Contextualizing Pan-Tropical Allometric Models for Biomass Estimation

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Abstract. Allometric Models (AMs) play a central role in monitoring and mitigating climate change as they provide accurate estimation of biomass and carbon sequestered by trees from non-destructive, easy to obtain physical measurements. Unfortunately, practitioners spend considerable effort in researching, qualifying and choosing AMs for specific growth conditions. To overcome this situation Chave et al. (2014) developed a pan-tropical AM with equivalent accuracy to local, site-specific AMs. We ameliorate this result by incorporating contextual information pertaining to growth conditions in a Machine Learning (ML) model, eventually achieving a reduction in Mean Average Error (MAE) of -17% as measured on hold-out data. This breakthrough shall have important impact in applications such as national forest inventories, carbon certifications and calibration of satellite based biomass maps to field data. To complete, we propose a principled method to estimate how much additional error one can expect when applying a given AM to shifting conditions and provide a data-driven safety check to practitioners.

1 Introduction

AMs support numerous important applications such as National Forest Inventories (NFIs), local afforestation and reforestation projects driven by carbon markets and large-scale scientific efforts to monitor the Earth system. In typical instances AMs are used directly to convert trees measurements in the field into biomass. Recently, this has been combined with Earth Observation (EO) systems that provide Above-Ground Biomass (AGB) maps. These maps rely indirectly on AMs as AGB models predicting AGB from air or space-borne canopy measurements are calibrated to AGB estimates provided by NFIs (Hunka et al., 2024; Santoro and Cartus, 2025). As such, even small improvement in AMs estimation accuracy can lead to potentially huge corrections, as the scale of application encompasses millions of tons of biomass and sequestered carbon (Blaufelder et al.).

Tree allometry has a long history, with early efforts dating from the work of Smith and Brand (1983). It has grown as a research field as modern computing and data became more and more accessible. Nowadays AMs are available for a variety of conditions affecting tree biomass including genetic (e.g. tree species), environmental (e.g. temperature, light, soil) and management-related (e.g. agroforestry system). Efforts have been made to consolidate them in general purpose databases (Henry et al., 2013) or more focused reviews, some of them encompassing several hundreds equations for a single country (Rojas-García et al., 2015). At the same time the scope of application of AMs expands, leading scientists to question how to properly select an AM for specific growth conditions (Van Breugel et al., 2011) as different AMs lead sometimes to very different estimations (Zhao et al., 2012). In practice, scientists and engineers select AMs developed in the closest possible conditions (Lovell and MacKenzie, 2014), as developing new AMs incur cutting, drying and weighing at least 40 trees, a cost that few projects can afford (Návar, 2010). However, there is to the best of our knowledge no quantitative method that can evaluate if one can safely use an AM developed for another context.

At this point we'd like to draw attention to recent developments that highlight an opportunity to improve applicability and accuracy of AMs. First, researchers have started exploring the use of ML models to replace classical power models and reported success in regional studies (Dutta Roy and Debbarma, 2024; Wongchai et al., 2022). Second, a pan-tropical AM introduced in (Chave et al., 2005) and enhanced in (Chave et al., 2014) has been showed to successfully replace site-specific AMs by a single model learned on pooled data from many sites encompassing different continents, for-

Table 1. Problem Variables

	Tree Measurements
D	diameter at breast height (cm)
H	total height (m)
W	wood density ($g.cm^{-3}$)
	Contextual Information
C	continent ($categorical$)
O	old growth ($binary$)
F	forest type ($categorical$)
A	altitude (m)
R	annual rainfall (mm)
M	dry months ($0-12$)
	Tree Biomass (prediction target)
AGB	above ground dry matter (kg)

est types and species. Their model only relies on 3 easily accessible parameters: diameter, height and wood density of trees. Considering AM development from a Statistical Learning point of view, we point out that prediction accuracy on unseen data is the result of a trade-off between data size and model complexity (Luxburg and Schoelkopf, 2008). The pan-tropical AM from Chave et al. (2014) pools data from 58 sites and learns a 3 parameter power model. Following this line of thought we postulate that incorporating additional *contextual information* in the model would grow complexity yet has a chance to enhance prediction accuracy by better describing factors affecting biomass production. This idea is illustrated in Figure 1.

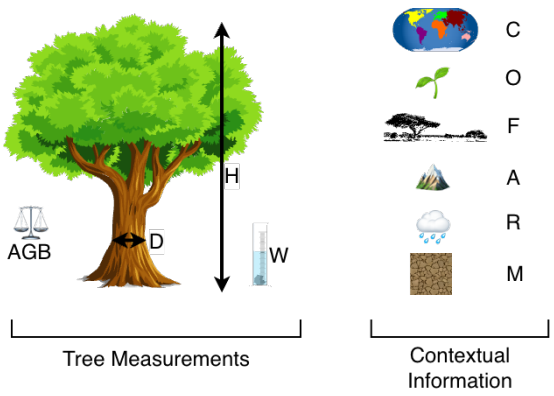

Figure 1. Tree measurements and contextual variables in the biomass estimation problem. Variables described in Table 1.

