

# A cascade of destabilizations: combining *Wolbachia* and Allee effects to eradicate insect pests

Julie C. Blackwood<sup>a</sup>, Roger Vargas Jr.<sup>a</sup> and Xavier Fauvergue<sup>\*b</sup>

<sup>a</sup>Department of Mathematics and Statistics, Williams College,  
Williamstown, MA 01267, USA

<sup>b</sup>INRA, CNRS, Université Nice Côte d'Azur, ISA, France

\*Corresponding author: [xavier.fauvergue@inra.fr](mailto:xavier.fauvergue@inra.fr)

## Summary

1. The management of insect pests has long been dominated by the use of chemical insecticides, with the aim of instantaneously killing enough individuals to limit their damage. To minimize unwanted consequences, environmentally-friendly approaches have been proposed that utilize biological control and

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\*[xavier.fauvergue@inra.fr](mailto:xavier.fauvergue@inra.fr)

take advantage of intrinsic demographic processes to reduce pest populations.

2. We address the feasibility of a novel pest management strategy based on the release of insects infected with *Wolbachia*, which causes cytoplasmic incompatibilities in its host population, into a population with a pre-existing Allee effect. We hypothesize that the transient decline in population size caused by a successful invasion of *Wolbachia* can bring the population below its Allee threshold and, consequently, trigger extinction.
3. We develop a stochastic population model that accounts for *Wolbachia*-induced cytoplasmic incompatibilities in addition to an Allee effect arising from mating failures at low population densities. Using our model, we identify conditions under which cytoplasmic incompatibilities and Allee effects successfully interact to drive insect pest populations toward extinction. Based on our results, we delineate control strategies based on introductions of *Wolbachia*-infected insects.
4. We extend this analysis to evaluate control strategies that implement successive introductions of two incompatible *Wolbachia* strains. Additionally, we consider methods that combine *Wolbachia* invasion with mating disruption tactics to enhance the pre-existing Allee effect.
5. We demonstrate that *Wolbachia*-induced cytoplasmic incompatibility and the Allee effect act independently from one another: the Allee effect does not modify the *Wolbachia*-invasion threshold, and cytoplasmic incompatibilities only have a marginal effect on the Allee threshold. However, the

interaction of these two processes can drive even large populations to extinction. The success of this method can be amplified by the introduction of multiple *Wolbachia* cytotypes as well as the addition of mating disruption.

6. Our study extends the existing literature by proposing the use of *Wolbachia* introductions to capitalize on pre-existing Allee effects and consequently eradicate insect pests. More generally, it highlights the importance of transient dynamics, and the relevance of manipulating a cascade of destabilizations for pest management.

## Keywords

biological control; cytoplasmic incompatibility; eradication; extinction; mating disruption; transient dynamics.

## Introduction

- 1 Although most insect species provide crucial ecosystem services (Losey & Vaughan,
- 2 2006), a minority of taxa that we consider pests ( $\sim 1\%$ ) have an overwhelming
- 3 influence on the development of population management in theory and in practice.
- 4 Among the various environmentally friendly approaches that have been envisaged
- 5 to control unwanted species, we focus on a research avenue that proposes the
- 6 exploitation of Allee effects, i.e., the decrease in survival or reproduction at small
- 7 population sizes and the consequent reduction in population growth (Dennis, 1989;
- 8 Courchamp *et al.*, 1999; Stephens *et al.*, 1999; Berec *et al.*, 2007; Liebhold & Tobin,
- 9 2008). The central ideas surrounding this work are twofold: management tactics

10 could be combined in order to (1) reduce a population size down below the Allee  
11 threshold – the population size at which the *per capita* growth rate decreases (in  
12 the case of “weak” Allee effects) or becomes negative (in the case of “strong” Allee  
13 effects) – which, in turn, increases the probability of stochastic extinction, and/or  
14 (2) amplify the mechanisms underpinning a pre-existing Allee effect to increase the  
15 Allee threshold itself (Tobin *et al.*, 2011). Capitalizing on Allee effects to manage  
16 undesirable species is particularly advantageous because it drives populations into  
17 extinction vortices without needing to eliminate every last individual.

18 Control methods centered on manipulating mating success as an alternative to  
19 chemical pesticides have long been recognized as desirable (e.g. Knipling, 1955;  
20 Baumhover *et al.*, 1955). The application of such control methods has been con-  
21 sidered for managing many insect pests including the Oriental fruit fly (Steiner  
22 *et al.*, 1970), Indian meal moth and almond moths (Sower & Whitmer, 1977), and  
23 Gypsy moths (Beroza & Knipling, 1972; Knipling, 1970). Importantly, mating dis-  
24 ruption has been successfully used to control populations with pre-existing Allee  
25 effects. The Gypsy moth (*Lymantria dispar*), for example, is one of the few insect  
26 species for which both a component (mate-finding) and demographic Allee effect  
27 have been explicitly identified (Tobin *et al.*, 2013, 2007; Johnson *et al.*, 2006). The  
28 Gypsy moth is an invasive forest pest in North America and triggered a major con-  
29 tainment program to slow the spread toward the western United States (Sharov  
30 *et al.*, 2002a; Liebhold *et al.*, 1992). Mating disruption has been a major tactic  
31 used to control newly established low-density populations along the invasion front,  
32 with evidence supporting that it is more efficient as well as economically cheaper  
33 than classic treatments with the pesticide *Bacillus thuringiensis* (Sharov *et al.*,  
34 2002a,b). This highlights the potential benefits of identifying other pest species

35 that have pre-existing Allee effects and determining whether environmentally de-  
36 sirable forms of control may similarly be effective.

37 Several recent theoretical developments have focused on taking advantage of  
38 Allee effects to promote pest eradication (e.g. Boukal & Berec, 2009; Liebhold &  
39 Bascompte, 2003; Blackwood *et al.*, 2012; Yamanaka & Liebhold, 2009). These  
40 models capture the underlying population dynamics of a pest and evaluate the suc-  
41 cess of population management tactics such as culling, release of sterile males, and  
42 mating disruption to determine whether these methods can create or enhance pre-  
43 existing Allee effects (Fauvergue, 2013 provides a comprehensive review). While  
44 there is evidence that such population management strategies will be successful for  
45 populations with pre-existing Allee effects, the range of species that might benefit  
46 from these tactics may be much greater than currently known. In a meta-analysis  
47 focused on the presence of Allee effects in natural animal populations (Kramer  
48 *et al.*, 2009), terrestrial arthropods were found associated with the largest number  
49 of studies (22) and the highest proportion (77%) exhibiting an Allee effect. Mating  
50 failure at low density appeared as the most frequent mechanism. Additionally, Fau-  
51 vergue (2013) found evidence supporting the presence of mate-finding Allee effects  
52 in 19 out of 34 published studies that investigated the interplay of population size  
53 and mating success in insects. Indirectly, the central role of Allee effects in insect  
54 population dynamics is supported by the efficiency of eradication programs based  
55 on the disruption of reproduction. Pest management based on the reduction of  
56 mating success via mass trapping, mating disruption with sex pheromones, or the  
57 release of sterile males has indeed proved successful in several instances (Knipling,  
58 1955; Baumhover *et al.*, 1955; Suckling *et al.*, 2012, 2014; Krafur, 1998).

59 In this article, we investigate *Wolbachia*-induced cytoplasmic incompatibility

60 (CI) as a novel method for triggering reproductive failures and consequently bring-  
61 ing a pest population below its Allee threshold. *Wolbachia* are endosymbiotic bac-  
62 teria that infect at least 20% of all insect species and up to two thirds in some  
63 estimations (Hilgenboecker *et al.*, 2008). *Wolbachia* have various effects on their  
64 insect hosts, the most widespread and prominent being cytoplasmic incompati-  
65 bility (Stouthamer *et al.*, 1999). Under CI, matings between an infected male  
66 and a female that is either uninfected or infected with an incompatible cytotype  
67 result in offspring mortality during embryonic development. Fitness advantages  
68 of infected females as well as maternal inheritance are key features that promote  
69 invasion of *Wolbachia* into a host population: above a threshold frequency, a given  
70 *Wolbachia* strain is expected to invade until near-fixation (Barton & Turelli, 2011;  
71 Hancock *et al.*, 2011; Caspari & Watson, 1959; Hoffmann & Turelli, 1997; Turelli &  
72 Hoffmann, 1991). As a result of the associated CI and subsequent reduction in re-  
73 productive rate, *Wolbachia* invasion via the release of infected hosts is a candidate  
74 biological control agent against arthropod pests (Bourtzis, 2008).

75 In practice, there are multiple ways to implement a management strategy cen-  
76 tered on inducing CIs via introduction of *Wolbachia*. For example, similar to the  
77 use of “Sterile Insect Technique” (SIT), males bearing a *Wolbachia* strain incom-  
78 patible with that of the target population can be released in large numbers. CIs  
79 arising from the mating of females and infected males would substantially limit  
80 the total offspring in the subsequent generation, resulting in a decrease in overall  
81 population growth rate and thereby increasing the possibility of local population  
82 extinction (Laven, 1967; Zabalou *et al.*, 2004; Atyame *et al.*, 2015). Incompatible  
83 males can be obtained via transfection, even between completely different species  
84 of host insects (e.g. Braig *et al.*, 1994; Hoffmann *et al.*, 2011). At the population

85 level, the conceptual underpinnings for mass-releases of incompatible males do not  
86 depart from that of SIT, for which interactions with the Allee effect have already  
87 been thoroughly analyzed (Boukal & Berec, 2009; Yamanaka & Liebhold, 2009;  
88 Fauvergue, 2013; Barclay & Mackauer, 1980; Barclay, 1982; Berec *et al.*, 2016;  
89 Lewis & Van Den Driessche, 1993).

