fishing mortality should be larger than $f_{MSY}$ for a marine reserve to improve the fishing yield. Therefore, marine reserves do not deliver further fisheries benefit to sustainably managed fisheries below the MSY level, as suggested previously [15, 21, 22]. If a marine reserve replaces a certain fraction of fishing ground, fishing mortality should be increased by the factor inversely proportional to the fraction of the fishing ground. In practice, this corresponds to the situation where all fishermen remain in the contracted fishing ground after the reserve implementation. Under these conditions, the fishing yields becomes as large as MSY, as the previously reported result [5, 16]. Our results suggest that the optimal reserve fraction spans $\alpha^* \in (0, 1)$, and the optimal reserve fraction approaches to 1 as the fishing mortality becomes sufficiently large. Our numerical results suggest that these theoretical predictions from the aggregated model give the upper boundary of the harvesting. That is, the maximum harvest under the management with a marine reserve is achieved when the species migration rate is large. This indicates that the reduce fishing ground can be compensated by increasing the fishing mortality when there is sufficient species exchange between a marine reserve and fishing ground. However, the intensified fishing mortality to achieve the maximum fishing yield given a migration rate $m$ often leads to a smaller total population than the MSY: $X_{ref}/X_{MSY} < 1$, except for the case $m \gg 1$ where $X_{ref}/X_{MSY} = 1$. These findings contrast with the conservation aspects of marine reserves. Namely, marine reserves may provide larger conservation benefits, such as a larger population recovery and reproductive capacity, at relatively low and moderate migration rate [6,13,30]. Our model also shows the larger population size at an intermediate migration rate (Fig. 3b and c, bottom).

The prediction of the equivalence in MSY and the yield from the management with a marine reserve has been revised by proceeding studies [8,22,34] However, this result derived from a general mathematical model has been a still useful benchmark to assess the effect of, for instance, age-structure, stochasticity, and more complex fishing regulations, in which mathematical analysis is usually not feasible. Similarly, the predictions of our model should be view as a benchmark to investigate further how and when more elaborated assumptions will revise the model prediction.

To remedy the harvesting is just one aspect of marine reserves, and one needs to mitigate underlying tradeoffs to establish marine reserves [20]. Marine reserves can provide a multitude of benefits of management and ecosystems, including enhancing ecological resilience [3, 30] and optimal profit [26], promoting biodiversity [24] and genetic diversity [4], and mitigating impacts of climatic change [12,23]. These form multiple tradeoffs of the management with marine reserves. The management under an optimal reserve size $\alpha^*$ discussed here can be used as a baseline to assess the strength of multiple tradeoffs, and integrative discussion of multiple management objects to mitigate the tradeoffs will promote an effective management decision making.

