

1     **Treadmilling FtsZ polymers drive the directional movement of sPG-**  
2             **synthesis enzymes *via* a Brownian ratchet mechanism**

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4     Joshua W. McCausland<sup>1</sup>, Xinxing Yang<sup>1</sup>, Georgia R. Squyres<sup>2</sup>, Zhixin Lyu<sup>1</sup>, Kevin E.  
5     Bruce<sup>3</sup>, Melissa M. Lamanna<sup>3</sup>, Bill Söderström<sup>4</sup>, Ethan C. Garner<sup>2</sup>, Malcolm E. Winkler<sup>3</sup>,  
6             Jie Xiao<sup>1,\*</sup>, and Jian Liu<sup>5,\*</sup>

7     1. Department of Biophysics and Biophysical Chemistry, Johns Hopkins School of  
8             Medicine, Baltimore, MD, 21205, USA;

9     2. Department of Molecular and Cellular Biology, Harvard University, Cambridge, MA,  
10             02138, USA

11    3. Department of Biology, Indiana University Bloomington, Bloomington, IN, 47405 USA

12    4. Structural Cellular Biology Unit, Okinawa Institute of Science and Technology, Japan;  
13    and The ithree institute, University of Technology Sydney, Ultimo, NSW, 2007 Australia;

14    5. Department of Cell Biology, Johns Hopkins School of Medicine, Baltimore, MD,  
15             21205, USA.

16             (\*) Corresponding Authors: [xiao@jhmi.edu](mailto:xiao@jhmi.edu) and [jliu187@jhmi.edu](mailto:jliu187@jhmi.edu)

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18

## 19 **Abstract**

20

21 FtsZ, a highly conserved bacterial tubulin GTPase homolog, is a central component of  
22 the cell division machinery in nearly all walled bacteria. FtsZ polymerizes at the future  
23 division site and recruits greater than 30 proteins to assemble into a macromolecular  
24 complex termed the divisome. Many of these divisome proteins are involved in septal cell  
25 wall peptidoglycan (sPG) synthesis. Recent studies found that FtsZ polymers undergo  
26 GTP hydrolysis-coupled treadmilling dynamics along the circumference the division site,  
27 driving the processive movement of sPG synthesis enzymes. How FtsZ's treadmilling  
28 drives the directional transport of sPG enzymes and what its precise role is in bacterial  
29 cell division are unknown. Combining theoretical modeling and experimental testing, we  
30 show that FtsZ's treadmilling drives the directional movement of sPG-synthesis enzymes  
31 *via* a Brownian ratchet mechanism, where the shrinking end of FtsZ polymers introduces  
32 an asymmetry to rectify diffusions of single sPG enzymes into persistent end-tracking  
33 movement. Furthermore, we show that the processivity of this directional movement is  
34 dependent on the binding potential between FtsZ and the enzyme, and hinges on the  
35 balance between the enzyme's diffusion and FtsZ's treadmilling speed. This interplay  
36 could provide a mechanism to control the level of available enzymes for active sPG  
37 synthesis both in time and space, explaining the distinct roles of FtsZ treadmilling in  
38 modulating cell wall constriction rate observed in different bacterial species.

39

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41  
42           During cell wall constriction in most Gram-negative bacteria, new septal  
43 peptidoglycan (sPG) synthesis and old cell wall degradation occur simultaneously<sup>1</sup>. A  
44 large number of the cell wall enzymes involved in this process and their regulators have  
45 been identified. However, it remains unclear how these proteins are orchestrated in time  
46 and space to achieve successful cytokinesis and at the same time maintain the structural  
47 integrity of the septal cell wall<sup>2,3</sup>. Perturbations of PG remodeling at septum compromise  
48 cell division and often lead to cell lysis<sup>4</sup>.

49  
50           Recent studies have indicated that FtsZ, an essential component of the bacterial  
51 cell division machinery, may play a central part in regulating the spatiotemporal  
52 coordination of sPG synthesis enzymes. FtsZ is a highly conserved bacterial tubulin  
53 homologue and GTPase<sup>5-7</sup>. During cell division, FtsZ polymerizes at the cytoplasmic face  
54 of the inner membrane to form a ring-like structure (Z-ring) at mid-cell<sup>8-10</sup>. The Z-ring then  
55 locally recruits an ensemble of more than 30 proteins, many of which are sPG-remodeling  
56 enzymes<sup>1,11</sup>, to initiate septal cell wall constriction. New studies employing super-  
57 resolution and single-molecule imaging *in vitro* and *in vivo* have demonstrated that the  
58 FtsZ polymers exhibit GTP hydrolysis-driven treadmilling dynamics, which are the  
59 continuous polymerization at one end and depolymerization at the other end, with  
60 individual FtsZ monomers remaining stationary in the middle<sup>12-15</sup>. Most interestingly, it  
61 was found that FtsZ's treadmilling dynamics drive processive movements of the essential  
62 sPG transpeptidase (TPase, FtsI in *E. coli* and PBP2B in *B. subtilis*)<sup>12,13</sup> and  
63 glycosyltransferase FtsW<sup>16</sup>. Consequently, it was proposed that FtsZ's treadmilling

64 dynamics spatially and temporally distribute sPG synthesis enzymes along the septum  
65 plane to ensure smooth septum morphogenesis<sup>13</sup>. However, it is unknown how FtsZ's  
66 treadmilling dynamics with stationary monomers in the cytoplasm are transduced into the  
67 periplasm to drive the persistent and directional movement of cell wall synthesis enzymes.  
68 The role of FtsZ's treadmilling dynamics in modulating sPG synthesis activity also  
69 remains elusive, as it was shown that the cell wall constriction rate is dependent on FtsZ's  
70 treadmilling speed in *B. subtilis*<sup>12</sup> but not in *E. coli*<sup>13</sup>, or *S. aureus*<sup>17</sup>.

71  
72 In this work, we combined agent-based theoretical modeling with single-molecule  
73 imaging-based experimental testing to address the mechanism of the FtsZ treadmilling-  
74 dependent processive movement of sPG enzymes and its associated role in bacterial cell  
75 division. We found that a Brownian ratchet mechanism underlies the persistent and  
76 directional movement of single sPG synthesis enzyme molecules driven by FtsZ's  
77 treadmilling dynamics. Using FtsI as a model sPG enzyme, we found that the processivity  
78 of the Brownian ratchet is dependent on the (indirect) binding potential between FtsI and  
79 FtsZ and modulated by the balance between FtsI's random diffusion and FtsZ's  
80 treadmilling speed. This finding offers predictions about how different bacterial species  
81 could harness the same FtsZ treadmilling machinery to achieve distinct processivities of  
82 sPG enzymes so that the available level of sPG synthases for cell wall constriction can  
83 be controlled differentially. Given the lack of linear stepping motors in the prokaryotic  
84 world, our work suggests a general framework for how polymer dynamics coupled with  
85 Brownian ratcheting could underlie directional transport of cargos and be shaped by  
86 evolution to meet the needs of different cellular milieus.

## 87 Results

### 88 Model description

89

90 Our model is based on the concept of a Brownian ratchet, where FtsZ's treadmilling  
91 introduces an asymmetry to bias the random diffusion of FtsI molecules in the periplasm,  
92 upon which FtsI persistently follows the shrinking end of a treadmilling FtsZ filament (Fig.  
93 1). The quantitative details of the model are rooted in the physical and chemical properties  
94 of key components of the system, which can be characterized by experiments.

95

96 As shown in Fig. 1, the model describes the movement of a free FtsI molecule at the  
97 septum as quasi-1D. The model assumes that FtsI, an essential TPase with a single  
98 transmembrane domain and a cytoplasmic tail, can freely diffuse along the inner  
99 membrane at the septum or interact indirectly with a treadmilling FtsZ filament underneath  
100 the inner membrane (Fig. 1A). The dynamics of a single FtsI molecule at the septum are  
101 therefore determined by three parameters: the constant of FtsI's free diffusion ( $D$ ), the  
102 treadmilling speed ( $V_z$ ) of FtsZ filaments, and the attraction force determined by the  
103 binding potential ( $U$ ) between FtsI and FtsZ (Fig. 1B and C).