90 An alternative management tactic using CI relies on the inoculation of a rela-  
91 tively small number of insects of both sexes with a *Wolbachia* strain incompatible  
92 with that of the target population. This method is investigated in the model  
93 introduced in Dobson *et al.* (2002), which combines insect population dynamics  
94 with releases of individuals infected with *Wolbachia*. During a successful invasion of  
95 *Wolbachia*, a transient reduction in the insect population size occurs. This decline  
96 results from the temporary increase in the fraction of incompatible matings, which  
97 peaks in the midst of the invasion process. Therefore, systematic introductions of  
98 different *Wolbachia* cytotypes could be applied to artificially sustain an unstable  
99 coexistence of multiple incompatible infections within an insect population, allow-  
100 ing the population size to be reduced and maintained at low levels (Dobson *et al.*,  
101 2002).

102 Our goal is to determine when the latter implementation of *Wolbachia* in-  
103 troductions can drive a population to extinction in the presence of Allee effects.  
104 Specifically, we derive a mathematical model built upon Dobson *et al.*'s (2002)  
105 approach of CI management that additionally accounts for Allee effects as well as  
106 environmental and demographic stochasticity. We also consider mating disruption  
107 in our model as a potential complementary tactic. We use this model to address  
108 three primary questions: (1) What is the influence of Allee effects present within  
109 a host population on *Wolbachia* invasion dynamics? (2) What is the influence of

110 cytoplasmic incompatibility on the demographic Allee effect? (3) What is the in-  
111 fluence of a combination of *Wolbachia*-induced CI, Allee effects, mating disruption,  
112 and stochasticity on the probability of host extinction?

## 113 **Methods**

### 114 **Population model**

115 Our model extends the framework introduced by Dobson *et al.* (2002) by account-  
116 ing for pre-existing Allee effects, the release of pheromone sources as a method of  
117 mating disruption, as well as both demographic and environmental stochasticity.  
118 In this section, we first introduce a model that considers the population dynamics  
119 in the absence of individuals infected with *Wolbachia*.

120 We considered populations such that the dynamics can be modeled in discrete  
121 time with non-overlapping generations. The population model explicitly tracks  
122 the total population size at each time  $t$ , given by  $N_t$ . Our population model can  
123 be expressed in terms of either census size or density and hereafter, we refer to  
124 our model in terms of size. Therefore, while the deterministic model can take non-  
125 integer values, population sizes less than one are considered extinct. In contrast,  
126 the stochastic model forces integer population sizes. We assume that each time  
127 step can be broken into two stages: the first (at time  $t+0.5$ ) captures reproduction,  
128 and the second (at time  $t+1$ ) captures density dependent survivorship of offspring  
129 to adults. The total number of offspring is given by:

$$N_{t+0.5} = \tilde{m} F_t g_1(M_t) g_2(F_t) \quad (1)$$



130 where  $\tilde{m}$  is the maximum *per capita* female fecundity,  $F_t$  is the number of females in  
131 the population, and  $M_t$  is the number of males.  $g_1(M_t)$  captures a component Allee  
132 effect that results from the failure of mates finding one another at low densities  
133 such that:

$$g_1(M_t) = \frac{M_t}{M_t + \tilde{\theta}}. \quad (2)$$

134 Here,  $\tilde{\theta}$  measures the strength of the Allee effect; a convenient interpretation of  
135 this term is that  $\tilde{\theta}$  represents the number of males at which half of the females  
136 successfully find a mate or, equivalently, the maximum mating rate is reduced by  
137 half (Boukal & Berec, 2009).

138 The function  $g_2(F_t)$  in Eqn. 1 captures the decline in fecundity resulting  
139 from techniques to control populations via mating disruption. We assume that  
140 pheromone sources are introduced into the environment and maintained in the  
141 population at a fixed number  $\tilde{P}$  and only a fraction  $F_t/(F_t + \tilde{P})$  males successfully  
142 find a mate (Fauvergue, 2013), or:

$$g_2(F_t) = \frac{F_t}{F_t + \tilde{P}}. \quad (3)$$

We now assume that there is a 50:50 sex ratio (i.e.  $F_t = M_t = N_t/2$ ) and define constants  $m = \tilde{m}/2$  (which is now the overall *per capita* fecundity),  $\theta = 2\tilde{\theta}$ , and

$P = 2\tilde{P}$ . Now, we can write both  $g_1$  and  $g_2$  as functions of  $N_t$  and Eqn. 1 is now:

$$N_{t+0.5} = mN_t \frac{N_t}{N_t + \theta} \frac{N_t}{N_t + P}. \quad (4)$$

143 For a simpler biological interpretation, we consider  $\theta$  relative to the carrying  
144 capacity of the population in the absence of any control (denoted as  $K$ ) as an  
145 indicator of the intensity of the Allee effect (i.e.  $\theta/K$ ). Now, when  $\theta = K$  (or,  
146 equivalently,  $\theta/K = 1$ ) half of the *total* population successfully mates at its car-  
147 rying capacity. Hereafter, we refer to  $\theta/K$  as the “relative strength of the Allee  
148 effect” and restrict its values to the range (0 - 0.5), with 0.5 likely on the upper end  
149 of biologically reasonable values for  $\theta/K$ . Similarly, we consider  $P$ , the number of  
150 pheromone sources, relative to  $K$ .  
151 Finally, we assume that survivorship of offspring to adults is density dependent so  
152 that:

$$N_{t+1} = N_{t+0.5} S_N \quad (5)$$

153 where

$$S_N = \frac{S_0}{1 + (\alpha N_{t+0.5})^\gamma} \quad (6)$$

and the constant  $\alpha$  is related to the carrying capacity (more details provided in the Supplementary Information S1),  $\gamma$  is related to intraspecific competition, and  $S_0$  is survivorship in the absence of intraspecific competition (Slatkin & Smith,

1979). Combining equations (4)-(7), we are left with the model:

$$N_{t+1} = mN_t \left( \frac{N_t}{N_t + \theta} \right) \left( \frac{N_t}{N_t + P} \right) \frac{S_0}{1 + (\alpha N_{t+0.5})^\gamma}. \quad (7)$$

Our goal is to consider the dynamical consequences of population control methods (i.e. *Wolbachia* introductions and the release of pheromone sources). Therefore, we analytically determine the equilibrium values for the population model (7) in the absence of these management tactics ( $P = 0$ ). Throughout the remainder of the paper, we distinguish between the “maximum reproductive rate” (which is given by  $mS_0$  and therefore accounts for both fecundity and density independent survivorship) and the overall “reproductive rate” (which is the reproductive rate in the presence of all other demographic processes in our model, namely the Allee effect). As detailed in the Supplementary Information (S1), we find three equilibria (assuming the maximum reproductive rate is greater than one and setting  $\gamma = 1$ ). The first equilibrium corresponds to population extinction, and the second two are given by:

$$\frac{mS_0 - 1 \pm \sqrt{(mS_0 - 1)^2 - 4\alpha m\theta}}{2\alpha m}. \quad (8)$$

154 When Allee effects are not present ( $\theta = 0$ ) these equilibria collapse to a single  
155 equilibrium (the carrying capacity  $K$ ). In contrast, for sufficiently large Allee  
156 effects these equilibria no longer exist and the reproductive rate is always less  
157 than one; consequently, the population will be driven to extinction independent  
158 of its initial size. However, when the value of  $\theta$  is between these extremes, the  
159 smaller equilibrium corresponds to the Allee threshold and the larger equilibrium

160 corresponds to the carrying capacity. The structure of these equilibria are integral  
161 to the insect species that we are considering: there is a carrying capacity and a  
162 strong Allee effect for positive values of  $\theta$  up to a threshold. We therefore conjecture  
163 that an alternative form of density dependence that captures these properties will  
164 yield qualitatively similar results.

165 Based on this deterministic framework, we build in environmental and demo-  
166 graphic stochasticity. We assume that environmental stochasticity leads to vari-  
167 ation in the population's fecundity between generations (Melbourne & Hastings,  
168 2008). Therefore, we adapt the methods of Schoener *et al.* (2003) and account for  
169 environmental stochasticity by rewriting Eqn. 4 as:

$$N_{t+0.5} = m_t N_t g_1(N_t) g_2(N_t) \quad (9)$$

170 where the fecundity  $m_t$  is drawn at each generation from a normal distribution  
171 with mean  $m$  (that is truncated so that  $m \geq 0$ ) and variance ( $\sigma^2$ ) that scales  
172 with the mean. In the main text, we fix the variance so that it is equal to the  
173 mean; however, a sensitivity analysis of the magnitude of the variance is provided  
174 in the Supplementary Information (S3). Further, we note that gamma-distributed  
175 environmental stochasticity is a commonly used alternative choice; under our pa-  
176 rameterization, the probability density function (pdf) is nearly identical for both  
177 normal- and gamma-distributed stochasticity and would therefore yield similar  
178 results.