On the question of choice between available AMs, another insight from statistical learning is that the shift in data distributions between the calibration site (used to fit model coefficients) and application site (where inference takes place) might derail generalization of a model and produce inaccurate estimates. This situation is illustrated in Figure 2 and

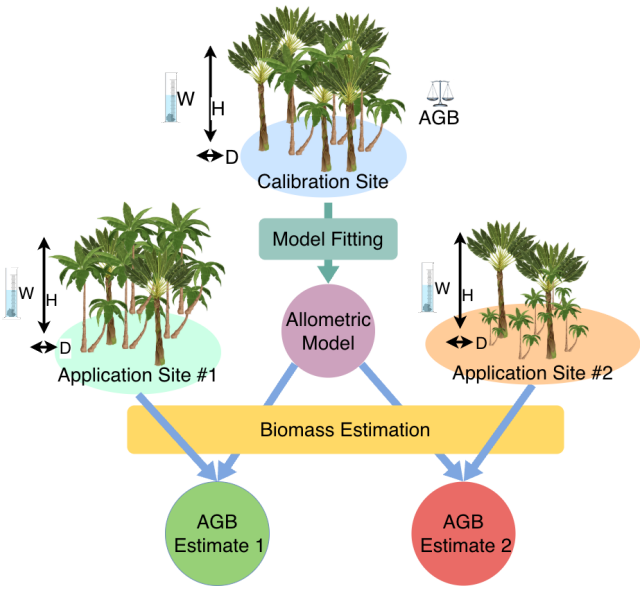

Figure 2. Illustration of how data distribution shift between Calibration site and two Application sites may affect estimated Biomass: a severe shift at site 2 jeopardizes estimated AGB Accuracy while a mild shift doesn't affect site 1 accuracy.

commonly known in statistical learning as Domain Adaptation (DA) (Cortes and Mohri, 2014). Here, we propose to use distribution distance metrics to quantify the shift (Ben-David et al., 2010; Richard et al., 2020) between calibration and application sites and learn a simple approximation of the incurred, additional biomass prediction error.

Our contributions can be summarized as follows:

1. *COFARM-NN*, a deep learning model achieving a reduction in MAE of -17% vs Chave et al. (2014)

2. a practical, data-driven method to estimate additional error when using AMs on new application sites

2 Material and methods

2.1 Data

The dataset from Chave et al. (2014) contains measurements of trunk diameter at breast height D (cm), total tree height H (m), wood density W ($g.cm^{-3}$), and total oven-dry above ground dry matter AGB (kg) for 4004 trees across 58 sites worldwide. We filter the dataset to focus on the portion of sites for which additional covariates are provided in (Chave et al., 2005) and described in Table 1. We exclude sites that were found to contain unreliable data and later removed in Chave et al. (2014) to obtain a contextual dataset of 1,481 trees that is used for all experiments.

2.2 Contextual AM Models

2.2.1 Contextual Models Definition

We compare different models against Model 1 from Chave et al. (2014):

$$AGB = \alpha(D^2HW)^\beta \tag{1}$$

trained as explained in Eq. 2 of the original paper.

First, we select context-agnostic baselines. We train *LogReg*, a simple log-log linear regression of

$$logAGB = \alpha logD + \beta logH + \gamma logW \tag{2}$$

to which we adjunct an L^2 regularization. We also train *HGBRT*, a Gradient Boosted Decision Trees regression model with a maximum of 1,000 iterations, access to all covariates (tree measurements & contextual information) (Shi et al., 2022) and hyper-parameter optimization to find relevant regularization parameters.

Then, assuming contextual information can provide valuable insights to the model, we add *ContextualChave*:

$$AGB = \alpha(D^2HW)^{\beta_0}\tilde{C}^{\beta_1}\tilde{O}^{\beta_2}\tilde{F}^{\beta_3}A^{\beta_4}R^{\beta_5}M^{\beta_6} \tag{3}$$

where $\tilde{}$ designates Target Encoding (Pargent et al., 2022), a popular and efficient method to encode categorical variables.

Finally, we propose 3 deep learning models:

- *LogReg-NN*, an evolution of the log-log regression on D, H, W with 2 additional fully connected layers of 3 neurons before the final regressor
- *COFARM* a model that embeds categorical covariates C, O, F in 2 dimensions before pooling them with D, H, W, A, R, M in a log-log linear regressor
- *COFARM-NN*, inspired by wide and deep architectures (Cheng et al., 2016) pools embedded categorical covariates C, O, F with A, R, M numerical features and passes resulting information through 3 layers of 3 fully connected neurons and ReLu blocks before the final regressor, as depicted on Figure 3.

These models are implemented in PyTorch (Paszke et al., 2019) and trained with the Adam optimizer (Kingma and Ba, 2017).

2.2.2 Contextual Models Evaluation

We perform 30 random train/test splits of proportion .8/.2 on the dataset of Chave et al. (2014), learning each model on train data and evaluating on hold-out test data. We report average value over the 30 splits and 90%-tile confidence intervals (Sileshi, 2014). Note that this methodology incorporates randomness of both data distribution and learning for non-convex methods such as deep neural networks.