104

105 To set the ranges of the three parameters, we consider the following. First, we set the  
106 diffusion constant to range from  $10^{-3}$  to  $10^{-1}$   $\mu\text{m}^2/\text{s}$ , which is of a typical inner membrane  
107 protein in bacterial cells<sup>18,19</sup>. For example, PBP2, the counterpart of FtsI in cell wall  
108 elongation, was measured at  $\sim 0.06$   $\mu\text{m}^2/\text{s}$ <sup>18,19</sup>. Second, the average treadmilling speed  
109 of FtsZ was at  $\sim 20 - 40$  nm/s *in vivo* but can be a few-fold faster *in vitro*, therefore we

110 set a large range of 10 – 100 nm/s<sup>14,15</sup>. Third, FtsI interacts with FtsZ at the septum  
111 indirectly through a relay of protein-protein interactions that include FtsN, FtsA, and/or  
112 FtsEX<sub>1,20</sub>. For simplicity we omit the details of the protein-protein interaction relay and  
113 refer to it as the interaction between FtsI and FtsZ. The indirect interaction between an  
114 FtsZ monomer and a nearby FtsI molecule constitutes an attractive force for each other  
115 and can be described as a short-ranged harmonic binding potential (Fig. 1C). We  
116 assumed the potential range was  $\pm 2.5$  nm, commensurate with the size of an FtsZ  
117 monomer ( $\sim 5$  nm)<sup>21-24</sup>. The potential's magnitude was  $\sim 10 k_B T$ , corresponding to a  $K_d$  in  
118 the  $\mu M$  range, which is typical for protein-protein interactions in the bacterial divisome  
119 system<sup>25-31</sup>. Note that here we use FtsI in *E. coli* as the model sPG enzyme, but the same  
120 analysis can be applied to any other sPG enzyme or divisome proteins as well.

121  
122 To numerically compute the model, we describe the dynamics of FtsI by a Langevin-type  
123 equation (Equation (1)) where the viscous drag force on the molecule is in balance with  
124 a driving force  $f$  and a force from thermal noise  $\xi$ :

$$125 \quad \lambda \frac{dx(t)}{dt} = f(x(t)) + \xi(t) \quad \text{Equation (1)}$$

126  
127 Here,  $x(t)$  represents the location of a FtsI molecule at time  $t$  along the 1D septum.  $\lambda$  is  
128 the effective viscous drag coefficient for FtsI's movement with  $\lambda = k_B T/D$ , where  $D$  is the  
129 diffusion constant of free FtsI molecules on the inner membrane when it is not interacting  
130 with FtsZ.  $f(x(t))$  is the attractive force exerted upon the FtsI molecule by the FtsZ's binding  
131 potential,  $U(x, t)$  (Fig. 1C). Specifically,  $f(x(t)) = -\partial U(x, t)/\partial x$  at time  $t$ . The last term  $\xi(t)$

132 reflects the random diffusive motion of the FtsI on the inner membrane with  $\langle \xi(t) \cdot \xi(t') \rangle =$   
133  $2D \cdot \Delta t \delta(t-t')$ , where  $\Delta t$  is the unitary time step in simulation.

134

135 Next, we simulate the treadmilling of an FtsZ filament. The model depicts the filament  
136 shrinking and growing according to Equations (2) and (3), which respectively describe  
137 how the positions of the shrinking and growing ends of a treadmilling FtsZ filament at time  
138  $t$ ,  $x_S(t)$  and  $x_G(t)$  are related to the treadmilling rate,  $V_Z$ :

139 
$$\frac{dx_S(t)}{dt} = V_Z \quad \text{Equation (2)}$$

140 
$$\frac{dx_G(t)}{dt} = V_Z \quad \text{Equation (3)}$$

141 The model simulates the treadmilling events in a discretized sense, which occur every  
142  $5\text{nm}/(V_Z \cdot \Delta t)$  simulation time step. Each time an FtsZ subunit falls off the shrinking end of  
143 the filament, the associated binding potential vanishes with it; likewise, when an FtsZ  
144 subunit adds onto the growing end, the associated binding potential appears with it. The  
145 treadmilling speed  $V_Z$  of each FtsZ filament is drawn stochastically from an experimentally  
146 measured distribution<sup>13</sup>. The FtsZ filament length is set at 50 monomers (250 nm) and  
147 the treadmilling speed is independent of the filament length, in accordance to previous  
148 biochemical studies and a recent *in vivo* study<sup>20,32-35</sup>. To discern principal interactions, the  
149 model considers one FtsI molecule and one FtsZ filament in a self-contained septal  
150 section. It can be easily expanded to include multiple FtsI molecules per FtsZ filament  
151 (Fig. S1).

152

153 Assuming that the FtsZ filament treadmills from left to right with a steady-state length of  
154 ~ 250 nm, the model implements open boundary conditions on the FtsI molecule at both  
155 the left and right edges of the system and the right-ward FtsZ treadmilling is not limited.  
156 The model results presented below reflect the nominal case, whose essence remains  
157 robust against variations of model parameters within the physical range constrained by  
158 existing experimental data.

159

### 160 **A Brownian ratchet mechanism couples FtsI's directional movement to FtsZ's** 161 **treadmilling dynamics at the shrinking end**

162

163 As we described above, Brownian ratcheting hinges on the diffusion of FtsI, its interaction  
164 with FtsZ, and FtsZ's treadmilling speed. To examine how the movement of FtsI depends  
165 on FtsI's diffusion and the binding potential between FtsI and FtsZ, we kept FtsZ's  
166 treadmilling speed constant at an experimentally measured speed of 25 nm/s and carried  
167 out a phase diagram study using stochastic simulations. As described above, we chose  
168 a parameter range of 0.0001 to 0.1  $\mu\text{m}^2/\text{s}$  for FtsI's diffusion<sup>18,19</sup>. The upper limit of the  
169 binding potential was set ~ 20  $k_B T$ , which corresponded to a dissociation constant  $K_d$  in  
170 the nM-range.

171

172 We considered an initial condition in which both the shrinking end of an FtsZ filament and  
173 an FtsI molecule were at the left boundary of the septal section. To be commensurate  
174 with our experimental analysis, we counted an FtsI trajectory as moving directionally if it  
175 tracked the shrinking end of a treadmilling FtsZ filament persistently and unidirectionally



176 for at least 4 seconds. Because of the stochastic nature of Brownian ratcheting, we  
177 characterized the state of FtsI under this parameter set condition as persistent end-  
178 tracking in the phase diagram if 50% or more of simulated FtsI trajectories displayed such  
179 a persistent directional movement.

180  
181 As shown in the phase diagram in Fig. 2A, the model showed that when the binding  
182 potential between FtsZ and FtsI was weak ( $< 5 k_B T$ ,  $\sim$  mM Kd), FtsI largely displayed  
183 random diffusion without directional movements along the septum. When the attraction  
184 potential was sufficiently strong ( $> 5 k_B T$ ), strong binding quenched free diffusion and  
185 confined FtsI to the end of an FtsZ filament. As the FtsZ subunit at the shrinking end of  
186 the filament fell off, the next one in the row attracted and coupled to FtsI, which pulled  
187 FtsI to the right by  $\sim 5$  nm. With the subsequent FtsZ subunits falling off one after the  
188 other from the shrinking end, the FtsI molecule ratcheted forward and persistently tracked  
189 the end of the treadmilling FtsZ filament. These consecutive movements resulted in a  
190 persistent and directional trajectory of FtsI (Fig. 2B). Consequently, the speed of FtsI  
191 directional movement was tightly coupled to FtsZ's treadmilling speed (Fig. 2C),  
192 recapitulating the experimentally measured near-linear correlation between FtsI's  
193 directional motion with FtsZ's treadmilling speeds in both wildtype and FtsZ GTPase  
194 mutants<sup>13</sup>.

195  
196 The phase diagram (Fig. 2A) also showed that at a constant binding potential between  
197 FtsZ and FtsI, persistent end-tracking of FtsI required an appropriate range of diffusion  
198 constants. If FtsI diffused too rapidly, it could not be confined by the binding potential of

199 the shrinking end of the FtsZ filament. Conversely, when FtsI diffused too slowly, it was  
200 not able to keep up with the speed of departing FtsZ subunits at the shrinking end. Once  
201 it fell behind, the FtsI molecule lost contact with the left most FtsZ subunits permanently.

202

203 We note that an alternative initial condition, in which an FtsI molecule binds in the middle  
204 of a FtsZ filament, will result in the same end-tracking behavior. The bound FtsI molecule,  
205 if has not dissociated, will start end-tracking when the shrinking end of the FtsZ polymer  
206 approaches and mobilizes it (Fig. S1).