179 Demographic stochasticity results from variation in fecundity at the individual  
180 level (Melbourne & Hastings, 2008). In the absence of demographic stochasticity,

181 the total number of individuals that successfully reproduce is given by:

$$B = N_t g_1(N_t) g_2(N_t). \quad (10)$$

182 We assume that each of these individuals at a given time  $t$  reproduces with fecun-  
183 dity  $m_t$  (as described above), and the total number of eggs produced is a Poisson  
184 random variable (Melbourne & Hastings, 2008). Since the sum of independent  
185 Poisson random variables is also a Poisson random variable, the total offspring of  
186 all adults is:

$$N_{t+0.5} \sim \text{Poisson}(m_t B). \quad (11)$$

187 Finally, we include stochasticity in density dependent survivorship, again fol-  
188 lowing Melbourne & Hastings (2008). Given that  $S_N$  (as defined in Eqn. 7) is  
189 the probability that offspring survive to adults, we assume that survivorship is  
190 binomially distributed so that:

$$N_{t+1} \sim \text{Binomial}(N_{t+0.5}, S_N). \quad (12)$$

## 191 Infection dynamics

192 We consider the infection dynamics of up to two different cytotypes of *Wolbachia*  
193 (referred to as cytotypes  $X$  and  $Y$ ) and denote the number of uninfected individuals

194 as  $W$ . Note that all variables and parameters with subscripts  $X$  (or  $Y$ ) are related  
195 to cytotype  $X$  (or  $Y$ ). This model is adapted from Dobson *et al.* (2002); therefore,  
196 we use similar notation throughout.

197 In the presence of a single cytotype of *Wolbachia*, there are only unidirec-  
198 tional cytoplasmic incompatibilities (CI); in contrast, in the presence of multiple  
199 cytotypes there may be bi-directional CI. We first introduce the case of a single  
200 cytotype and then extend the model to include two cytotypes. Below we describe  
201 the mathematical formulation of the infection dynamics, and Table 2 summarizes  
202 these processes.

203

#### 204 *One cytotype*

205 To capture the *Wolbachia* dynamics, we first determine the proportions of  
206 infected and uninfected individuals in the population at time  $t$ . For example, if  
207 there are  $W_t$  uninfected individuals and  $X_t$  infected with cytotype  $X$  then, under  
208 the assumption that there is a 50:50 sex ratio, the fraction of females infected with  
209 cytotype  $X$  at  $t + 0.5$  is given by:

$$a_t = \frac{X_t}{W_t + X_t}.$$

210 Similarly, we find the fraction  $i_t$  of all males that are infected (where  $i_t = a_t$ ), the  
211 fraction  $q_t$  of all males that are uninfected, and the fraction  $c_t$  of all females that  
212 are uninfected (again  $q_t = c_t$ ).

213 Based on the proportions of uninfected and infected individuals in the popula-  
214 tion, we can now determine the fraction of offspring that are infected. Crosses be-

215 tween pairs with an infected female suffer a fecundity loss due to infection ( $1 - F_X$ ),  
216 where  $F_X$  is the relative fecundity of infected individuals. Vertical transmission of  
217 *Wolbachia* occurs maternally and we assume that transmission is successful with  
218 probability  $(1 - \mu_X)$ , where  $\mu_X$  is the probability of transmission failure. In the  
219 instance of *Wolbachia*-induced CIs, crosses between infected females and unin-  
220 fected males in addition to crosses between infected males and infected females  
221 give rise to infected offspring. The proportion of viable offspring that are infected  
222 with cytotype  $X$  after reproduction (i.e. at time  $t + 0.5$ ) is therefore given by:

$$x_{t+0.5} = a_t (1 - \mu_X) F_X (i_t + q_t) \quad (13)$$

223 where a lowercase  $x$  is used to denote proportion rather than number. Second,  
224 we can identify the proportion of viable offspring that are uninfected ( $w_{t+0.5}$ ).  
225 Uninfected individuals can arise from crosses between both uninfected females and  
226 uninfected males. Further, matings between both infected females and infected  
227 males can yield viable uninfected offspring. This results from failure to vertically  
228 transmit *Wolbachia* to their offspring. When one type of *Wolbachia* is present  
229 within a population, then only unidirectional cytoplasmic incompatibility (CI) is  
230 possible. This type of CI occurs through matings between infected males and  
231 uninfected females. Therefore, we assume that all but a fraction  $H_X$  of pairings  
232 between infected males and uninfected females undergo CI. Additionally, offspring  
233 from pairings between infected males and infected females that fail to transmit  
234 *Wolbachia* are subject to CI-induced mortality. The proportion of viable offspring  
235 that are not infected with *Wolbachia* following reproduction is therefore given by:

$$w_{t+0.5} = (\mu_X F_X a_t + c_t) \times (i_t H_X + q_t). \quad (14)$$

236 Notice that due to cytoplasmic incompatibilities and the fecundity cost due to  
237 infection with *Wolbachia*, the fraction of the offspring that are viable ( $x_{t+0.5} +$   
238  $w_{t+0.5}$ ) is less than one. Therefore, the total number of offspring as governed by  
239 Eqn. 4 can be rewritten as:

$$N_{t+0.5} = m (x_{t+0.5} + w_{t+0.5}) N_t g_1(N_t) g_2(N_t). \quad (15)$$

240 In other words, the product  $g_1(N_t)g_2(N_t)$  captures the total fraction of adults at  
241 time  $t$  who successfully find a mate, and the sum  $x_{t+0.5} + w_{t+0.5}$  is the fraction of  
242 all offspring that are viable. Finally, as described in the previous section, density  
243 dependent mortality limits the total number of adults at time  $t + 1$  (Eqn. 7).

244

#### 245 *Two cytotypes*

246 In addition to releasing a single cytotpe of *Wolbachia*, we consider a scenario  
247 in which a second cytotpe is introduced. When two cytotypes of *Wolbachia* are  
248 present within a population, bidirectional CI occurs when a male with one cytotpe  
249 mates with a female infected with an incompatible *Wolbachia* cytotpe. Similar  
250 to the previous section, we assume that a fraction  $H_X$  (or  $H_Y$ , depending on the  
251 infection type of the male and female) of offspring survives. Offspring of pairings  
252 between infected males and females of either cytotpe that fail to transmit are



253 again subject to CI-induced mortality.

254 Therefore, in the presence of two strains we rewrite Eqn. 13 as:

$$x_{t+0.5} = a_t (1 - \mu_X) F_X (i_t + j_t H_Y + q_t) \quad (16)$$

255 where  $j_t$  is the fraction of males infected with cytotype  $Y$ . Similarly, the proportion  
256 of viable offspring infected with cytotype  $Y$  following reproduction is given by:

$$y_{t+0.5} = b_t (1 - \mu_Y) F_Y (i_t H_X + j_t + q_t) \quad (17)$$

257 where  $b_t$  is the fraction of females infected with cytotype  $Y$ . The proportion of  
258 viable uninfected offspring is now given by:

$$w_{t+0.5} = (\mu_X F_X a_t + \mu_Y F_Y b_t + c_t) (i_t H_X + j_t H_Y + q_t). \quad (18)$$

## 259 **Parameterization**

260 Here, we define the maximum reproductive rate as the product of the *per capita*  
261 fecundity and density independent survivorship (i.e.  $mS_0$ ). In other words, this  
262 is the reproductive rate in the absence of Allee effects and in the limit as the  
263 population size approaches zero. Our parameterization of the population model  
264 is based on both the parameterization used in Dobson *et al.* (2002) and common  
265 ranges for insect populations (see Table 1). For example, Hassell *et al.* (1976) cites

266 finite net rates of increase (defined as fecundity discounted by density independent  
267 mortality and therefore equivalent to  $mS_0$  here) ranging from 1.3 – –75 (with 22  
268 of the 24 species investigated in the range 1.3 – –13.5) which is consistent with  
269 our parameterization (e.g. see Fig. 2 caption and the associated results as  $mS_0$   
270 is varied). Additionally, several parameters vary for our analysis including the  
271 strength of the Allee effect,  $\theta$ , and the initial population size. However, our results  
272 are intended to assess the general qualitative behavior of *Wolbachia* introductions  
273 and consequently the actual implementation of such management tactics would  
274 require a detailed analysis and parameterization specific to the target population  
275 and cytotype.

276 Our parameterization for the infection dynamics is based on values that are rea-  
277 sonable for *Wolbachia* cytotypes (Hoffmann & Turelli, 1997; Dobson *et al.* (2002);  
278 Charlat *et al.*, 2005). In the main text, we assume that fecundity loss, trans-  
279 mission failure, and survival of CI are equal between all cytotypes. However, the  
280 Supplementary Information (S3) provides an analysis of the dynamics when the  
281 introduced cytotypes are not identical. S3 additionally provides sensitivity anal-  
282 yses for parameters related to the population model, including the strength of  
283 environmental stochasticity and the maximum reproductive rate.

## 284 Results

285 In the following sections we first test our model against well-established results  
286 related to *Wolbachia* invasion as a method of model validation, establish the re-  
287 lationship between *Wolbachia* and the strength of the Allee effect, and finally  
288 evaluate the potential for the release of infected insects to control a population.