2.3 Additional Error Prediction in Shifting Conditions

2.3.1 Preliminary: Unsupervised Domain Adaptation

The application of AMs to new sites can be seen as a problem of Unsupervised Domain Adaptation (DA). We observe that coefficients of an AM fitted on a calibration site may be suboptimal compared to ones that would have been fitted directly on the application site because of the difference in specific growth conditions between both sites. A key observation is that a shift in the distribution of covariates is indicative of such a situation. For instance, suppose an AM developed for a given biome and species mix in a primary forest is applied to plots from a similar biome and species mix but in a secondary forest. It is likely in this case that the AM will make a larger error in predicting AGB from covariates as its assumptions would be violated to a certain point when applying it on data from the application site. It is also likely that a shift in the data distribution between datasets collected at both sites will be observed, for instance larger trees could be observed in the primary forest compared to the secondary one.

We now formalize the DA problem in statistical terms following notation from (Wilson and Cook, 2020). We define as *source* distribution the distribution of covariates in the calibration site and *target* distribution the one of the application site. We consider a predictor $h : \mathcal{X} \to \mathcal{Y}$, source and target datasets $\mathcal{D}_s = (X_s, f_s)$ and $\mathcal{D}_t = (X_t, f_t)$ and a loss function $L : (\mathcal{Y}, \mathcal{Y}) \to \mathbb{R}$. We are interested in the expected error in the target domain of a hypothesis (or predictor) $h \in \mathcal{H}$: $\epsilon_t(h) = \mathbb{E}_{x\sim p_t}[L(f_t, h(x))]$. To develop a practical estimator $\hat{h}$ we assume access to an empirical source dataset $\hat{\mathcal{D}}_s = \{x_s, f_s\}_{i=1\ldots n}$ and target covariates $\hat{X}_t$; that allows to estimate the expected error of h on source $\epsilon_s(h) = \mathbb{E}_{x\sim p_s}[L(f_s, h(x))]$ while the true target label f_t remains unobserved.

Many approaches have been proposed to tackle this problem, mostly in the classification case and often applied to image related tasks (Gulrajani and Lopez-Paz, 2020). For regression, we rely on (Richard et al., 2020) that establish a theoretical upper bound on the expected target error $\epsilon_t(f)$ using tools from statistical learning theory. We restate their Proposition 1 for convenience:

$$\epsilon_t(h) \leq \epsilon_s(h) + \eta_{\mathcal{H}}(f_s, f_t) + \mathcal{H}Disc_{\mathcal{H},L}(p_t, p_s, h) \tag{4}$$

where $\eta_{\mathcal{H}}(f_s, f_t)$ is the sum of the errors made by the ideal predictor on both source and target domains and $\mathcal{H}Disc_{\mathcal{H},L}(p_t, p_s, h)$ is the hypothesis-discrepancy associated with h and defined as

$$\mathcal{H}Disc_{\mathcal{H},L}(p_t, p_s, h) = \max_{h'\in\mathcal{H}} |\mathbb{E}_{x\sim p_t}[L(h'(x), h(x))] - \mathbb{E}_{x\sim p_s}[L(h'(x), h(x))]| \tag{5}$$

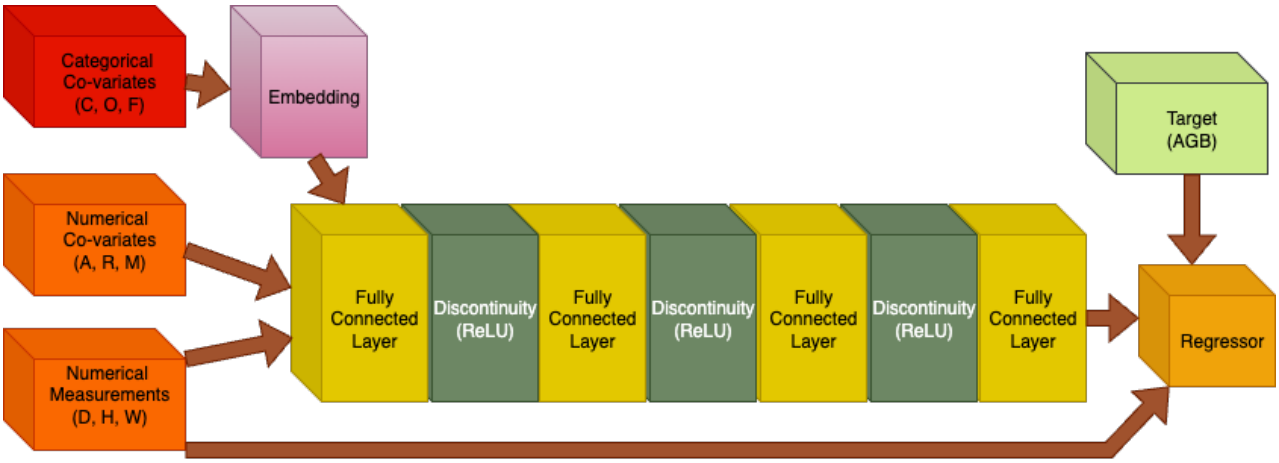

Figure 3. *COFARM-NN* model architecture for AGB prediction. Refer to Table 1 for description of variables.

While η is purely related to the labeling discrepancy, $\mathcal{H}Disc$ measures the discrepancy between source and target distributions, relatively to the model h. In other words, this bound suggests that the additional error made when applying a regression model learned on a source dataset to a target dataset can be quantified by measuring the shift between source and target covariates distributions.