207

208 Our modeling also found that coupling to the growing end of an FtsZ filament is unable to  
209 produce the directional movement of FtsI molecules (Fig. S2). As there is no biochemical  
210 evidence showing that the FtsI-binding potential of a newly added FtsZ subunit at the  
211 growing tip will be higher than the ones in the middle of the filament, the addition of a new  
212 FtsZ subunit at the growing end does not bias the diffusion of the FtsI molecule bound at  
213 the original tip to dissociate and re-associate with the new FtsZ subunit. Consequently,  
214 slow-diffusing FtsI molecules will be stuck in the local binding potential, unable to catch  
215 up with the addition of new FtsZ subunits (Fig. S2A), whereas fast-diffusing FtsI  
216 molecules have a high probability of escaping from the tip, because there are no FtsZ  
217 subunits beyond the growing tip to keep it within the vicinity as that in the shrinking tip-  
218 tracking scenario (Fig. S2B). Therefore, FtsI cannot persistently track the growing end of  
219 a FtsZ filament.

220

221 Taken together, our analysis showed that the end-tracking Brownian ratchet mechanism  
222 was able to couple FtsI's directional movement to FtsZ's shrinking end within the  
223 parameter range that is well consistent with experimentally measured data. Furthermore,  
224 the same model could explain the nondirectional movement of the cytoplasmic tail of FtsN,  
225 another divisome protein, in a recent *in vitro* study<sup>25</sup>. In this study, the cytoplasmic tail of  
226 FtsN was reported to follow the tracks of treadmilling FtsZ filaments on a supported lipid  
227 bilayer at the ensemble level. At the single molecule level, however, the FtsN tail only  
228 binds and unbinds FtsZ filaments transiently but does not exhibit directional movement<sup>25</sup>.  
229 Such a scenario could be explained by to our Brownian ratchet model in that the diffusion  
230 of free FtsN cytoplasmic tail anchored on the membrane was too large ( $0.3 - 0.6 \mu\text{m}^2/\text{s}$ )<sup>25</sup>.

231

### 232 **FtsZ's treadmilling speed modulates processivity of FtsI's end-tracking**

233

234 Next, we investigated how FtsZ's treadmilling speed impacts the processivity of FtsI's  
235 directional movement at the shrinking end. Addressing this question will help us  
236 understand the role of FtsZ's treadmilling dynamics in the spatial organization and/or  
237 regulation of sPG synthesis activity. We focused on three features that collectively define  
238 the processivity of FtsI's end-tracking: (1) the propensity, (2) the run distance, and (3) the  
239 duration time of persistent end-tracking trajectories.

240

241 We first examined how the relative propensity of FtsI's persistent end-tracking was  
242 modulated by FtsZ treadmilling speed. The relative propensity is defined as the  
243 percentage of the number of FtsI persistent end-tracking trajectories at each FtsZ

244 treadmill speed, normalized by the total number of FtsI persistent end-tracking  
245 trajectories of all the simulated FtsZ treadmill speeds. Keeping the diffusion constant  
246 of FtsI at  $0.04 \mu\text{m}^2/\text{s}$  and the binding potential at  $10 k_B T$ , stochastic simulations of the  
247 Brownian ratchet model predicted that the relative propensity of persistent end-tracking  
248 trajectories of FtsI dropped off with increasing FtsZ's treadmill speed (Fig. 3A). That is,  
249 when FtsZ treads too fast, FtsI could not persistently track the FtsZ shrinking end in  
250 most cases and became largely diffusive.

251  
252 To further this point, we calculated the phase diagram of FtsI's persistent end-tracking  
253 propensity as a function of both FtsI's diffusion constant and FtsZ's treadmill speed  
254 (Fig. 3B), while keeping the binding potential fixed at  $10 k_B T$ . Again, we used a threshold  
255 of 50% FtsI persistent end-tracking trajectories as the criterion for the phase boundary.  
256 As shown in Fig. 3B, for a fixed diffusion constant of FtsI, there was an upper limit of  
257 FtsZ's treadmill speed that FtsI could persistently track. Conversely, for a fixed FtsZ  
258 treadmill speed, persistent end-tracking of FtsI required an appropriate range of  
259 diffusion constants. Importantly, very large diffusion constants of FtsI ( $> 0.1 \mu\text{m}^2/\text{s}$ ) did  
260 not support persistent end-tracking irrespective of FtsZ's treadmill speed. These  
261 results were consistent with the phase diagram in Fig. 2A and again the recent *in vitro*  
262 study of FtsN's cytoplasmic tail<sup>25</sup>.

263  
264 Next, we investigated how FtsZ's treadmill speed modulates the run distance and  
265 duration time of FtsI's persistent end-tracking. The Brownian ratchet model predicted that  
266 both the run length and duration time of FtsI's persistent end-tracking should display

267 broad distributions due to the stochastic nature of FtsI's diffusion and the interaction  
268 between FtsI and FtsZ. Moreover, the model predicts that when FtsZ's treadmilling speed  
269 increases, the duration time of FtsI's persistent end-tracking will decrease (Fig. 3C),  
270 whereas the run distance will display a biphasic dependence – it increases to peak around  
271 an intermediate FtsZ's treadmilling speed (~ 30 nm/s at the current parameter setting),  
272 and then decreases when FtsZ's treadmilling speed increases further (Fig. 3D).  
273 Importantly, such distinctive dependences of duration time and run distance on FtsZ's  
274 treadmilling speed is a natural consequence of the Brownian ratchet mechanism (SI).

275

276 Qualitatively speaking, when an FtsZ subunit falls off to the cytoplasm from the shrinking  
277 end of the FtsZ filament, the associated FtsI molecule will dissociate from the FtsZ subunit,  
278 either diffuse away on the membrane, or catch up with the next FtsZ subunit in the row to  
279 continue end-tracking, the latter depending on how fast FtsZ treadmills. When FtsZ  
280 treadmills too fast (for example > 30 nm/s), it will be difficult for FtsI to catch up (Fig. 3A),  
281 resulting in early termination of end-tracking, and hence both the persistence run distance  
282 and duration time will be short (right sides of Fig. 3C and D). When FtsZ treadmills  
283 relatively slowly (< 30 nm/s), the probability of FtsI catching up with the shrinking end of  
284 the FtsZ filament is high (Fig. 3A). Therefore, the slower FtsZ treadmills, the fewer number  
285 of dissociation events of an end-tracking FtsI molecule would face, and hence the lower  
286 the chance for FtsI to diffuse away, leading to a longer time duration of persistence run  
287 (Fig. 3C). Within the same time window, however, the persistence run distance will be  
288 proportional to the FtsZ's treadmilling speed as predicted in Fig. 3D, that is, the slower  
289 FtsZ treadmills, the shorter FtsI's persistence run distance is. One can imagine in the

290 extreme case where FtsZ does not treadmill at all ( $V_z = 0$ ), the duration time of persistence  
291 runs would then be the longest and mainly dictated by the intrinsic dissociation rate of  
292 FtsI from FtsZ, and the persistence run distance would be the shortest (i.e., the size of a  
293 single FtsZ subunit). An analytical proof of these relationships is provided in the SI and  
294 Fig. S3.

295

### 296 **Single-molecule tracking of FtsI confirms model predictions**

297 To experimentally examine the model's predictions on the modulation of the processivity  
298 of FtsI's directional movement by FtsZ's treadmilling speed, we performed single-  
299 molecule tracking (SMT) of a functional sandwich fusion protein Halo-FtsI<sub>sw</sub> labeled with  
300 JF646 in live *E. coli* cells<sup>36,37</sup>. To avoid disrupting the cytoplasmic interactions of FtsI's N-  
301 terminal tail with other divisome proteins, we inserted the Halo tag between the last  
302 residue (18) of the N-terminal cytoplasmic tail and the first residue of the inner membrane  
303 helix (19) of FtsI (Fig. 4A). We integrated the *halo-ftsI<sub>sw</sub>* fusion gene into the chromosome  
304 replacing the endogenous *ftsI* gene and showed that it was expressed as a full-length  
305 fusion protein and supported normal cell division as a sole cellular source of FtsI similar  
306 to wild-type (WT) cells (Fig. S4).

307

308 To obtain precise measurements of the persistent run distance and duration time of single  
309 Halo-FtsI<sub>sw</sub> molecules, we trapped individual *E. coli* cells vertically in agarose microholes  
310 made using cell-shaped nanopillar molds as previously described<sup>38,39</sup> so that the entire  
311 circumference of the septum could be visualized at the same focal plane (Fig. 4B). To  
312 determine whether a Halo-FtsI<sub>sw</sub> molecule was at a septum, we labeled the FtsZ-ring

313 using an ectopically expressed GFP-ZapA fusion protein, which we and others have  
314 previously shown as a faithful marker of the Z-ring localization and dynamics<sup>40</sup>. The GFP-  
315 ZapA image also allowed us to unwrap the circular trajectories of FtsI-Halo molecules  
316 along the septum (Fig. 4D) to linear displacements along the circumference of the septum  
317 (Fig. 4E), from which we could measure the persistent run speed, distance, and duration  
318 time (Fig. 4F, G and H).