## 289 Model validation

290 We first determine whether our model captures the same features of the important  
291 earlier work (Hoffmann *et al.*, 1990; Turelli & Hoffmann, 1991; Hancock *et al.*,  
292 2011). Hoffmann *et al.* (1990) derived an analytic expression for the expected  
293 equilibrium infection frequencies. After adjusting their notation to match ours  
294 and simplifying, the equilibrium infection frequency (which we denote as  $p$ ) for a  
295 single cytotype of *Wolbachia X* should satisfy the equation:

$$(1 - H_X)(1 - \mu_X F_X) p^2 + (F_X + H_X - 2)p + 1 - F_X(1 - \mu_X) = 0.$$

296 Their work predicts that there is an unstable equilibrium referred to as the  
297 *Wolbachia* invasion threshold; cytotypes with initial infection frequencies above  
298 this threshold will increase until reaching the higher stable equilibrium, indicating  
299 a successful invasion. Following Charlat *et al.* (2005), we considered invasion dy-  
300 namics by estimating the infection frequency at generation  $t + 1$  as a function of  
301 the frequency at generation  $t$ . Doing so allows us to create a simple graphical rep-  
302 resentation of the stable and unstable equilibria (Fig. 1). Unless stated otherwise,  
303 the default parameter values are listed in Table 1.

304 Our simulation results are consistent with the analytically derived equilibrium  
305 infection frequencies (Fig. 1). This verifies that our simulations are in line with  
306 the behavior that we would like to capture from our model and are consistent with  
307 the results observed in Charlat *et al.* (2005). This is not surprising given that our  
308 model makes similar assumptions on the mechanisms driving *Wolbachia* invasion  
309 dynamics (e.g. fecundity loss and cytoplasmic incompatibilities). In contrast to

310 earlier studies, our population model is dynamically different because of the inclu-  
311 sion of Allee effects and pheromones. Therefore, we determined the relationship  
312 between the invasion threshold and these features of the model. We found that the  
313 *Wolbachia* invasion threshold is not affected by Allee effects nor by the application  
314 of pheromones to the host insect (not shown). This is important to note because  
315 in all of our simulations and analyses, the invasion threshold does not vary as  $\theta$   
316 and  $P$  are adjusted. Finally, we note that the invasion threshold is not affected by  
317 any of our demographic parameters (i.e.  $m$ ,  $\alpha$ ,  $\gamma$ , and  $S_0$ ; not shown.)

### 318 **The effect of *Wolbachia* on the Allee threshold**

319 To determine the dynamical effects of the presence of *Wolbachia* infection within a  
320 population, we find the Allee threshold in insect populations both in the presence  
321 and absence of infection. In this section, we ignore stochasticity as well as the  
322 release of pheromones ( $P = 0$ ). For a given proportion of infected individuals, we  
323 calculate the reproductive rate between two consecutive generations (i.e.  $N_{t+1}/N_t$ )  
324 across all population sizes (Fig. 2). The equilibria for our population model occur  
325 when  $N_{t+1} = N_t$ , and there are three equilibria for the parameters used to produce  
326 this figure: the first corresponds to population extinction (stable), the second is the  
327 Allee threshold (unstable), and finally the third is the carrying capacity (stable).

328 In addition to considering the population model in the absence of *Wolbachia*-  
329 infected individuals, we calculated the reproductive rates when the population is  
330 comprised of 10%, 50%, and 90% infected individuals (Fig. 2). Given our pa-  
331 rameterization, the frequency of infected individuals is chosen to lie above the  
332 invasion threshold (which is  $\sim 8.5\%$ ); therefore, this figure captures the dynamics

333 between two consecutive generations during the replacement process when the pop-  
334 ulation contains the specified distribution of infected and uninfected individuals.  
335 As a consequence of cytoplasmic incompatibilities, there is an increase in the Allee  
336 threshold. Additionally, there is a slight decrease in the carrying capacity that  
337 results from the fecundity loss associated with *Wolbachia* infection. Consequently,  
338 the maximum reproductive rate decreases as the proportion of infected individuals  
339 increases. Finally, the changes in reproductive rate are most significant during  
340 the replacement process (e.g. when 50% is infected) and they diminish when the  
341 infection is close to fixation (e.g. 90%). The proportion of *Wolbachia*-infected  
342 individuals has a significantly smaller effect on the location of the Allee threshold  
343 than the strength of the mate finding Allee effect itself (Fig. 2).

## 344 **Implications for population management**

### 345 *Deterministic results*

346 In this section, we characterize implications for population management through  
347 the release of *Wolbachia*-infected individuals into an insect population. As ob-  
348 served by Dobson *et al.* (2002), there should be a transient decline in the popu-  
349 lation size during the replacement of uninfected hosts by *Wolbachia*-infected indi-  
350 viduals. Therefore, we find the magnitude of this decline in the presence of Allee  
351 effects to determine the conditions under which the replacement process brings  
352 the population size below the Allee threshold in a deterministic setting, thereby  
353 forcing extinction. This is achieved by running our model over a range of values  
354 for the strength of the mate-finding Allee effect ( $\theta$ ). We assume that the initial  
355 population size is at its carrying capacity (which is found analytically, as shown

356 in the Supplementary Information S1).

357 We find the minimum population size (relative to  $K$ ) over 50 generations fol-  
358 lowing the introduction of one cytotype (Fig. 3B) and two cytotypes (Fig. 3C).  
359 Here, values of zero for the minimal population size indicate that the transient  
360 reduction in population size brought the population below the Allee threshold,  
361 therefore leading to deterministic extinction. The first cytotype is always released  
362 in the second generation, and the release of the second cytotype is optimized so  
363 that the release occurs in the generation that causes the largest decline in pop-  
364 ulation size (see Supplementary Information S2 for a detailed explanation of the  
365 optimization scheme). While in the main text we assume that all cytotypes have  
366 the same infection properties, this assumption is challenged in the Supplementary  
367 Information (S3) and our qualitative results are unchanged. To ensure that the  
368 introduction size is above the invasion threshold, in all simulations we assume that  
369 the introduction is large enough so that the initial infection frequency is 10%. This  
370 value lies just above the actual threshold of  $\sim 8.5\%$  associated with our parameter  
371 values. Therefore, the number of infected individuals introduced in our simulations  
372 directly depends on the current host population size.

373 To determine the success of releases under varying reproductive rates, we repli-  
374 cated the results for four different values of  $S_0$ . For all reproductive rates, the  
375 release of two incompatible cytotypes of *Wolbachia* is substantially more effective  
376 than a release of a single cytotype. In fact, release of a single cytotype is only  
377 effective in driving population extinction when reproductive rates are small. This  
378 observation holds more generally: the effectiveness of *Wolbachia* introductions in-  
379 creases as the maximum reproductive rate decreases (Fig. 3B-C).

380 These results suggest that CI management may fail in species with high repro-

381 ductive rates or alternatively in these species, complementary tactics that either  
382 decrease the population size or increase the Allee threshold may be required to  
383 amplify the effects of *Wolbachia* introductions. Therefore, we consider the use of  
384 mating disruption through the release of sex pheromones ( $P$ ) as a supplemental  
385 management tactic (as in Eqns. (4) and (3)). We determine the minimum pop-  
386 ulation size following no introduction, one introduction, and the introduction of  
387 two cytotypes while additionally assuming there is a fixed number of pheromone  
388 sources present. These results suggest that combining both methods is significantly  
389 more effective than either tactic alone (Fig. 3D-F). The associated dynamics can  
390 be visualized by considering a plot of the population size with time (Fig. 4).

#### 391 *Stochastic results*

392 The interplay of Allee effects and stochasticity can be especially important at  
393 low population sizes, when the population is at higher risk of stochastic extinc-  
394 tion. Therefore, in this section we determine the ability of *Wolbachia* and mating  
395 disruption to drive populations with variable initial population sizes to extinction  
396 in the presence of Allee effects and stochasticity.

397 To achieve this, we determine the probability of extinction based on 500 real-  
398 izations of the stochastic model (i.e. Eqns. 9-12) over all relevant combinations  
399 of the initial population size and strength of the Allee effect (i.e. the initial pop-  
400 ulation size is at most at 80% of carrying capacity and the relative Allee effect,  
401  $\theta/K$  is no greater than 0.5). To determine the relative roles of environmental and  
402 demographic stochasticity, we simulate the model while including both types of  
403 stochasticity as well as demographic stochasticity alone. Further, we find the ex-  
404 tinction probability under three scenarios: no introduction of *Wolbachia*-infected  
405 individuals, introduction of one cytotype, and the introduction of two incompati-

406 ble cytotypes. As in the deterministic setting, we assume that the introduction of  
407 the first cytotpe occurs at the second generation. When two cytotypes are intro-  
408 duced, the generation of the second release is determined in the same way as it is  
409 found in the deterministic setting: the second introduction is optimized for each  
410 realization so that it occurs in the generation (up to 25 generations) that creates  
411 the largest transient decrease in population size resulting from the *Wolbachia* in-  
412 troduction. The number of generations between releases increases as the strength  
413 of the mate-finding Allee effect decreases (see Supplementary Information S2). As  
414 before, each release is implemented so that the proportion of infected individuals  
415 of a given cytotpe is 10% (just above the invasion threshold).