2.3.2 Additional Error Model

Considering that the prediction error in the target domain can be related to the shift between source and target covariates, we postulate that the suitability of an AM to new, potentially shifting conditions can be modeled. Let us consider $\Delta_{s,t} = \epsilon_t(h) - \epsilon_s(h)$, the additional error made when applying an AM to new, shifted conditions. We consider from now on L to be the MAE which is both telling for practical usage and respects the triangle inequality and symmetry assumptions in Richard et al. (2020). We propose a quadratic model of the additional error

$$\begin{aligned} \Delta_{s,t} MAE(h) &= \alpha \times \mathcal{H}Disc_{\mathcal{H},L}(p_t, p_s, h) \\ +\beta \times sign(\Delta_{s,t} Z) &+ \gamma \times \mathcal{H}Disc_{\mathcal{H},L}(p_t, p_s, h)^2 \\ +\delta \times (sign(\Delta_{s,t} Z) &\times \mathcal{H}Disc_{\mathcal{H},L}(p_t, p_s, h)) \end{aligned} \quad (6)$$

where $\Delta_{s,t} Z = \mathbb{E}_{X \sim p_s}[D^2HW] - \mathbb{E}_{X \sim p_t}[D^2HW]$. The model is learned as a Ridge regression with L^2 penalty strength optimized by internal cross-validation. To estimate $\mathcal{H}Disc$ in practice we solve the following constrained optimization problem

$$\max_{\alpha', \beta'} \left| \left(\hat{\epsilon}_s(f_{\alpha,\beta}) - \hat{\epsilon}_s(f_{\alpha',\beta'})\right) - \left(\hat{\epsilon}_t(f_{\alpha,\beta}) - \hat{\epsilon}_t(f_{\alpha',\beta'})\right) \right| \quad \text{s.t. } |\alpha' - \alpha| < \delta_1 \text{ and } |\beta' - \beta| < \delta_2 \quad (7)$$

, where $\hat{\epsilon}$ is the empirical MAE. In other words we assume $\mathcal{H}$ to be the family of models which parameters reside inside a ball of fixed radius centered on coefficients of the Chave model fitted on the source site data. The solution is found by solving successive quadratic optimization with the *SLS-QP* algorithm (Nocedal and Wright, 2006).

2.3.3 Synthetic Distribution Shifts

A central piece of development of the additional error mode is to be able to create distribution shifts experimentally. We do so by splitting data according to contextual information to produce fake calibration and application sites. This ensures existence of a shift in tree measurement and biomass distributions between the two sub-parts of the data.

For each numerical variable (A, R, M), we compute the 25th and 75th percentiles and use them as thresholds to divide the distribution into two sub-distributions, corresponding to source and target datasets. Table 2 shows the thresholds used to implement the shifts. For example, for altitude, the 25th percentile is 42.0 m ("altitude-low" shift). The source dataset is thus composed of trees located < 42 m, and the target dataset encompasses the trees located >= 42 m. C (Asia or Americas) and O (0 or 1) have only two categories. Source and target datasets are thus created by separating the trees based on one of the two values. F encompasses four categories. Each one of them has been used to create a supplementary shift in distributions. We also consider two random shifts by randomly separating the dataset, resulting to a total of 14 shift types. Finally, we also implement reversed shifts (indicated by $\longleftarrow$) by switching source and target datasets leading to a total of 28 shifts. For each of them, the objective is to train the AM from Chave et al. (2014) on the source dataset and to predict AGB and the additional error on the target dataset without observing target AGB

To illustrate the magnitude of the discrepancy between source and target datasets for each shift we compute the Wasserstein distance on D^2HW (Panaretos and Zemel (2019)). Moreover, a significant shift in distributions implies

Table 2. Reference values used to experimentally create the shifts with Wasserstein distance and $\mathcal{H}Disc$ between source/calibration and target/application.

Var.	Shift	Reference	Wass. Dist	$\mathcal{H}Disc$
A	altitude-high$\longrightarrow$	100.0	8954.7	1070.9
A	altitude-low$\longrightarrow$	42.0	9736.9	2198.8
C	continent-asia$\longrightarrow$	Asia	7019.4	1611.0
M	drymonths-few$\longrightarrow$	1.0	25083.8	7225.7
M	drymonths-many$\longrightarrow$	4.0	11524.2	1718.7
F	foresttype-dry$\longrightarrow$	Dry	11524.2	1718.7
F	foresttype-mangrove$\longrightarrow$	MoistMangrove	10632.2	6656.6
F	foresttype-moist$\longrightarrow$	Moist	3986.4	489.1
F	foresttype-wet$\longrightarrow$	Wet	18408.5	3090.1
O	oldgrowth-0$\longrightarrow$	0	11702.0	4628.4
R	rainfall-high$\longrightarrow$	3125.0	16162.3	2824.7
R	rainfall-low$\longrightarrow$	1862.0	11816.2	3965.9
-	random-1$\longrightarrow$	-	2400.3	126.8
-	random-2$\longrightarrow$	-	3954.3	1488.6

that it would be possible to predict the origin of a tree (source or target distribution) based on its covariates values (D^2, H, and W). To illustrate this statement, we train a Gradient Boosted Decision Trees classifier model (Friedman, 2002) on the Chave dataset to predict whether a tree belongs to the source dataset or not, for each shift ((Lopez-Paz and Oquab, 2018)). If it performs better than random it confirms that a shift exists and is significant.