319  
320 As shown in Fig. 4F, the directional motion speed of Halo-FtsI exhibited a wide distribution,  
321 similar to what we previously observed for FtsZ's treadmilling. The similarity between  
322 FtsI's directional motion speed distribution and FtsZ's treadmilling speed distribution  
323 suggest that at these speed ranges, *E. coli* FtsI can faithfully end-track treadmilling FtsZ  
324 filaments as the model predicted, a point that will become important in the section below.  
325 Most importantly, the persistence run distance and duration time exhibited largely the  
326 same trends as what were predicted by the model: while the run duration time decreased  
327 monotonically (Fig. 4G), the persistence run distance increased and then decreased  
328 when FtsI's speed increased (Fig. 4H). Note here that we inferred FtsZ's treadmilling  
329 speed from FtsI's directional motion speed due to the difficulty of a two-color co-tracking  
330 experiment in the same cells and because we have demonstrated previously that these  
331 two were linearly coupled<sup>13</sup>. Another potential caveat in these experiments was that a  
332 very fast FtsZ's treadmilling speed (*i.e.*, > 80 nm/s) is rare in wildtype *E. coli* cells as we  
333 showed previously. Therefore, given the relatively small dataset for high speed FtsZ  
334 treadmilling, our data cannot definitively determine whether FtsI could effectively end-  
335 track FtsZ filaments of very fast treadmilling speeds. Nevertheless, the agreement of our

336 experimental measurements with theoretical predictions supported the validity of the  
337 Brownian ratchet model.

338

339 **sPG synthase's diffusion and FtsZ-binding potential underlie the dependence of its**  
340 **enzymatic activity on FtsZ treadmilling**

341

342 In *E. coli*, the total amount of septal PG synthesis and the septum constriction rate are  
343 insensitive to perturbations in FtsZ's treadmilling speed from ~ 8 nm/s to ~ 30 nm/s in a  
344 series of FtsZ GTPase mutants<sup>13</sup>. This insensitivity suggests that end-tracking,  
345 directionally-moving FtsI molecules were inactive in sPG synthesis. Indeed, a second,  
346 slow-moving population of FtsW and FtsI (~ 8 nm/s) is found to move independently of  
347 FtsZ's treadmilling, and likely corresponds to the active population of sPG synthesis in *E.*  
348 *coli*<sup>16</sup>. Similarly, in *S. pneumoniae*, FtsW and its cognate TPase PBP2x were found to  
349 move completely independently of FtsZ's treadmilling<sup>41</sup>, likely representing the active  
350 population of sPG synthase as that in *E. coli*. In *B. subtilis*, however, it was shown that  
351 the Z-ring constriction speed is positively correlated with FtsZ's treadmilling speed,  
352 suggesting that the faster FtsZ treadmills, the higher the sPG synthesis activity<sup>12</sup>. How  
353 could the same FtsZ treadmilling dynamics result in different sPG synthesis activity in  
354 different species?

355

356 We propose that since FtsI molecules tracking FtsZ filaments are most likely inactive<sup>16</sup>,  
357 the population of FtsI molecules not tracking with treadmilling FtsZ polymers is then  
358 available for sPG synthesis. Therefore, FtsI's off-rate, or the reciprocal of the time a FtsI



359 molecule spends bound in the middle of FtsZ polymers and/or persistently end-tracking,  
360 represents the rate at which an FtsI molecule becomes available for sPG synthesis, and  
361 therefore is proportional to the sPG synthesis rate. As such, FtsZ's treadmilling speed  
362 could modulate the rate of sPG synthesis in different bacterial species depending on the  
363 unique combination of the enzyme's diffusion coefficient and binding potentials. This  
364 modulation could explain the difference observed between *E. coli* and *B. subtilis*.

365

366 As shown in Fig. 5A, the model suggests that when FtsI diffuses relatively fast ( $\sim 0.05$   
367  $\mu\text{m}^2/\text{s}$ , blue line), the lifetime of FtsZ-bound FtsI is largely insensitive to FtsZ's treadmilling  
368 speed in a 3-fold range from  $\sim 8$  nm/s to 25 nm/s. In contrast, when FtsI diffuses relatively  
369 slowly, the lifetime of FtsZ-bound FtsI is critically dependent on FtsZ treadmilling speed.  
370 For example, at a diffusion constant of  $0.005 \mu\text{m}^2/\text{s}$ , the relative lifetime of FtsI molecules  
371 decreased by  $\sim 70\%$  when FtsZ treadmilling speed increased from  $\sim 8$  nm/s to 25 nm/s  
372 (Fig. 5A, green line).

373

374 The physical reason behind this drastic difference between fast and slow FtsI diffusion  
375 lies at the core of Brownian ratchet mechanism. Fast diffusion will allow FtsI to catch up  
376 with the shrinking end of an FtsZ filament in a very short time (Fig. 5B). When FtsI's  
377 diffusion becomes slower and slower, it eventually becomes the rate-limiting factor in the  
378 Brownian ratchet – a slow FtsI molecule falls behind the FtsZ shrinking end and takes a  
379 long time to catch up with the departing FtsZ filament, or simply diffuses away and  
380 become lost (Fig. 5C). As such, further increasing the FtsZ treadmilling speed in the latter  
381 case will significantly reduce the chance of FtsI keeping up with the FtsZ shrinking end

382 and, hence the lifetime of the FtsZ-bound FtsI. Crucially, this diffusion-modulated  
383 dependence of sPG synthesis on FtsZ treadmilling speed hinges on the binding between  
384 FtsI and FtsZ. When the binding potential is reduced, the lifetime of FtsZ-bound FtsI is  
385 less sensitive to FtsZ speed than its higher-potential counterpart (compare Figs. 5A, 5D  
386 and 5E). Therefore, how FtsZ's treadmilling speed modulates the rate of sPG synthesis  
387 depends on the combined effects of the sPG synthase's diffusion coefficient and FtsZ-  
388 binding potential.

389  
390 To examine this hypothesis, we performed fast frame-rate single-molecule tracking to  
391 measure the diffusion coefficients of free sPG synthase molecules outside the septum  
392 (FtsI in *E. coli* and PBP2b in *B. subtilis*, Fig. 5F). As FtsZ only localizes to the midcell  
393 during cell division, sPG synthase molecules not localized to the midcell are considered  
394 free and not interacting with FtsZ. Using the measured diffusion coefficients, we then fit  
395 the experimentally observed dependence of cell wall constriction rate on FtsZ's  
396 treadmilling speed in *E. coli* and *B. subtilis*<sup>12,42</sup> with the normalized off-rate calculated from  
397 the model. The only free parameter in the model fitting is the binding potential, which was  
398 not possible to measure accurately in live cells with available experimental methods. As  
399 shown in Figs. 5G and 5H, we found that with the apparent diffusion constants of FtsI in  
400 *E. coli* and PBP2b in *B. subtilis* measured at  $\sim 0.041 \pm 0.0051$  (mean  $\pm$  S.E.M., N = 5049  
401 trajectories) and  $0.038 \pm 0.0019 \mu\text{m}^2/\text{s}$  (mean  $\pm$  S.E.M., N = 6765 trajectories) respectively,  
402 and binding potentials set at 8-9 and 10-12  $k_B T$ , respectively, the model quantitatively  
403 recapitulated the differential dependence of cell wall constriction rate on FtsZ's  
404 treadmilling speed in the two species as previously measured. The higher binding

405 potential of PBP2b to FtsZ in *B. subtilis*, likely due to the significantly different protein-  
406 protein interactions in the Gram-positive bacteria, renders tighter coupling between end-  
407 tracking PBP2b molecules with FtsZ than that in *E. coli*, hence the fraction of end-tracking  
408 FtsI can be sensitively modulated by FtsZ's treadmilling speed in *B. subtilis*, but that of  
409 FtsI in *E. coli* cannot.