416 Here, we consider populations with relatively high reproductive rates (max-  
417 imum reproductive rate 3.75); the Supplementary Information (S3) provides a  
418 sensitivity analysis for lower reproductive rates. We find that the introduction  
419 of a single cytotpe of *Wolbachia* leads to negligible increases in the extinction  
420 probability across all values of the relative strength of the Allee effect (Fig. 5A-  
421 B). However, environmental stochasticity increases the uncertainty associated with  
422 extinction near the Allee threshold, and there are more apparent increases in ex-  
423 tinction probability for *Wolbachia* introductions for very strong Allee effects (Fig.  
424 5D-E). As described in the previous section, the success of *Wolbachia* releases in-  
425 creases for lower reproductive rates. This finding holds in the stochastic setting  
426 (see S3).

427 In contrast to a single introduction, the release of two incompatible cytotypes is  
428 much more effective (Fig. 5C,F). Interestingly, when  $\theta$  is relatively high, *Wolbachia*  
429 introductions succeed in driving population extinction independent of the initial  
430 population size. This result has the important implication that the success of



431 *Wolbachia* introductions in driving extinction do not necessarily rely on having a  
432 pest population at the initial stage, or at the front, of the invasion. Moreover, in  
433 settings with high levels of environmental stochasticity, extinction is possible for  
434 much smaller Allee effects than predicted by the deterministic model (Fig. 5F).

435 As explored in the deterministic framework, combining *Wolbachia* introduc-  
436 tions with other methods that increase the Allee threshold (e.g. mating disruption)  
437 will likely further increase the success of the overall management strategy. Here,  
438 we perform a more global exploration of management options that combine both  
439 *Wolbachia* introductions and pheromone releases. We again consider the popula-  
440 tion dynamics under three different management regimes: mating disruption only,  
441 mating disruption and the introduction of one cytotype, and mating disruption  
442 and the introduction of two cytotypes of *Wolbachia*. In this case, we assume that  
443 the relative strength of the Allee effect is fixed and relatively low so that  $\theta/K$  is  
444 0.1; under this condition, *Wolbachia* introductions alone do not drive the popu-  
445 lations to extinction in the deterministic setting (Fig. 3B-C). We again find that  
446 environmental stochasticity increases the extinction probability for a smaller num-  
447 ber of pheromone sources and, importantly, utilizing both mating disruption and  
448 CI is much more effective than using mating disruption alone (Fig. 6). Therefore,  
449 these two methods can serve as complementary tactics for pest management.

## 450 Discussion

451 We investigated a population management strategy that considers *Wolbachia*-  
452 induced cytoplasmic incompatibility in the presence of Allee effects. In particular,  
453 we developed a stochastic population model, building upon the seminal approach

454 of Dobson *et al.* (2002) and the continuously expanding body of literature investi-  
455 gating the use of Allee effects for the eradication of pest species (Liebhold & Bas-  
456 compte, 2003; Tobin *et al.*, 2011; Liebhold *et al.*, 2016). Our model demonstrates  
457 that the introduction of a relatively small number of incompatible individuals into  
458 a pest population that has a strong pre-existing Allee effect can drive the pest  
459 population to extinction with no further intervention. These methods are suc-  
460 cessful more broadly when multiple strains of *Wolbachia* are introduced. We also  
461 demonstrate that extinction is possible for surprisingly large pest populations and  
462 that combinations of more than one strain of *Wolbachia* and mating disruption via  
463 sex pheromones work synergistically to increase the population's extinction risk.  
464 Biological control has been studied for decades as an environmentally friendly al-  
465 ternative to pesticide use (e.g. Knipling, 1955; Baumhover *et al.*, 1955; Murdoch  
466 *et al.*, 1985; Bale *et al.*, 2008), and our study adds to this work by providing in-  
467 sight into ways that *Wolbachia* invasions can take advantage of intrinsic population  
468 processes – that is, Allee effects – to manipulate and control pest populations.

469 An important first step of our modeling work was to uncover the basic inter-  
470 actions between Allee effects and cytoplasmic incompatibility (CI). We show that  
471 these interactions are weak or non-existent: the *Wolbachia* invasion threshold does  
472 not depend on the strength of the Allee effect of its insect host, and the Allee  
473 threshold has only a marginal decrease in the presence of CI. Therefore, invasion  
474 of a particular *Wolbachia* strain into a population only depends on the critical in-  
475 fection frequency above which invasion succeeds in a deterministic setting (Barton  
476 & Turelli, 2011). This invasion threshold corresponds to a proportion of infected  
477 hosts above which infection spreads up to almost fixation, and is determined by  
478 parameters such as the reduction in egg hatch-rate caused by CI, the fitness costs

479 of *Wolbachia* carriage, and the fraction of offspring that inherit the bacteria from  
480 an infected mother (Turelli, 1994). The invasion threshold found with our sim-  
481 ulation model is consistent with that derived analytically (Turelli & Hoffmann,  
482 1991), and unaffected by the intensity of a mate-finding Allee effect. In addition  
483 to adding validation to our model, this result holds interest because most theoret-  
484 ical approaches of *Wolbachia* invasion dynamics are purely genetic and consider  
485 changes in infection frequency without considering host population dynamics. One  
486 exception is the work of Hancock *et al.* (2011) which suggested that *Wolbachia*  
487 invasion thresholds predicted analytically hold for closed populations. Our results  
488 are consistent even when, as assumed here, host reproductive rate is affected by  
489 both positive and negative density dependence.

490 In the presence of strong Allee effects a population below the Allee thresh-  
491 old will be forced to extinction in a deterministic setting, making the Allee effect  
492 a central paradigm for conservation (Deredec & Courchamp, 2007; Stephens &  
493 Sutherland, 1999), invasions (Taylor & Hastings, 2005), biological control intro-  
494 duction (Fauvergue *et al.*, 2007, 2012), and as hypothesized in the present work,  
495 eradication (Tobin *et al.*, 2011). Whether an Allee effect is weak or strong (and the  
496 value of the Allee threshold in the latter case) depends on the strength of the un-  
497 derlying component Allee effect(s) relative to other density-dependent processes.  
498 Our simulations of various levels of cytoplasmic incompatibilities in a population  
499 with a pre-existing mate finding Allee effect suggest that the Allee threshold is  
500 much less sensitive to variations in the frequency of *Wolbachia*-infected individu-  
501 als (0-90% infected individuals) than it is to variations in mating success (Fig. 2B).  
502 *Wolbachia*-induced cytoplasmic incompatibility does decrease population growth  
503 rate, as expected, but it has a minimal effect on the extinction threshold. There-

504 fore, *Wolbachia*-induced CI may be considered a culling population management  
505 tactic where population size is temporarily decreased as a result of cytoplasmic  
506 incompatibilities.

507 Despite their initial apparent independence, cytoplasmic incompatibility and  
508 the Allee effect yield interesting properties when acting in concert. Our first anal-  
509 ysis that considered the combined occurrence of Allee effects and CI in a determin-  
510 istic context reveals that the transient decrease in population size is large enough  
511 to trigger extinction when the reproductive rate of the host species is relatively  
512 low. Extinction caused by the introduction of a single *Wolbachia* cytotype in  
513 populations with higher reproductive rates is only observed for very strong Allee  
514 effects (Fig. 3A). However, the strength of the Allee effect required for extinction  
515 lowers with the introduction of an additional incompatible *Wolbachia* strain. The  
516 resulting insect extinction probability, estimated by simulating the model in the  
517 presence of stochasticity, confirmed the interaction between the two processes. In  
518 the absence of *Wolbachia*, we determined the population's extinction probability  
519 as it varies with its population size and the strength of the component Allee ef-  
520 fect (Fig 5A and 5D). Introducing infected individuals results in the extinction of  
521 populations that would have persisted otherwise (i.e., a population that is above  
522 its Allee threshold can be brought to extinction). Introducing a second incompat-  
523 ible cytotype of *Wolbachia* increases CIs within the population and, consequently,  
524 increases the extinction domain by reducing the severity of Allee effect necessary  
525 to trigger extinction (Fig 5).

526 Nonetheless, our model predicts that although Allee effects and CI combine  
527 to drive populations to extinction – even surprisingly large populations – these  
528 extinctions may occur for unrealistically severe Allee effects. For instance, after

529 the introduction of two incompatible *Wolbachia* strains into a population with a  
530 maximum reproductive rate of 3.75, extinction is expected when  $\theta/K$  exceeds 0.25  
531 (Fig. 5C-F); that is, extinction occurs if only 25% of all females successfully mate  
532 when the population is at 50% of the carrying capacity. Field estimations of mate-  
533 finding Allee effects in insects are rare, but it is probable that mating failures only  
534 occur at low densities. For instance, in the Gypsy moth *Lymantria dispar*, mating  
535 failures occurred below a density of  $\sim 4$  (estimated via the rate of male captures  
536 on sex-pheromone traps) whereas the carrying capacity was estimated similarly  
537 around 800 (Tobin *et al.*, 2007, 2013), so that estimation of  $\theta/K$  in this species  
538 could be one or two orders of magnitude lower than that yielding extinction in our  
539 model.