2.3.4 Additional Error Model Training and Evaluation

In order to evaluate the proposed model we first randomly split the 1481 trees of our data into $\mathcal{D}_{train}$ (741 trees) and $\mathcal{D}_{test}$ (740 trees). D_{train} is used to fit coefficients of Equation 8 while D_{test} is used to evaluate the capacity of the model to correctly predict additional error.

For the training we randomly split D_{train} into $i = 1 \dots n$ sub-datasets $\mathcal{D}^i_{train-train}$ and $\mathcal{D}^i_{train-valid}$. In turn, each of these n sub-datasets generates $s = 1 \dots S$ synthetic shifts by applying the procedure detailed in Section 2.3.3. A Chave model is then fitted on each $\mathcal{D}^{i,s}_{train-train}$ instance and inference performed on corresponding $\mathcal{D}^{i,s}_{train-valid}$. Every (i, s) pair thus contributes one training sample of the form $(\Delta MAE^{i,s}, \mathcal{H}Disc^{i,s}, sign(\Delta Z)^{i,s})$. With $n = 2000$ repeats and 14 shifts in 2 directions this procedure generates a dataset $\mathcal{D}^{augmented}_{train}$ of 56000 samples. The model is then trained on a standardized version of this dataset with a square loss and L^2 penalty strength optimized through cross-validation.

For the evaluation, $\mathcal{D}_{test}$ is first divided into $\mathcal{D}^s_{test-calibration}$ and $\mathcal{D}^s_{test-application}$ datasets based on the same $s = 1 \dots S$ shift definitions. A Chave model is trained on $\mathcal{D}^s_{test-calibration}$ and ΔMAE^s computed from $\mathcal{D}^s_{test-calibration}$ and $\mathcal{D}^s_{test-application}$. This value is then compared to the additional error predicted by Equation 8 based on $\mathcal{H}Disc$ and $sign(\Delta Z)$.

3 Results

3.1 Contextual Models Performance

As can be seen on Table 3, our proposed approach *COFARM-NN* reduces significantly the MAE by 17% (or 31.5 kg per tree) compared to the original AM and improves R^2. It also reduces RMSE yet the result is not significant, possibly due to data size and inherent variance of the metric.

The *COFARM* model also improves significantly the test MAE by 14%, which indicates that the deep part of the *COFARM-NN* model accounts for an additional 3% reduction and confirms the usefulness of additional contextual information. Including this information into *ContextualChave* improved a bit the test MAE, however its performance stayed behind *COFARM-NN* ones.

LogReg performs comparably to the original AM which is expected as the log-log regression is equivalent to learning the power model from Chave et al. (2014). However adding deep layers in *LogReg-NN* doesn't produce gains vs *LogReg*, indicating that just increasing representation power is not sufficient to improve performance.

HGBRT, a different kind of method but having access to the same contextual information behaves surprisingly badly (just on par with a simple log-log linear regression), even though the model is considered a strong baseline in many regression problems (Shi et al., 2022). While the root cause is unclear, we believe it could be linked to inefficient regularization that leads to over-fitting, as indicated by a large difference between train and test metrics.

All in all, these results provide evidence of the generalization power of the *COFARM* and *COFARM-NN* models. Besides, the origin of the accuracy gains of *COFARM-NN* comes from better, more conservative calibration of predictions as can be observed on Figure 4. Indeed, the *COFARM-NN* calibration curve is closer to the ideal value of 1.0, especially so for small but also large AGB (> exp(6) = 400 kgs), an area where the original AM is be suboptimal.

3.2 Evidence of Covariate Shift

Table 2 shows the Wasserstein distance for each shift type that has been implemented. The distance is the lowest for random shifts, meaning that there is little difference between source and target distributions for those shifts. $foresttype-moist$ shift shows also a low distance, meaning there is only a minor shift of distribution. $drymonths-few$ has the highest distance, highlighting a very different distribution for the trees located in places where there is no dry months. $\mathcal{H}Disc$ is also the highest for $drymonths-few$ showing the maximal discrepancy for this shift. However, there is no linear relationship between the Wasserstein distance and $\mathcal{H}Disc$, i.e. a higher distance is not always related to a higher $\mathcal{H}Disc$. This is probably because the distance is model-agnostic and

Table 3. Predictive power of different AMs (**bold** = best average, * = statistical significance at 90% vs Chave et al. (2014))

Dataset	Metric	Chave et al. (2014)	*ContextualChave*	*HGBRT*	*LogReg*	*LogReg-NN*	*COFARM*	*COFARM-NN*
train	RMSE	829.9 ± 23.3	822.8 ± 21.2	628.8 ± 24.8	762.3 ± 18.9	762.3 ± 18.9	692.2 ± 20.2	**590.0** ± 35.2
train	MAE	175.0 ± 2.9	163.4 ± 2.3	**83.2** ± 2.4	168.8 ± 2.6	168.8 ± 2.6	147.7 ± 2.3	130.6 ± 3.8
train	R^2	0.888 ± 0.007	0.890 ± 0.006	0.936 ± 0.004	0.905 ± 0.006	0.905 ± 0.006	0.922 ± 0.005	**0.942** ± 0.007
test	RMSE	804.9 ± 80.7	797.6 ± 78.9	785.4 ± 111.1	747.3 ± 69.8	747.2 ± 69.8	**694.6** ± 71.7	702.9 ± 91.7
test	MAE	180.4 ± 11.1	168.0 ± 9.3	170.5 ± 13.7	173.6 ± 10.2	173.6 ± 10.2	155.4 ± 9.2* [-14%]	**148.9** ± 9.4* [-17%]
test	R^2	0.859 ± 0.025	0.866 ± 0.027	0.813 ± 0.062	0.865 ± 0.030	0.865 ± 0.030	0.880 ± 0.029	**0.881** ± 0.026