410

411 As a comparison, we also measured the diffusion of FtsW in *S. pneumoniae* (Fig. 5F).  
412 We found that the apparent diffusion coefficient of FtsW was at  $0.028 \pm 0.0004 \mu\text{m}^2/\text{s}$   
413 (mean  $\pm$  S.E.M., N = 21 trajectories), in the same order of magnitude as that of *E. coli*  
414 and *B. subtilis*. As FtsW does not follow the treadmilling of FtsZ at all in *S. pneumoniae*  
415 at all, it is most likely that the binding potential between FtsW and FtsZ is significantly  
416 lower than  $5 k_B T$  as predicted by the model (Fig. 2A) under the experimental condition. It  
417 remains interesting to investigate in *S. pneumoniae* whether other divisome proteins are  
418 also independent of FtsZ's treadmilling, or they exhibit conditional dependence once the  
419 protein-protein interactions of the divisome are altered due to the presence or depletion  
420 of their binding partners. These possibilities will be further investigated in our future work.

421

## 422 Discussion

423

424 In this work, we presented data supporting a Brownian ratchet model that couples the  
425 directional movements of sPG synthases to FtsZ's treadmilling and underlies the  
426 differential sensitivity of sPG synthesis to FtsZ's treadmilling speed in *E. coli* and *B.*  
427 *subtilis*.

428  
429 We first show that an sPG synthase molecule (here using FtsI as the model enzyme) can  
430 follow a treadmilling FtsZ polymer by end-tracking its shrinking tip but not the growing tip,  
431 due to the intricate interplay between the sPG synthase molecule's diffusion and its  
432 binding potential to FtsZ. Only within a particular diffusion range ( $\sim 0.001$  to  $0.1 \mu\text{m}^2/\text{s}$ )  
433 and at a sufficient binding potential ( $> 5 k_{\text{B}}T$ ), a sPG synthase molecule can exhibit FtsZ-  
434 treadmilling-dependent directional movement (Fig. 2). Furthermore, we show that the  
435 persistence run duration and distance of FtsI exhibit different dependence on FtsZ's  
436 treadmilling speed, as predicated by the Brownian ratchet model (Fig. 3). Using single-  
437 molecule tracking, we confirmed these model predictions (Fig. 4). The ability of  
438 treadmilling FtsZ polymers to modulate the persistence run duration and distance of sPG  
439 synthase molecules could play an important role in regulating the spatial distribution of  
440 sPG synthases to ensure the correct septum shape. Finally, we show that the Brownian  
441 Ratchet model could explain the differential dependence of sPG synthesis activity on  
442 FtsZ's treadmilling speed in *E. coli*, *B. subtilis* and *S. pneumoniae*.

443  
444 Given experimentally measured diffusion coefficients of sPG synthases in different  
445 bacterial species, the Brownian ratchet model predicts that the tighter binding between  
446 PBP2b and FtsZ in *B. subtilis* could cause tighter coupling between them. Hence, the  
447 fraction of time a PBP2b molecule spends on FtsZ, and consequently the fraction of time  
448 it is off FtsZ to become available for sPG synthesis, can be sensitively modulated by  
449 FtsZ's treadmilling speed. In *E. coli* or *S. pneumoniae*, the binding potential between  
450 FtsI/FtsW and FtsZ may not be as high as that in *B. subtilis*, and hence the fraction of

451 time a FtsI/FtsW molecule remains end-tracking FtsZ exhibits much less sensitivity or no  
452 sensitivity at all to FtsZ's treadmilling speed. Different binding potentials between these  
453 different bacterial species are likely, as detailed molecular interactions among the septal  
454 ring complexes are distinct in each species. These results suggest that the same  
455 Brownian-ratchet machinery may be at work but operate in distinct regimes of the  
456 parameter space in different species. Consequently, sPG synthesis depends on FtsZ  
457 treadmilling differentially, reflecting different strategies to meet different functional needs.

458

459 We note that additional factors could also be at play. For example, the expression level  
460 of sPG synthase relative to that of FtsZ in *B. subtilis* is significantly higher than that in *E.*  
461 *coli* or *S. pneumoniae*<sup>43-51</sup> (Table S1). This suggests that the number of sPG synthases  
462 per FtsZ filament in *B. subtilis* would be higher than that in *E. coli* or *S. pneumoniae*. Our  
463 Brownian ratchet model predicts that this condition further enhances the sensitivity of sPG  
464 synthesis activity to FtsZ's treadmilling speed, as sPG synthase molecules bound to the  
465 inner positions of FtsZ polymer would “knock” the end-tracking one off FtsZ (or vice versa,  
466 Fig. S1). Therefore, the faster the FtsZ treadmills, the faster the FtsZ shrinking end  
467 catches up to the sPG synthase in the middle and the more sPG synthase molecules will  
468 be dissociated from FtsZ to become available for sPG synthesis. This is a stark contrast  
469 to the case of a single sPG synthase per FtsZ filament, which is mostly likely the case in  
470 *E. coli* or *S. pneumoniae* (Fig. S1).

471

472 Moreover, the level of cell wall synthesis precursors, for example, could be another  
473 important factor. It is possible that across bacterial species, sPG synthase molecules are

474 coupled to FtsZ's treadmilling dynamics and their lifetime on FtsZ can be sensitively  
475 modulated by FtsZ's treadmilling speed. However, if the level of a cell wall synthesis  
476 precursor is limiting, which is likely the case in *E. coli*, such a sensitivity could be further  
477 masked by the limited level of the precursor<sup>52-55</sup>. In *S. pneumoniae*, besides a low binding  
478 potential between the sPG synthase and FtsZ, cell wall synthesis precursor levels could  
479 also play a role in the independence of FtsZ's treadmilling. High enough levels of PG  
480 precursors could saturate all sPG synthase molecules so that no free ones are available  
481 to track with FtsZ polymers.

482

483 In summary, given the lack of linear stepper motors in prokaryotic world, Brownian  
484 ratcheting appears to be an ancient mechanism for directed cargo transportation in  
485 bacteria – another salient example is ParA-mediated DNA partitioning<sup>56-58</sup>. Interestingly,  
486 a similar Brownian ratchet mechanism also underlies the directional movement of mitotic  
487 chromosomes by end-tracking spindle microtubule in eukaryotes<sup>59</sup>. Can we distill unified  
488 fundamental principle(s) by which evolution shapes the same Brownian ratchet  
489 mechanism to meet distinct needs under different contexts? We will relegate these  
490 exciting questions to our future study.

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492

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504

## 505 **Author Contributions**

506

507 J.W.M., X.Y., G.R.S., K.E.B., M.M.L., and Z.L. performed experiments. J.L. carried out  
508 theoretical modeling. All authors contributed to concept development, data analysis, and  
509 manuscript writing.

510

## 511 **Competing interests**

512

513 The authors declare no competing interests.

514

515

## 516 **Figure Captions**

517

518 Figure 1. Model description. A) Schematic representation of sPG synthase complex's  
519 interaction with FtsZ treadmilling. FtsZ resides in the cytoplasm. FtsI is a transmembrane  
520 protein that does not dissociate from the membrane even when it dissociates from FtsZ.  
521 B) Model simplification of FtsZ – FtsI interaction at the septum. The FtsZ filament (purple)  
522 undergoes treadmilling by dissociating FtsZ subunit from the left end and associating new  
523 ones from the right end. While the FtsI complex (grey) intrinsically diffuses around, it has  
524 a binding affinity to FtsZ subunits. C) Schematics of FtsZ – FtsI binding potentials. Here,  
525 the binding potential is assumed to be harmonic and short-ranged (~ 5 nm), which is  
526 about the size of an individual FtsZ subunit. Note that there is no energy barrier for FtsI  
527 to bind to FtsZ. Once the FtsI binds to a FtsZ subunit, however, the binding potential  
528 presents an energetic barrier preventing the FtsI from diffusing away.

529

530

531 Figure 2. A FtsZ treadmilling-mediated Brownian ratchet mechanism drives FtsI's  
532 directional movement. A) Phase diagram depicting the dependence of FtsI's persistent  
533 end-tracking (blue-shaded region) on FtsI's diffusion constant and FtsI-FtsZ binding  
534 potential. B) A representative simulated trajectory of a FtsI molecule persistently end-  
535 tracking with a treadmilling FtsZ filament. Inset: zoom-in view of a boxed region of the  
536 trajectory. The model parameters for this simulation are: FtsZ's treadmilling speed  $V_z =$   
537 25 nm/s, FtsI diffusion constant  $D = 0.04 \mu\text{m}^2/\text{s}$ , FtsZ–FtsI binding potential  $U = 10 k_B T$ ,  
538 and the simulation time step =  $5 \times 10^{-6}$  s. The full trajectory is plotted every  $10^{-1}$  s, and the



539 zoom-in inset is plotted every  $5 \times 10^{-2}$  s. C) FtsI's directional speed tightly couples with  
540 FtsZ's treadmilling speed. Each of the data points was the average of >80 independent  
541 stochastic simulation trajectories using the segments that undergo directional movement.  
542 FtsZ's treadmilling speed was fixed within each trajectory but varied across the ensemble  
543 following a Gaussian distribution with a standard deviation (SD) of 30%, in line with the  
544 experimental measurements in FtsZ WT and GTPase mutants<sup>13,16</sup>.