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# Appendix figures

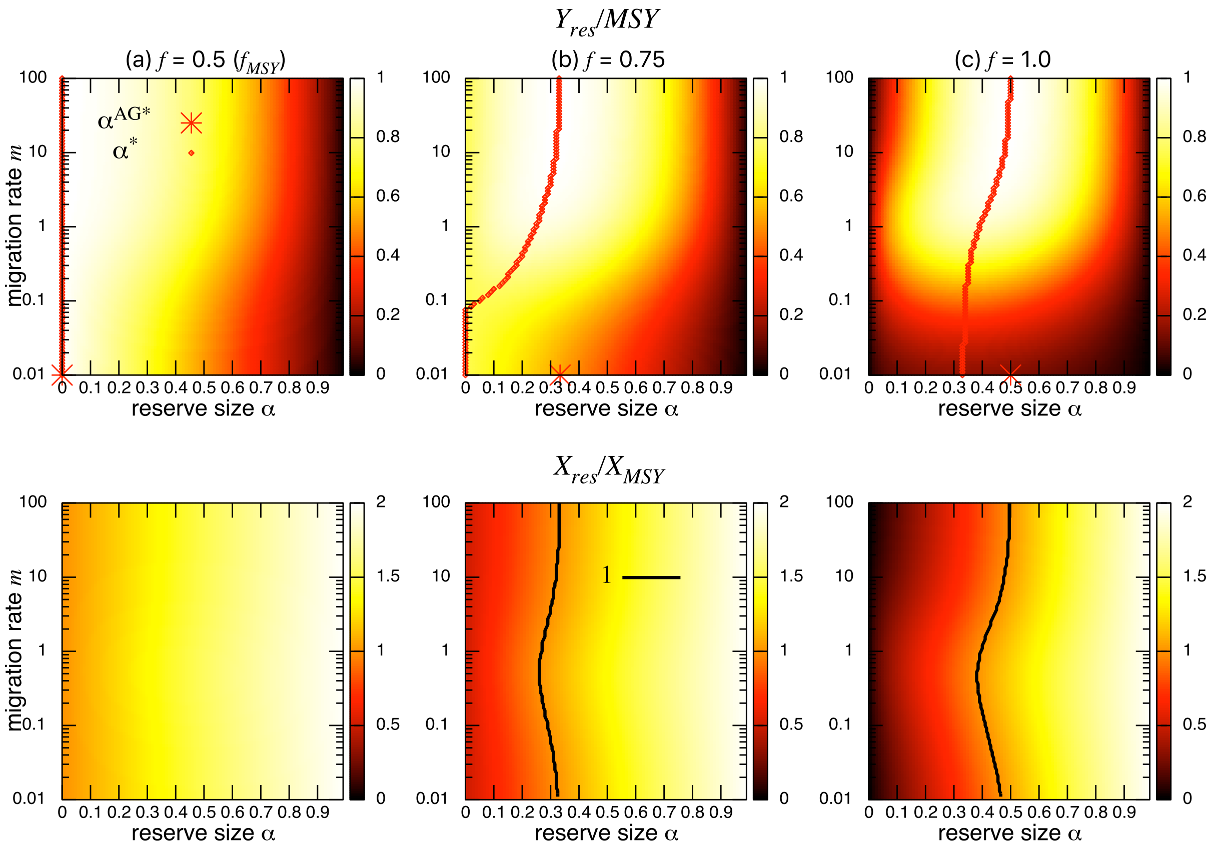


Figure A.1: (Top) the normalized fishing yield ($Y_{res}$/MSY) and optimal reserve size (diamond); and (bottom) the normalized population size ($X_{ref}/X_{MSY}$) under a density-dependent migration. Optimal reserve size predicted by the aggregated model Eq. (6) is also shown on $x$-axis (star). The parameter values used are $r = 1$, $K = 10$, and $s = 1$.

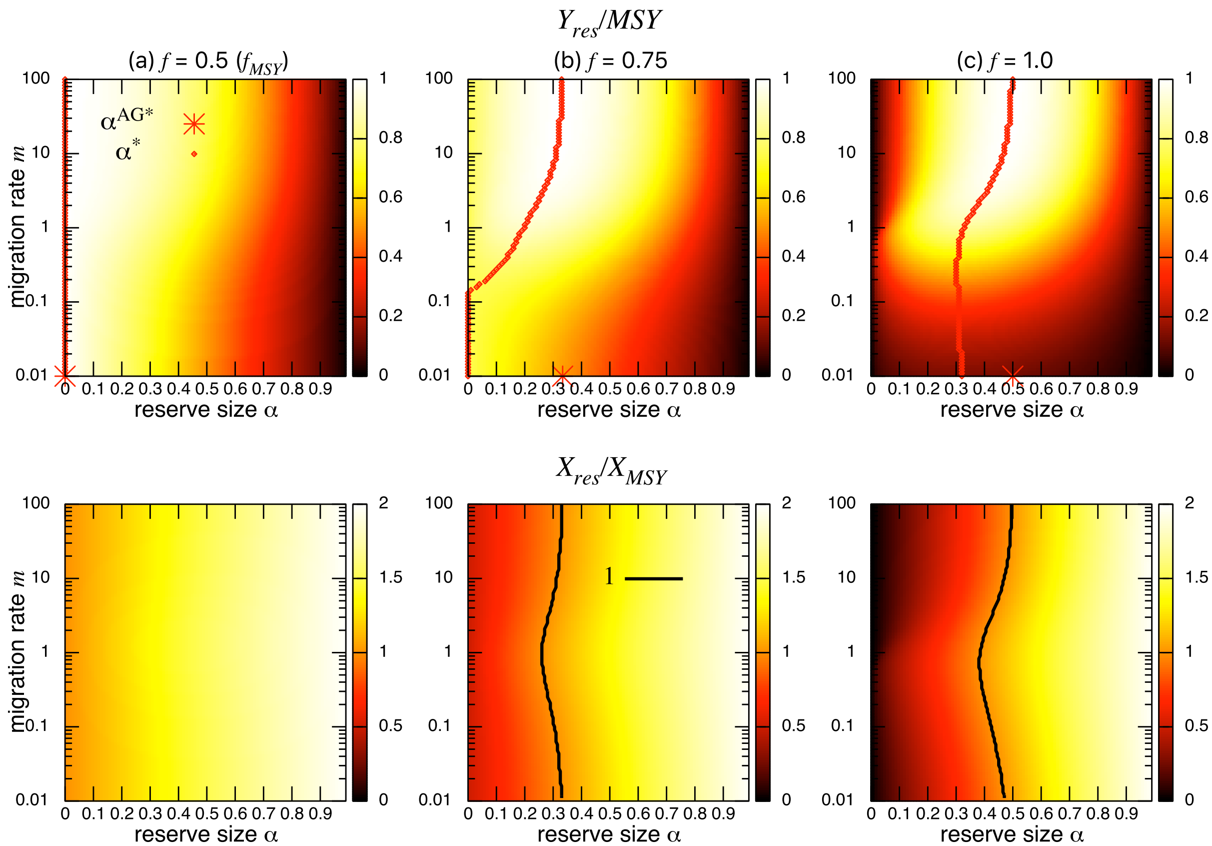


Figure A.2: (Top) the normalized fishing yield ($Y_{res}$/MSY) and optimal reserve size (diamond); and (bottom) the normalized population size ($X_{ref}/X_{MSY}$) under a negative density-dependent migration. Optimal reserve size predicted by the aggregated model Eq. (6) is also shown on $x$-axis (star). The parameter values used are $r = 1$, $K = 10$, and $s = -0.5$.