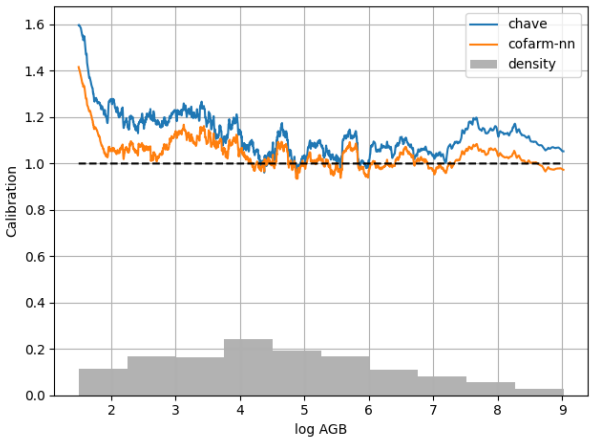

Figure 4. Calibration of AGB predictions

rely only on covariates distributions while $\mathcal{H}Disc$ is conditional to the fitted Chave model.

For all shifts except the random ones a Gradient Boosting Classifier is able to learn to predict when samples come from source or target datasets (data not shown, p-value < .0001). This highlights a clear shift in the covariates distributions between the source and target distributions even when the Wasserstein distance is low.

3.3 Generalization Error Prediction

Figure 5 shows the predicted additional MAE due to distributions shift as a function of the observed one. Our model was able to predict $\Delta_{s,t}MAE$ with a high precision (R^2 = 0.832) meaning that it was able to learn the additional error based only on $\mathcal{H}Disc$ and $sign(\Delta_{s,t}Z)$. The red area highlights where predictions would have been falsely optimistic (positive $\Delta_{s,t}MAE$ predicted as negative) and potentially lead to qualifying an AM when it should not. We observe only 1 data point in this case. These results show that our model is conservative and able to predict the sign and magnitude of the additional error correctly. More details about MAE values for each shift can be found in Appendix A1.

The final model equation stands as follows:

$$\begin{aligned}\Delta_{s,t}MAE(h) = -19.69 \times \mathcal{H}Disc_{\mathcal{H},L}(p_t, p_s, h) \\ +13.72 \times sign(\Delta_{s,t}Z) + 44.86 \times \mathcal{H}Disc_{\mathcal{H},L}(p_t, p_s, h)^2 \\ +153.80 \times (sign(\Delta_{s,t}Z) \times \mathcal{H}Disc_{\mathcal{H},L}(p_t, p_s, h))\end{aligned} \tag{8}$$

The coefficient for $\mathcal{H}Disc$ is negative, however the one for $\mathcal{H}Disc^2$ is twice as big and positive leading to a global positive effect of $\mathcal{H}Disc$ on $\Delta_{s,t}MAE$. This means that $\Delta_{s,t}MAE$ will get bigger when $\mathcal{H}Disc$ increases. The coefficient for $sign(\Delta_{s,t}Z)$ being positive, it means that $\Delta_{s,t}MAE$ will be higher when $\mathbb{E}_{X\sim p_s}[D^2HW] > \mathbb{E}_{X\sim p_t}[D^2HW]$, i.e. when source covariates are bigger than target covariates. The interaction term between $sign(\Delta_{s,t}Z)$ and $\mathcal{H}Disc$ shows the biggest coefficient highlighting that when $\mathbb{E}_{X\sim p_s}[D^2HW] > \mathbb{E}_{X\sim p_t}[D^2HW]$ a high $\mathcal{H}Disc$ will increase the $\Delta_{s,t}MAE$. On the contrary, when $\mathbb{E}_{X\sim p_s}[D^2HW] < \mathbb{E}_{X\sim p_t}[D^2HW]$ a high $\mathcal{H}Disc$ will decrease $\Delta_{s,t}MAE$.

4 Discussion

4.1 Summary of Key Findings

We present a novel deep-learning model to predict AGB from easy to obtain tree measurements and contextual information. This model proves to reduce significantly predictions errors, especially for large trees. The proposed model is also able to incorporate additional information related to growth conditions. We also note that reduced error in AGB estimation should decrease uncertainty when estimated using Monte-Carlo noise injection methods as in (Réjou-Méchain et al., 2017).

Moreover, we propose a principled method to predict the additional error incurred by changing growth conditions between the site where a given AM is learned and the one where it is applied. This method can be used with any AM given that one has access to tree measurement data from both sites (AGB data is not required), allowing practitioners to test applicability and judge it from observed data.