545

546 Figure 3. Model predictions for the processivity of FtsI directional movement modulated  
547 by FtsZ treadmilling speed. A) Predicted relative propensity of FtsI persistent end-  
548 tracking as a function of FtsZ's treadmilling speed. For each FtsZ's treadmilling speed,  
549 100 independent stochastic FtsI trajectories were simulated, from which the number of  
550 FtsI persistent end-tracking trajectories were counted. In line with our experimental  
551 measurement, the criteria for persistent end-tracking were as follows: (1) the distance  
552 between the FtsZ shrinking end and the FtsI is less than 100 nm and (2) FtsI persistently  
553 follows the FtsZ shrinking end for greater than 4 seconds. A total of 664 out of 1000  
554 simulated FtsI trajectories were scored as persistent end-tracking, and the number of the  
555 trajectories at each FtsZ's treadmilling speed was normalized as a relative probability of  
556 end-tracking. B) Calculated phase diagram of FtsI persistent end-tracking characterized  
557 by FtsI diffusion constant and FtsZ treadmilling speed. C) Predicted FtsZ treadmilling  
558 speed-dependence of the duration of FtsI persistent end-tracking. D) Predicted FtsZ  
559 treadmilling speed-dependence of run distance of FtsI persistent end-tracking. For the  
560 model calculations in (A-D), the FtsZ–FtsI binding potential is set to be  $10 k_B T$ .

561

562 Figure 4. Experimental characterization of FtsI directional motion. A) Schematic of the  
563 functional sandwich fusion of FtsI. The Halo tag is inserted between residue 18 and 19 of  
564 FtsI, immediately before the first residue of the TM domain. B) Diagram of individual *E.*  
565 *coli* cells loaded in microholes made by nanopillars. C) Brightfield and fluorescence  
566 images of microholes loaded with *E. coli* cells labeled with GFP-ZapA and Halo-FtsI<sub>sw</sub>  
567 fusion proteins. Inset shows the zoomed image of one cell in the yellow box. Blue circles  
568 indicated the pixels used for the fit of ZapA-GFP circle. D) Circle-fitting of GFP-ZapA  
569 image super-imposed with the trajectory of a single FtsI molecule, colored in time. E) the  
570 unwrapped trajectory from D with fitted lines at each segment to extract directional speeds,  
571 persistent run duration and distance. F) Histogram of FtsI's directional movement speeds  
572 (N = 77 trajectories, 74 directional events), with the distribution of FtsZ's treadmilling  
573 speed presented in grey for reference (from Yang et al.<sup>13</sup>). For the purpose of this study,  
574 we only plotted the fast-moving population of FtsI that follows FtsZ's treadmilling and  
575 leave out the slow-moving population of FtsI that we show to be independent of FtsZ's  
576 treadmilling<sup>13</sup>. Both FtsI's and FtsZ's histograms were bootstrapped 100 times for error  
577 bars (showing standard error) and a KS test showed no significant difference between  
578 the two ( $p = 0.1852$ ). G) Dependence of the duration of FtsI's persistent run on its speed.  
579 H) Dependence of the distance of FtsI's persistent run on its speed.

580

581 Figure 5. Dependence of sPG synthase's processivity on FtsZ's treadmilling speed is  
582 modulated by the synthase's diffusion and binding potential. A) Predicted dependence of  
583 FtsZ-bound FtsI's lifetime on FtsZ treadmilling speed with a binding potential of 10  $k_B T$ .  
584 B) A representative trajectory of FtsI's persistent end-tracking when FtsI diffusion is fast

585 (0.05  $\mu\text{m}^2/\text{s}$ ). C) A representative trajectory of FtsI's persistent end-tracking when the FtsI  
586 diffusion is slow (0.005  $\mu\text{m}^2/\text{s}$ ). D) Predicted dependence of FtsZ-bound FtsI lifetime on  
587 FtsZ treadmilling speed with a weaker binding potential (8  $k_B T$ ). E) Predicted dependence  
588 of FtsZ-bound FtsI lifetime on FtsZ treadmilling speed with a stronger binding potential  
589 (12  $k_B T$ ). For A, D, and E, the initial condition is FtsI being randomly positioned along a  
590 FtsZ filament. The lifetime of a FtsZ-bound FtsI molecule was defined as the first time  
591 that the FtsI molecule escapes from the either end of the FtsZ filament for greater than  
592 100 nm. For B and C, a fast diffusion allows FtsI to catch up with the shrinking end of  
593 FtsZ almost immediately, whereas it takes a long time for FtsI to catch up (if it eventually  
594 catches up) when it diffuses slowly. Here, the simulation time step is  $10^{-5}$  s, the model  
595 results are plotted every  $2 \times 10^{-2}$  s. F) Measured mean-squared displacement  
596 (MSD) of FtsI, PBP2b and FtsW in wildtype *E. coli*, *B. subtilis*, and *S. pneumoniae* cells  
597 outside of the septal regions respectively. All three MSD curves were fitted using the  
598 function  $\text{MSD} = 4Dt_\alpha$  since these molecules diffuse on a 2D curved membrane surface.  
599 From these data, the diffusion constants of FtsI, Pbp2b, and FtsW were extracted at 0.041,  
600 0.038, and 0.028  $\mu\text{m}^2/\text{s}$ , with  $\alpha = 0.29, 0.51, \text{ and } 0.71$ , respectively. Error bars represent  
601 standard error of the mean. G) Model fitting of the dependence of sPG synthesis activity  
602 on FtsZ treadmilling speed in *E. coli*. The relative sPG synthesis activity was taken from  
603 previous constriction rate measurements in Coltharp et al<sup>42</sup>. and Yang et al<sup>13</sup>. H) Model  
604 fitting of the dependence of sPG synthesis activity on FtsZ treadmilling speed in *B. subtilis*.  
605 The relative sPG synthesis activity was taken from previous cell division time  
606 measurements in Bisson-Filho et al<sup>12</sup>. For G and H, the model fitting used the measured  
607 diffusion constants as that in F, varied the binding potentials, and determined their upper

608 and lower limits (8 and 9  $k_B T$  for *E. coli*., and 10 and 12  $k_B T$  for *B. subtilis*), which mark  
609 the boundaries of the shaded regions.

610

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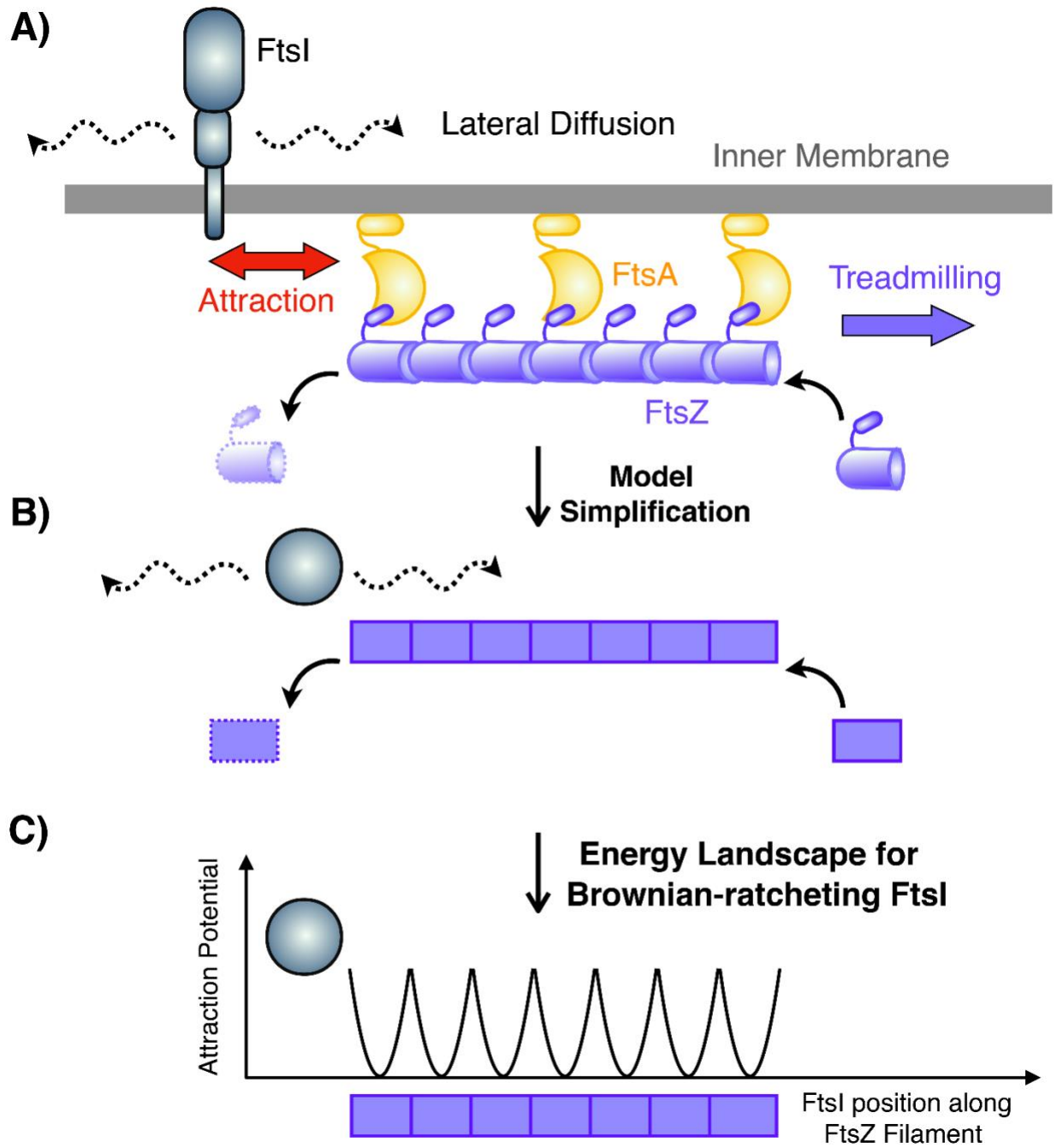
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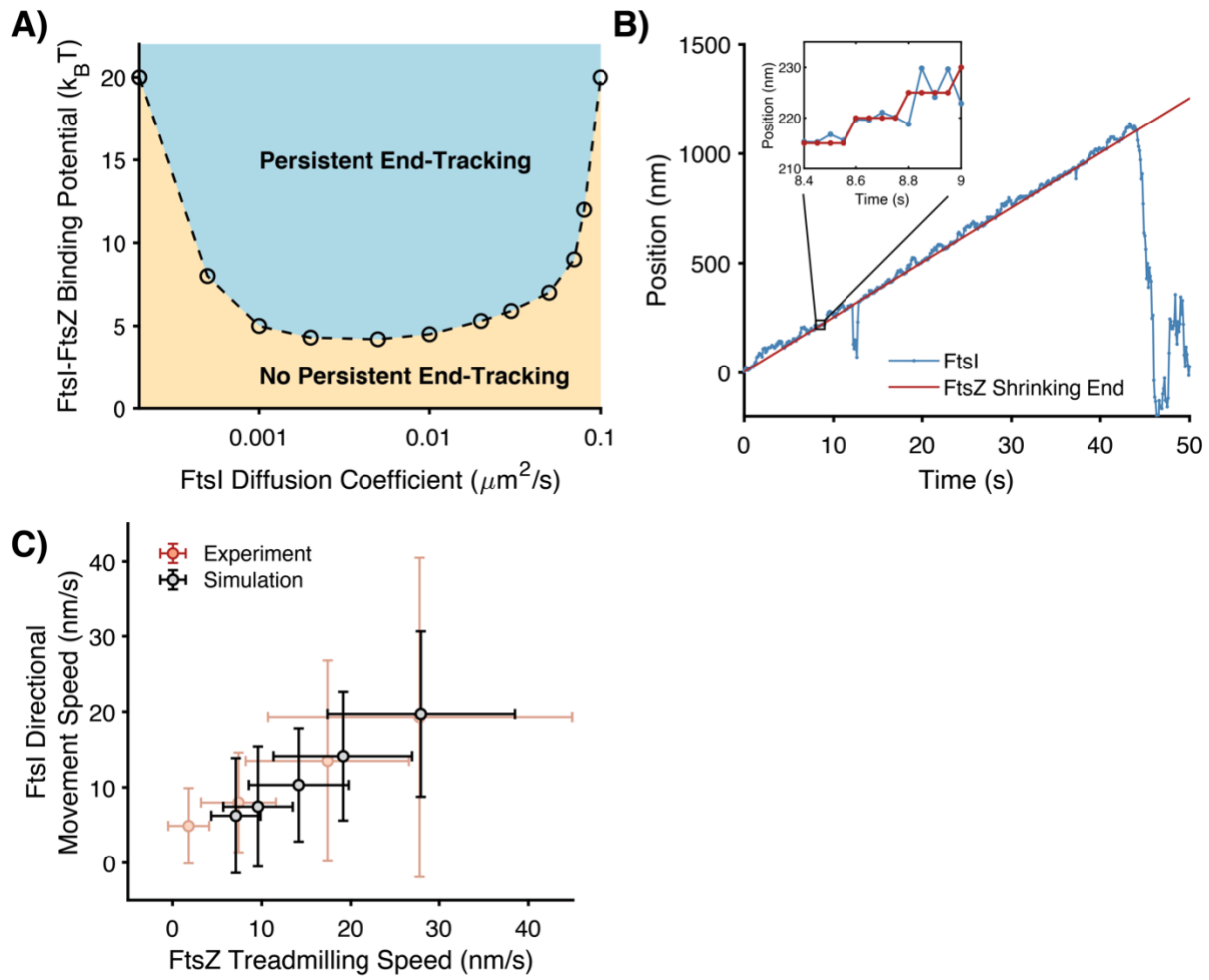
793 **Figure 1**