540 Our simulations demonstrate that eradication is much more likely if the intro-  
541 duction of *Wolbachia*-infected individuals is combined with mating disruption via  
542 the release of sex pheromone sources (Fig. 6). Interestingly, eradication is not  
543 restricted to small populations, but also applies to populations that have reached  
544 carrying capacity. Moreover, our results suggest that the two tactics act synergis-  
545 tically: the decrease in population size obtained when CI and mating disruption  
546 are combined is higher than the cumulative decrease obtained with each tactic  
547 applied separately. Our model therefore supports previous studies that highlight  
548 the potential benefit of simultaneously using multiple complementary management  
549 tactics (Blackwood *et al.*, 2012; Suckling *et al.*, 2012; Berec *et al.*, 2016). If dif-  
550 ferent tactics benefit from one another, additional methods for controlling a pest  
551 population should also be considered. For example, other methods for popula-  
552 tion control such as parasitism or predation by native natural enemies may also  
553 be complementary. Additionally, while our focus was on cytoplasmic incompati-

554 bilities, there is evidence that *Wolbachia* and other bacteria are capable of other  
555 reproductive manipulations including male-killing (Dyer, 2004; Richardson *et al.*,  
556 2016). Similar conclusions were also made in the recent modeling study of Berc  
557 *et al.* (2016), who suggest that sterile insect technique is improved when combined  
558 with male-killing bacteria. This suggests the existence of additional avenues for  
559 utilizing *Wolbachia* in the context of pest management.

560 Theoretical models of population management are currently flourishing much  
561 faster than empirical evidence can be obtained; therefore, it is important to dis-  
562 cuss the relevance of our predictions and the feasibility of our proposed methods.  
563 First, we have shown that extinction in insect species with high reproductive rates  
564 may not be feasible because extinction would require an unrealistic amount of sex  
565 pheromone lures to successfully complement the *Wolbachia*-induced transient de-  
566 crease in population size. However, in the majority of our simulations we assigned  
567 the maximum reproductive rate a value of 3.75 or, equivalently, *per capita* fecun-  
568 dity as  $m = 25$  and density independent survivorship of larvae as  $S_0 = 0.15$ . This  
569 value is just below the median of the reproductive rates estimated in Hassel *et al.*  
570 (1976) across 24 different insect species. Therefore, we conjecture that these meth-  
571 ods would still apply to a variety of pest species. Second, the reproductive rate is  
572 just one of the parameters in our model. Although the main text is supplemented  
573 with sensitivity analyses, our work is not intended to provide robust quantitative  
574 guidelines for a practical application of our proposed management strategies. In-  
575 stead, our analysis provides general properties of the interactions between Allee  
576 effects and cytoplasmic incompatibilities on the population dynamics of an insect  
577 pest species. Should our proposed management strategy be applied to a specific  
578 insect pest that exhibits Allee effects, thorough investigations would be needed to

579 better quantify parameters that dictate the strength of the processes that underpin  
580 the model, including *Wolbachia* diversity and incompatibility as well as component  
581 and demographic Allee effects.

582 To date, the potential use of *Wolbachia*-induced CI for pest management is  
583 supported by a few but important studies on mosquitoes (Hoffmann *et al.*, 2011;  
584 Laven, 1967) and fruit flies (Zabalou *et al.*, 2004). Furthermore, evidence for the  
585 existence of both a component and a strong demographic Allee effect exists in  
586 the Gypsy moth, which could explain the relative success of mating disruption  
587 in this species (Tobin *et al.*, 2013, 2007). Although such empirical advances are  
588 indeed promising, they deserve a much stronger body of data and robust cause  
589 and effect demonstrations (Fauvergue, 2013). From this perspective, our model  
590 was not built as a predictive tool for a specific species. It was rather developed as  
591 a heuristic theory yielding qualitative predictions which will hopefully encourage  
592 future experimental approaches on the consequences of cytoplasmic incompatibility  
593 and Allee effects on population extinction.

594 There is a long and prolific body of research in population dynamics that  
595 focuses on understanding the mechanisms stabilizing species near their carrying  
596 capacities (e.g. Hassell & May, 1973; Robert M. May, 1978; Bernstein, 2000).  
597 More recently, global climate change and the biodiversity crisis, including popu-  
598 lation declines, extinctions, or biological invasions, points towards the increasing  
599 relevance of nonequilibrium ecology (Rohde, 2006) and the biology of small popu-  
600 lations (Fauvergue *et al.*, 2012). Transient dynamics are increasingly emphasized  
601 (Hastings, 2004) and sometimes considered in the specific context of population  
602 management (Ezard *et al.*, 2010; Kidd & Amarasekare, 2012). As first highlighted  
603 by Dobson *et al.* (2002), cytotype replacement which occurs in the course a suc-

604 cessful *Wolbachia* invasion yields a transient coexistence of incompatible infections  
605 within a host population, and as a consequence, a transient decrease in reproduc-  
606 tive rate and population size. Here, the transients only last a few generations and  
607 this perturbation of the population's microbiome is the first step in a destabi-  
608 lizing cascade. We show here that the population can then be pushed toward a  
609 second step of destabilization, triggered by a mate-finding Allee effect that can be  
610 reinforced by the application of mating disruption, which potentially drives the  
611 population to extinction.

## 612 Acknowledgements

613 We thank Fabrice Vavre for his enthusiasm in preliminary discussions, and Sylvain  
614 Charlat for comments in earlier stages of this work. We also thank the Associate  
615 Editor and two anonymous reviewers for their helpful and constructive feedback.

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801 *ences of the United States of America*, **101**, 15042–15045.

802 **Tables**

Parameter	Description	Value	Source
$N_0$	initial population size	varies 1– $K$	
$m, m_t$	<i>per capita</i> fecundity (realized, expected)	25	
$\sigma^*$	standard deviation of fecundity	5	
$S_0^*$	survivorship in absence of competition	varies 0.08–0.2	
$\alpha$	related to carrying capacity	0.00002	[1]
$\gamma$	related to intraspecific competition	1	[1]
$\mu_X^*, \mu_Y$	maternal transmission failure	0.03	[1]
$F_X^*, F_Y$	relative fecundity of infected individuals	0.95	[1]
$H_X, H_Y$	proportion of offspring surviving CI	0.05	[2]
$\theta$	strength of Allee effect	varies 1–0.5 $K$	
$P$	number of pheromone sources	varies 0–0.15 $K$	

Table 1: List of model parameters. All figures use these parameter values, unless otherwise stated. Parameter values with a “\*” have associated sensitivity analyses (as discussed in the main text) in the Supplementary Information (S3). [1] refers to the reference Dobson *et al.* 2002, and [2] refers to Charlat *et al.* (2005).

$\begin{array}{c} \text{♀} \\ \text{♀} \end{array} \backslash \begin{array}{c} \text{♂} \\ \text{♂} \end{array}$	$W$	$X$	$Y$
$W$	$W: c_t q_t$ $X: 0$ $Y: 0$	$W: H_X c_t i_t$ $X: 0$ $Y: 0$	$W: H_Y c_t j_t$ $X: 0$ $Y: 0$
$X$	$W: \mu_X F_X a_t q_t$ $X: (1 - \mu_X) F_X a_t q_t$ $Y: 0$	$W: \mu_X F_X H_X a_t i_t$ $X: (1 - \mu_X) F_X a_t i_t$ $Y: 0$	$W: \mu_X F_X H_Y a_t j_t$ $X: (1 - \mu_X) F_X H_Y a_t j_t$ $Y: 0$
$Y$	$W: \mu_Y F_Y b_t q_t$ $X: 0$ $Y: (1 - \mu_Y) F_Y b_t q_t$	$W: \mu_Y F_Y H_X b_t i_t$ $X: 0$ $Y: (1 - \mu_Y) F_Y H_X b_t i_t$	$W: \mu_Y F_Y H_Y b_t j_t$ $X: 0$ $Y: (1 - \mu_Y) F_Y b_t j_t$

Table 2: Summary of *Wolbachia* transmission and its effects on reproduction in its host population. The first column states the maternal *Wolbachia* cytotype and the first row states the paternal *Wolbachia* cytotype. Each box in the table corresponding to a particular pairing between a female and male provides that proportion of the offspring from that pair that are uninfected ( $W$ ), infected with cytotype  $X$ , and infected with cytotype  $Y$ .

## 803 Figure captions

804 **Figure 1.** Verification that our model accurately predicts the invasion threshold  
805 as analytically determined in Hoffman *et al.* 1990. Here, we ignore Allee effects,  
806 stochasticity, and assume  $P = 0$ . The thick black curve is the frequency of infected  
807 individuals at time  $t + 1$  given the frequency at  $t$ ; equilibria occur when this curve  
808 and the gray line (which corresponds to the case that the frequency at generations  
809  $t$  and  $t + 1$  are equal) intersect. When the black curve lies above the gray line,  
810 the infection frequency is increasing; similarly, the infection frequency is decreas-  
811 ing when the black curve falls below the gray line. The dashed lines indicate the  
812 analytically predicted equilibrium. The smaller intersection is an unstable equi-  
813 librium that defines the invasion threshold: individuals introduced at a frequency  
814 higher than this threshold will successfully invade the population and approach  
815 the higher stable equilibrium. In this figure, we use more extreme values for pa-  
816 rameters related to CI to more clearly demonstrate the location of the invasion  
817 threshold (specifically,  $\mu_X = 0.2$ ,  $H_X = 0.1$ ). Here,  $S_0 = 0.15$ .

818  
819 **Figure 2.** Reproductive rate as a function of population size when  $\theta = 100$  (gray)  
820 and  $\theta = 1500$  (red). Values above one correspond to population growth, and values  
821 below one correspond to decline. The populations corresponding to the solid lines  
822 have no *Wolbachia*-infected individuals, populations with dashed lines have 10%  
823 of the population infected, dash-dotted lines have 50% of the population infected,  
824 and dotted lines have 90% of the population infected. Here,  $S_0 = 0.15$ .