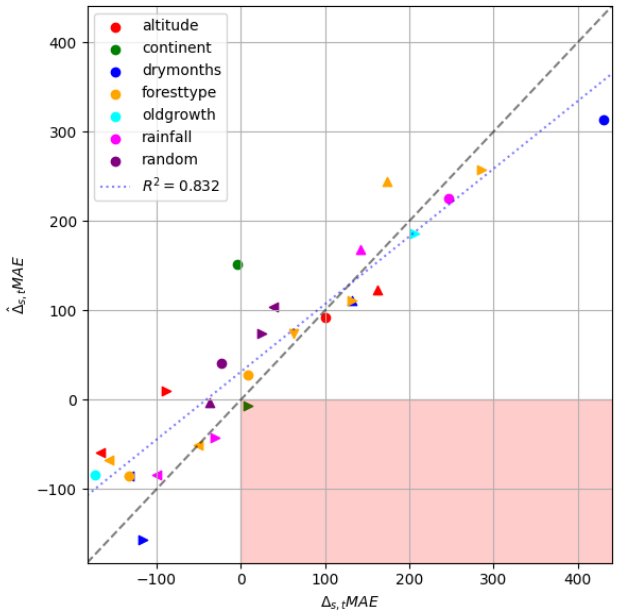

Figure 5. Predicted vs Actual Additional Error due to Covariate Shift. Color indicates the variable chosen to simulate the shift while marker shapes indicate different shifts magnitudes and directions.

4.2 Limitations

Limitations of present work include data, modeling and methodological considerations. First, we'd like to praise the authors of Chave et al. (2014) for the constitution of this incredibly helpful dataset. Yet, the datasets we use are still imperfect and may potentially misrepresent characteristics of tropical tree growth as they can be found in the wide area of tropical forests. In that respect we inherit the limitations of the original work, including the applicable range of D (5-156cm) and the exclusion of palms and liana species. More diverse open source biomass datasets would be required to develop more general pan-tropical AMs. Experiments on adding contextual information to AMs could be conducted only on the portion of the dataset that contains such information, totaling 1,481 trees. Second, even though we took care to evaluate a significant number of ML models, including linear, tree-based and deep neural networks, we possibly missed more accurate models. Finally, our proposed error prediction method is based on established statistical learning theory, yet the generalization bounds tightness can be questioned. Even if our experiments suggest a strong link and predictability in practice, only a larger scale use of the technique would give necessary feedback. Additional studies on other potential covariate shifts between conditions for learning and applying an AM would be of great value to the community.

4.3 Recommendation on Applicability of Allometric Models

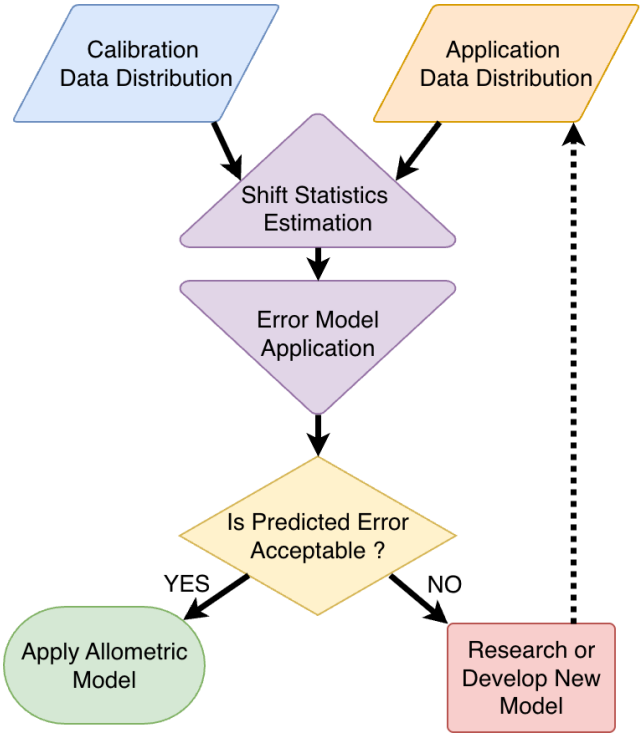

Figure 6. Recommended Process for Qualifying or Adjusting Allometric Models based on Predicted Additional Error.

In light of the experiments in Section 3.3, we propose a data-driven sanity check to decide if a given AM is suitable to be applied to a new dataset of tree measurements. The process is depicted in Figure 6 and starts with the computation of statistics on both the calibration dataset used to fit the AM and the newly considered dataset. It follows by injection of these statistics in the model of Equation 8 an estimation of the error incurred by the use of this AM in the new application site. The principal can then declare if the forecasted error falls within her pre-defined acceptable error range. If the answer is positive, the biomass predictions of the AM are judged satisfying and can be used. Conversely, an alternative AM should be considered; and if unfortunately none is available a case made to develop a new AM using destructive measurements. The advantage of this process are three-fold. First, it bases AM selection on objective, data-driven criteria. Second, it provides trust and visibility as shift statistics may be openly communicated to stakeholders. Finally, it motivates creation of new AM and the incurred costs on a strictly needed basis. We note that the proposed sanity check should be understood as a necessary yet not sufficient condition and by no means qualifies a given AM by itself, but rather adds an objective criteria of applicability. Indeed, the absence of a distribution shift on tree measurements doesn't guarantee

that the AM captured the relation of the former with AGB and we still recommend in addition to follow the prescriptions in each AM reference publication.

4.4 Perspectives

We foresee exciting future works in many directions. On the allometric modeling side, we see recent developments in Geospatial Foundation Models (Brown et al., 2025) as an opportunity to bring even more contextual information to AMs in an efficient manner. We also anticipate that deep embeddings used to represent contextual information could well incorporate species information when it is available. On the methodological side, we encourage researchers in the field to get more into the habit of releasing primary data pertaining to AM development. Not only would it allow more modeling research to happen, but also we could increase reliability of AM choices by generalizing the kind of safety checks we propose to all AMs. More importantly, as a scientific field, open data is a formidable opportunity to develop better, unified models that one day could cover the whole spectrum of tropical forests, both primary and managed.

Author contributions. Conceptualization: E.D.; Methodology: E.D.; Software: A.D., E.D.; Experiments: A.D., E.D.; Formal analysis: E.D.; Writing: A.D., E.D.

Competing interests. A.D. and E.D are employed by PUR, a leading project implementer of Nature Based Solutions in which allometric equations may be used in the process of generating carbon credits under different certification standards. Note that the proposed model produces more conservative biomass estimates compared to established works.

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Appendix A: Additional Results

Table A1. Forecast of Additional Biomass Prediction Error due to Covariate Shift (test dataset)

Shift	$\mathrm{E}[AGB_s]$	$\mathrm{E}[AGB_t]$	ΔZ	$\mathcal{H}Disc$	η	MAE_s^c	MAE_t^c	$\Delta_{s,t}MAE$	$\hat{\Delta}_{s,t}MAE$
altitude-high⟵	661.9	857.6	+3598.9	913.7	447.9	171.7	271.6	+99.8	+92.2
altitude-high⟶	857.6	661.9	-3598.9	1070.9	447.9	296.1	208.1	-87.9	+10.0
altitude-low⟵	772.4	371.0	-8793.6	3414.6	276.9	220.8	53.8	-167.0	-59.3
altitude-low⟶	371.0	772.4	+8793.6	2198.8	276.9	53.0	215.4	+162.4	+122.6
continent-asia⟵	594.4	912.7	+6569.2	3369.5	359.8	197.3	193.7	-3.6	+152.1
continent-asia⟶	912.7	594.4	-6569.2	1611.0	359.8	165.0	174.1	+9.1	-6.6
drymonths-few⟵	621.0	1210.6	+21406.9	8809.3	638.8	167.7	598.3	+430.6	+312.8
drymonths-few⟶	1210.6	621.0	-21406.9	7225.7	638.8	291.1	175.4	-115.7	-156.7
drymonths-many⟵	760.7	257.7	-11134.2	4350.1	273.7	200.7	67.5	-133.2	-85.0
drymonths-many⟶	257.7	760.7	+11134.2	1718.7	273.7	67.8	199.1	+131.3	+111.0
foresttype-dry⟵	760.7	257.7	-11134.2	4350.1	273.7	200.7	67.5	-133.2	-85.0
foresttype-dry⟶	257.7	760.7	+11134.2	1718.7	273.7	67.8	199.1	+131.3	+111.0
foresttype-mangrove⟵	704.5	222.7	-10262.3	3695.8	226.9	191.6	35.6	-156.0	-67.1
foresttype-mangrove⟶	222.7	704.5	+10262.3	6656.6	226.9	33.6	207.4	+173.9	+244.6
foresttype-moist⟵	573.3	737.8	+313.1	92.6	363.2	136.9	199.6	+62.7	+73.9
foresttype-moist⟶	737.8	573.3	-313.1	489.1	363.2	207.6	216.3	+8.7	+28.3
foresttype-wet⟵	615.5	1064.0	+16042.0	7068.6	506.7	172.5	458.0	+285.5	+257.2
foresttype-wet⟶	1064.0	615.5	-16042.0	3090.1	506.7	235.8	185.4	-50.4	-50.1
oldgrowth-0⟵	763.5	234.5	-11685.8	4332.3	247.2	208.3	35.9	-172.4	-84.5
oldgrowth-0⟶	234.5	763.5	+11685.8	4628.4	247.2	43.2	250.0	+206.8	+185.9
rainfall-high⟵	621.5	986.1	+13746.6	6027.8	477.7	175.6	421.6	+246.0	+225.8
rainfall-high⟶	986.1	621.5	-13746.6	2824.7	477.7	217.1	186.7	-30.5	-42.5
rainfall-low⟵	788.9	322.2	-11514.4	4312.0	308.9	199.9	100.5	-99.3	-83.9
rainfall-low⟶	322.2	788.9	+11514.4	3965.9	308.9	101.0	242.9	+141.9	+167.9
random-1⟵	538.3	588.3	-99.3	106.8	341.6	183.0	159.5	-23.5	+40.6
random-1⟶	588.3	538.3	+99.3	126.8	341.6	158.3	183.0	+24.7	+74.6
random-2⟵	676.3	754.5	+3138.5	1399.9	386.3	175.8	214.0	+38.2	+103.4
random-2⟶	754.5	676.3	-3138.5	1488.6	386.3	210.4	173.8	-36.6	-2.8