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796 **Figure 2**

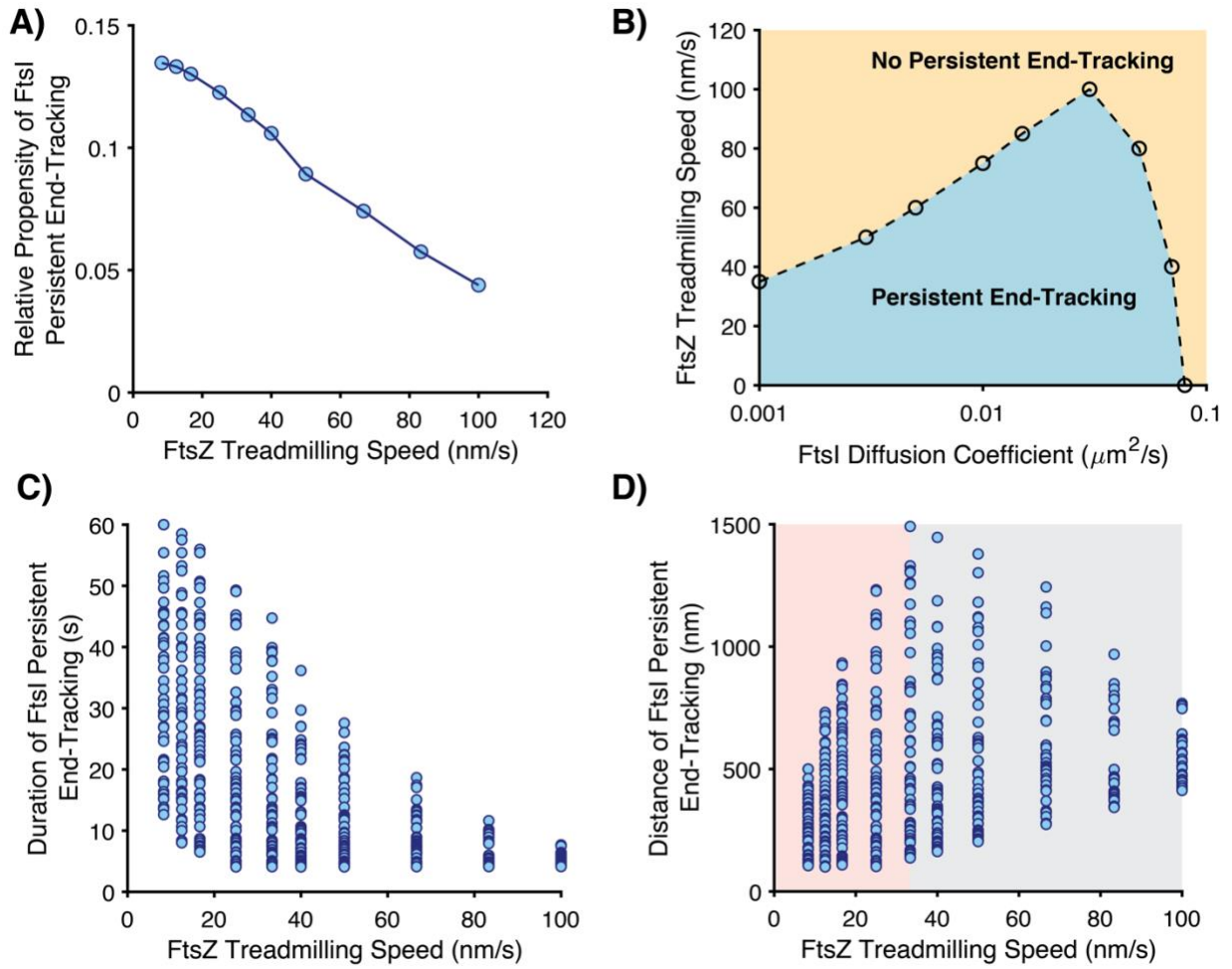


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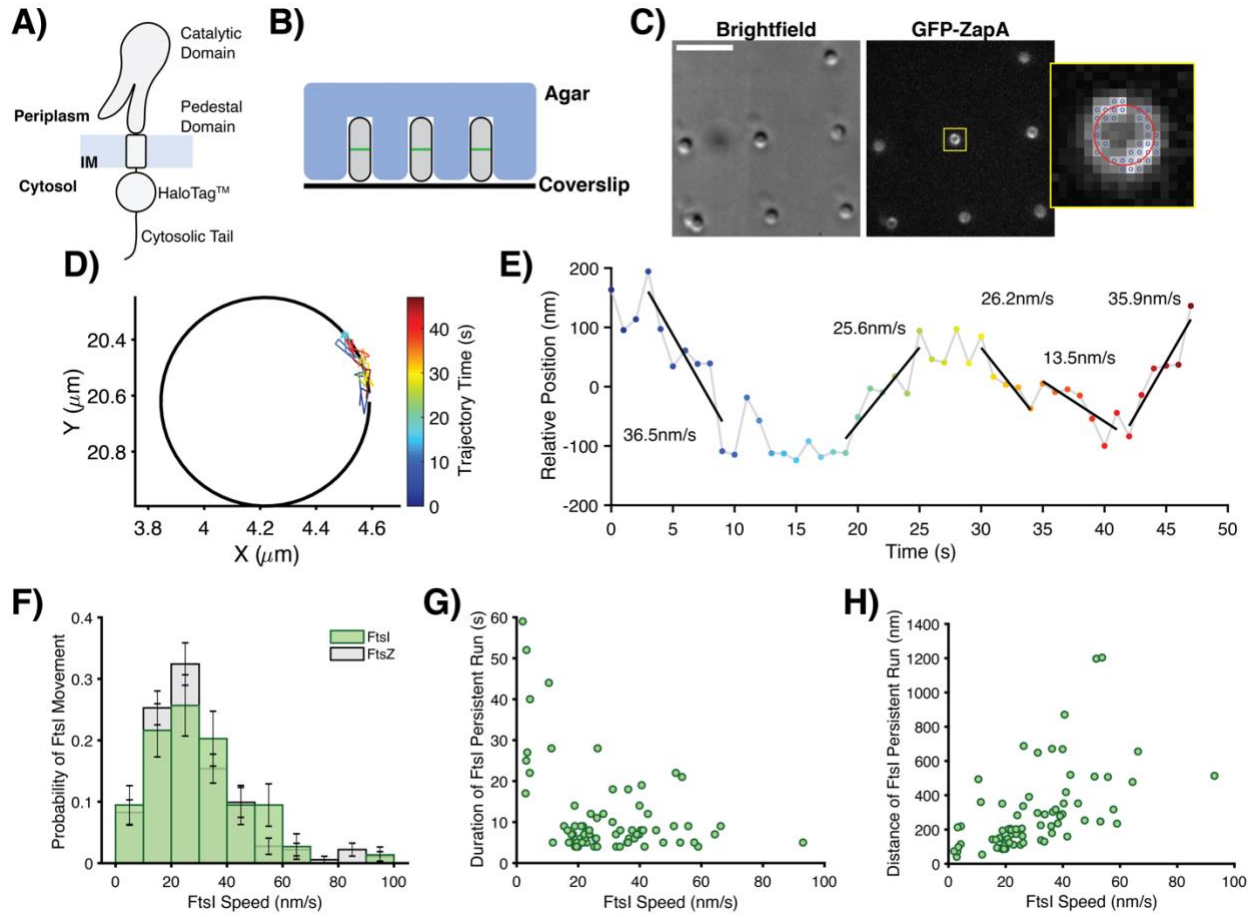
800 **Figure 3**



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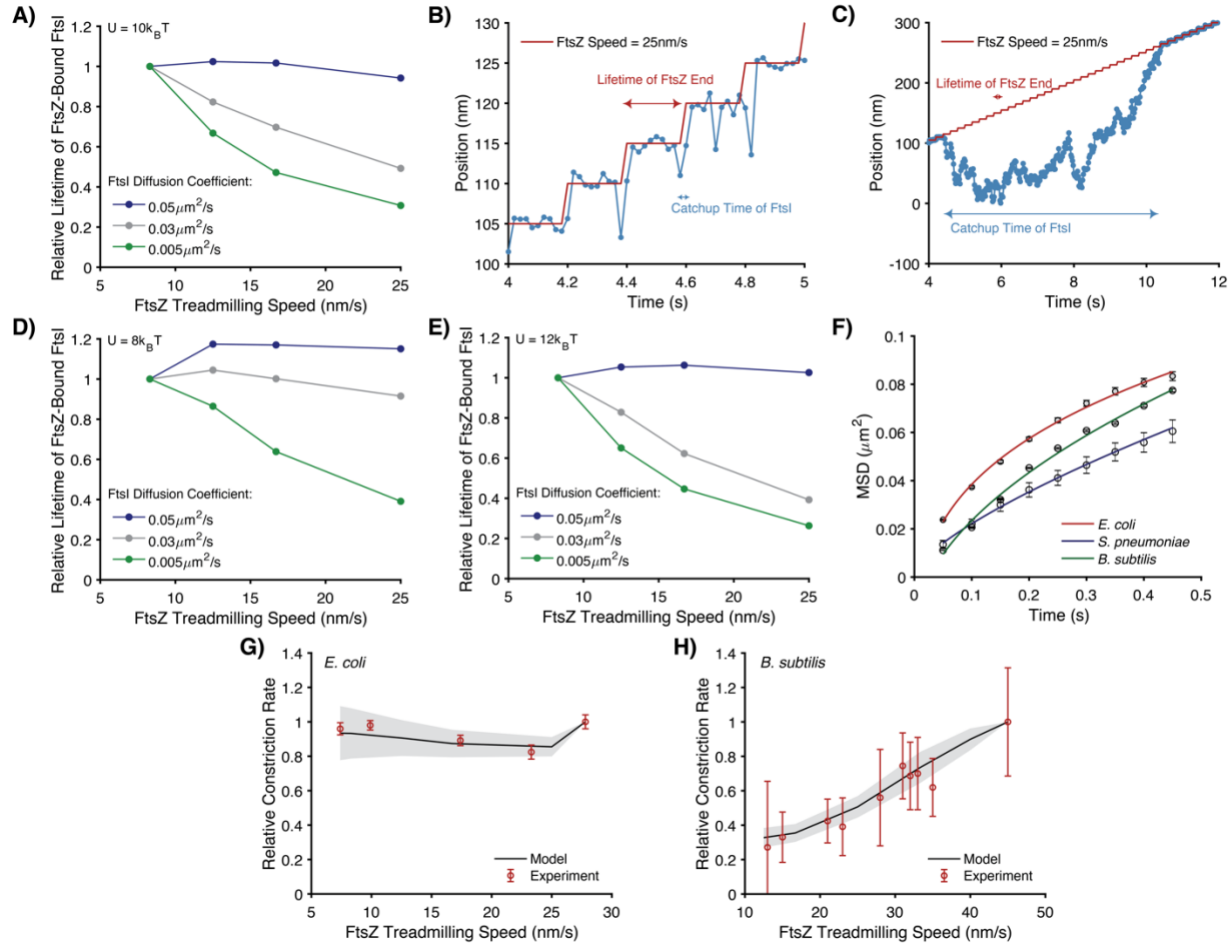
803 **Figure 4**



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806 **Figure 5**



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