825  
826 **Figure 3.** Deterministic results. (A) no control (so the population size remains

827 at  $K$ ); (B) single *Wolbachia* introduction; (C) two introductions. Plot displays the  
828 minimum population size relative to  $K$  over 50 generations assuming that  $N_0 = K$   
829 versus the relative strength of the Allee effect ( $\theta/K$ ). The solid line has  $S_0 = 0.08$   
830 (maximum reproductive rate of 2 in the absence of Allee effects as in Dobson *et.*  
831 *al.*, 2002), the dashed red line has  $S_0 = 0.15$  (maximum reproductive rate of 3.75 in  
832 the absence of Allee effects), dash-dotted line has  $S_0 = 0.2$  (maximum reproductive  
833 rate of 5 in the absence of Allee effects), and dotted line has  $S_0 = 0.25$  (maximum  
834 reproductive rate of 6.25 in absence of Allee effects). (D)-(E) are identical to (A)-  
835 (C), respectively, but it is additionally assumed that pheromone sources are held  
836 at a fixed level such that  $P/K = 0.1$ . In all plots, each release is created so the  
837 initial infection frequency of that cytotype is 10%. The first release is at genera-  
838 tion one, and the second release is determined by the optimization scheme detailed  
839 in Section S2 in the ESM. In (E)-(F), the absence of the solid line indicates that  
840 extinction is achieved for all values of  $\theta/K$ . Curves corresponding to  $S_0 = 0.15$ ,  
841 the value used in all subsequent plots, are highlighted in red to further emphasize  
842 the synergy between management tactics.

843

844 **Figure 4.** Sample trajectories of the population dynamics (relative to  $K$ ) when  
845 two incompatible cytotypes of *Wolbachia* are introduced into a population when  
846  $S_0 = 0.15$ . The relative strength of the Allee effect is set to  $\theta/K = 0.15$ , which  
847 corresponds to an Allee threshold of  $\sim 5.8\%$  of the carrying capacity (as indicated  
848 by the horizontal dashed gray line). The black dashed line assumes that there  
849 are no pheromone sources, and the solid line assumes that the relative number of  
850 pheromone sources  $P/K$  is 0.1. As also evidenced in Fig. 3 (e.g. by considering  
851 the dashed curve in Fig. 3A-B when  $\theta/K = 0.15$ ), intervention with *Wolbachia*

852 introductions only under these parameter conditions is not successful. A combina-  
853 tion of *Wolbachia* introduction and pheromone sources, however, is successful in  
854 achieving population extinction.

855

856 **Figure 5.** The gray scale in each plot represents the extinction probability for  
857 a given parameter combination based on 500 realizations of the model. In each  
858 plot, the initial population size and the strength of the Allee effect  $\theta$ , both relative  
859 to  $K$ , are varied. We note that the carrying capacity of the population in the  
860 absence of Allee effects is 5500 with these parameters; therefore, the introduction  
861 sizes – which adjust the population size so that there is a 10% infection frequency  
862 – do not exceed  $\sim 612$  insects. Top row: demographic stochasticity only. Bottom  
863 row: both demographic and environmental stochasticity. First column: no intro-  
864 duction. Second column: introduction such that infection frequency is at 10%.  
865 Third column: two subsequent introductions, both of which ensure the infection  
866 frequency is 10% for each cytotype at time of introduction (see Supplementary  
867 Information S2 for generation of second introduction). The dashed gray curve is  
868 the Allee threshold (i.e. initial populations below the gray curve go to extinction  
869 in the deterministic model). Here,  $S_0 = 0.15$ .

870

871 **Figure 6.** Fixing the relative strength of the Allee effect  $\theta/K = 0.1$ , the gray scale  
872 in each plot represent the extinction probability for a given parameter combination  
873 based on 500 realizations of the model. In each plot, the initial population size  
874 and the number of pheromone sources ( $P$ ) relative to  $K$  are varied. Top row: de-  
875 mographic stochasticity only. Bottom row: both demographic and environmental  
876 stochasticity. First column: no introduction. Second column: introduction such

877 that infection frequency is at 10%. Third column: two subsequent introductions.

878 Here,  $S_0 = 0.15$ .

879

Figure 1:

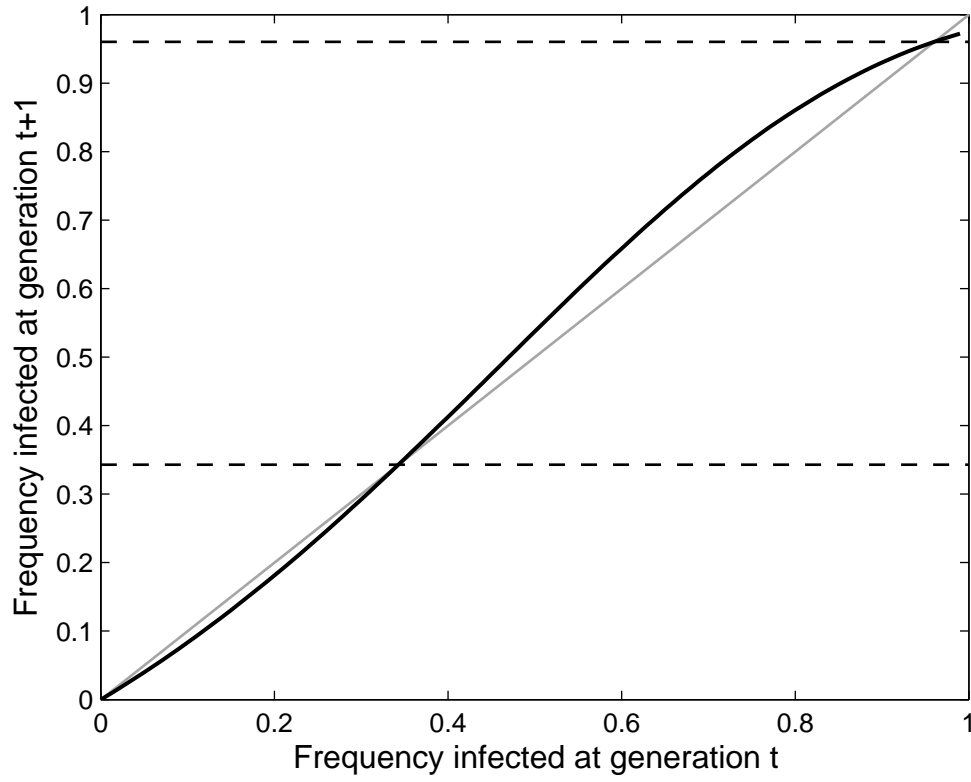




Figure 2:

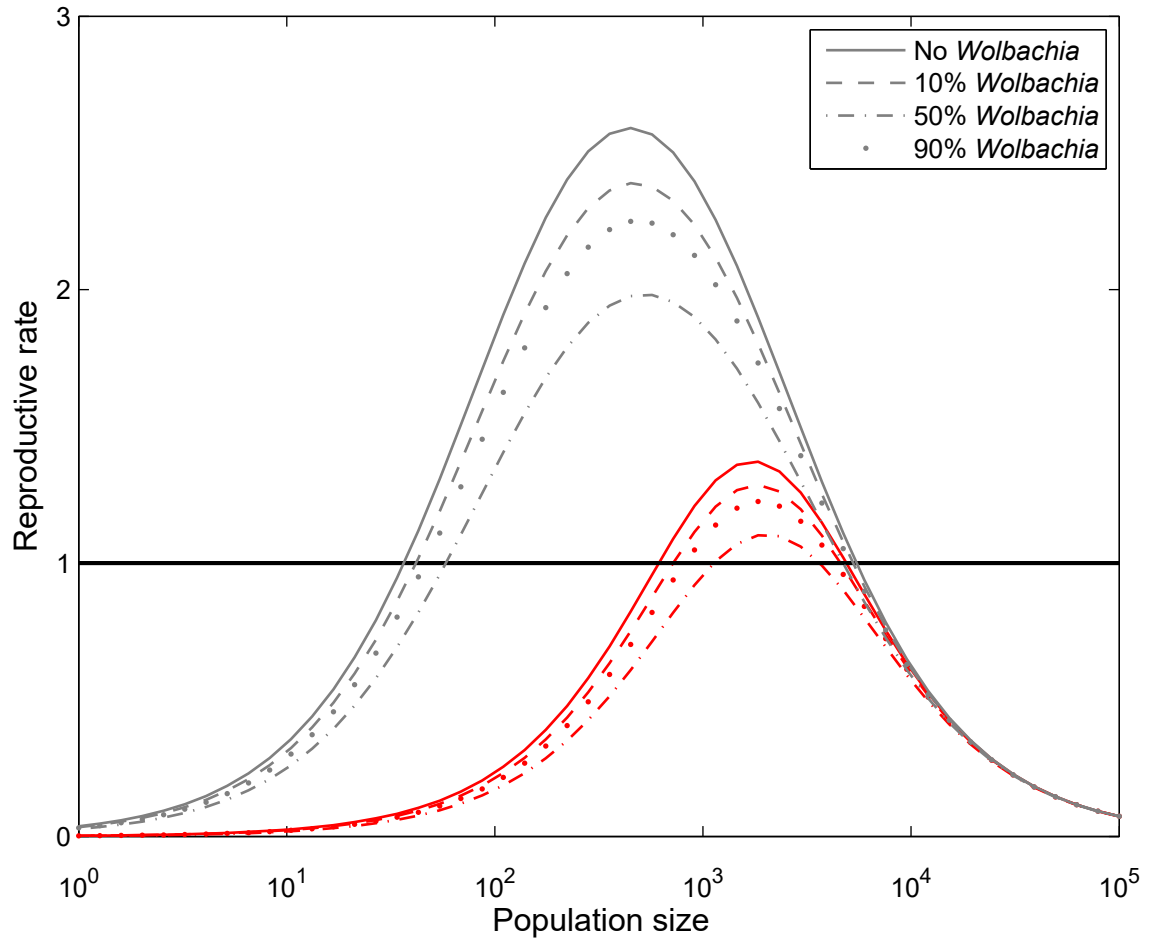


Figure 3:

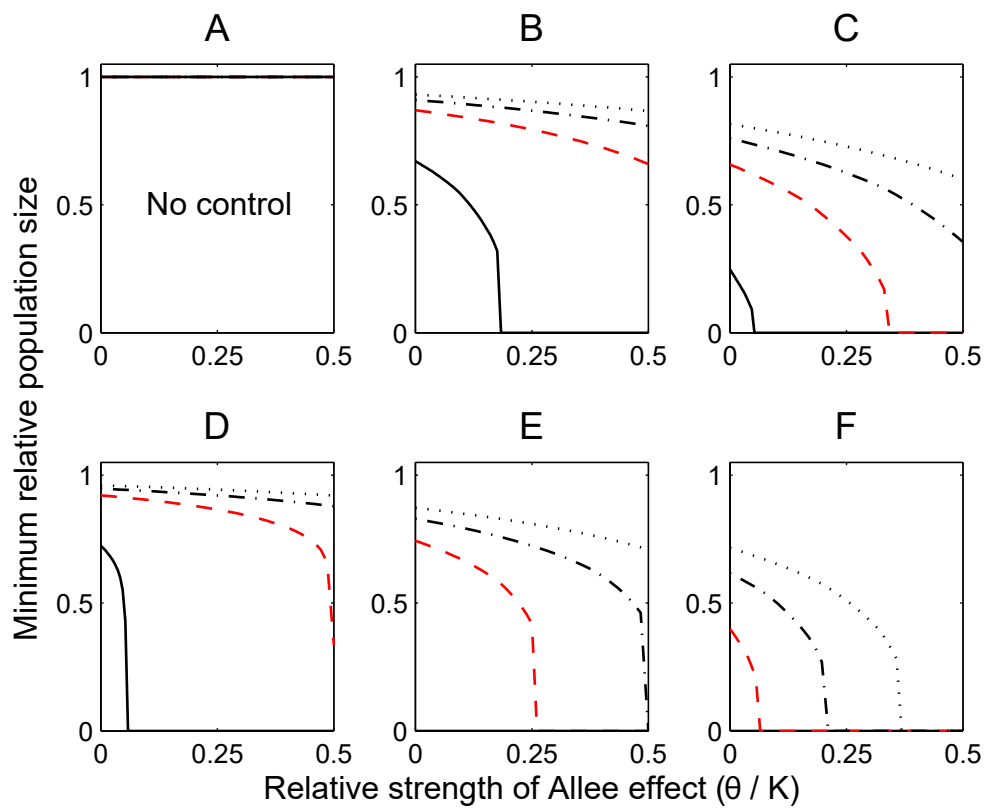


Figure 4:

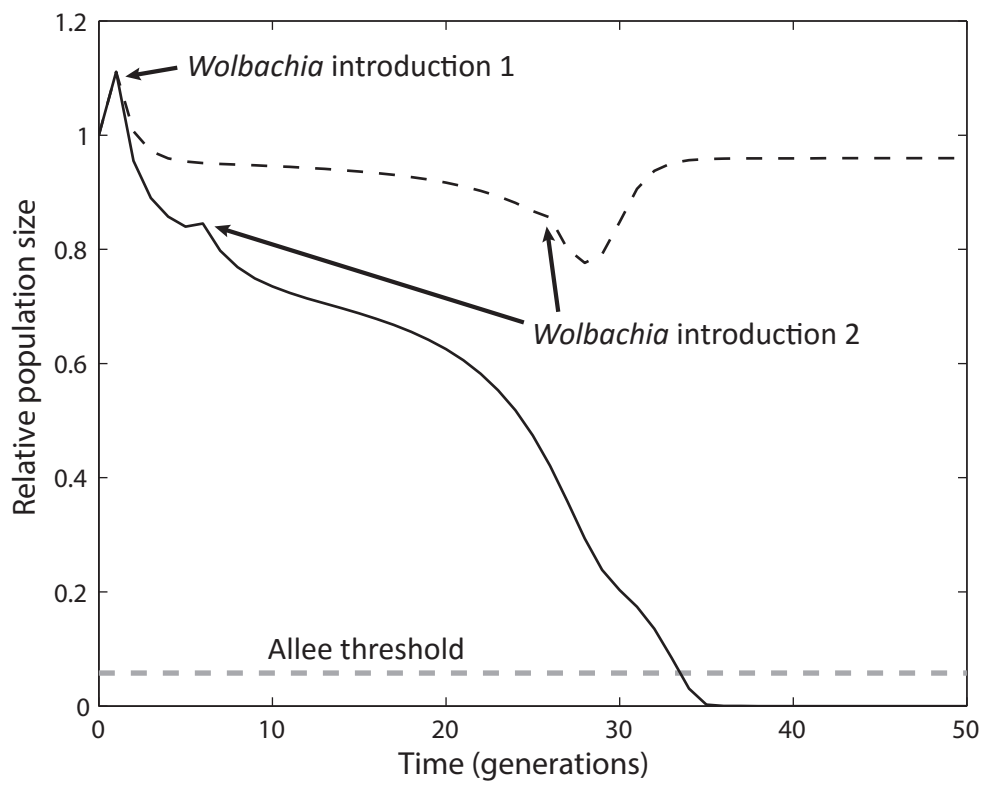


Figure 5:

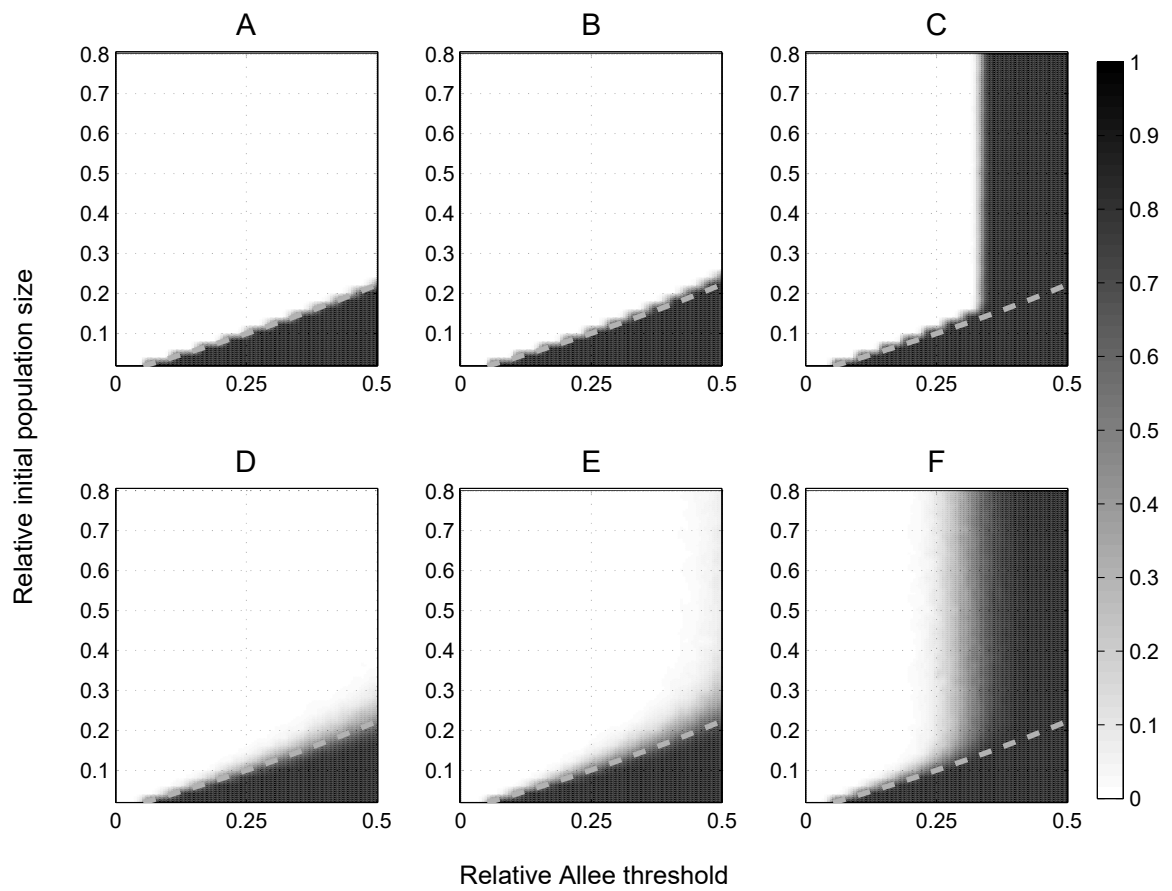


Figure 6:

