

1 **Can we obtain *in vivo* transmural mean hoop stress of the aortic**
2 **wall without knowing patient-specific material properties and**
3 **residual deformations?**

4
5
6 Minliang Liu¹, Liang Liang¹, Haofei Liu², Ming Zhang², Caitlin Martin¹, Wei Sun¹

7
8
9
10 *¹Tissue Mechanics Laboratory*
11 *The Wallace H. Coulter Department of Biomedical Engineering*
12 *Georgia Institute of Technology and Emory University, Atlanta, GA*

13
14 *²Department of Mechanics, Tianjin University*
15 *92 Weijin Road, Tianjin, China 300072*

16
17
18 Submitted to

19 *Some Journal*

20 7/10/2018

21
22
23 For correspondence:

24 Wei Sun, Ph.D.

25 The Wallace H. Coulter Department of Biomedical Engineering
26 Georgia Institute of Technology and Emory University
27 Technology Enterprise Park, Room 206
28 387 Technology Circle, Atlanta, GA 30313-2412
29 Tel: [\(404\) 385-1245](tel:(404)385-1245); Email: wei.sun@bme.gatech.edu

30
31

32 **Abstract**

33 It is well known that residual deformations/stresses alter the mechanical behavior of arteries, e.g.
34 the pressure-diameter curves. In an effort to enable personalized analysis of the aortic wall stress,
35 approaches have been developed to incorporate experimentally-derived residual deformations into
36 *in vivo* loaded geometries in finite element simulations using thick-walled models. Solid elements
37 are typically used to account for “bending-like” residual deformations. Yet, the difficulty in
38 obtaining patient-specific residual deformations and material properties has become one of the
39 biggest challenges of these thick-walled models. In thin-walled models, fortunately, static
40 determinacy offers an appealing prospect that allows for the calculation of the thin-walled
41 membrane stress without patient-specific material properties. The membrane stress can be
42 computed using forward analysis by enforcing an extremely stiff material property as penalty
43 treatment, which is referred to as the forward penalty approach. However, thin-walled membrane
44 elements, which have zero bending stiffness, are incompatible with the residual deformations, and
45 therefore, it is often stated as a limitation of thin-walled models. In this paper, by comparing the
46 predicted stresses from thin-walled models and thick-walled models, we demonstrate that the
47 transmural mean hoop stress is the same for the two models and can be readily obtained from *in*
48 *vivo* clinical images without knowing the patient-specific material properties and residual
49 deformations. Computation of patient-specific mean hoop stress can be greatly simplified by using
50 membrane model and the forward penalty approach, which may be clinically valuable.

51 **Keywords:** residual stress; transmural mean hoop stress; membrane stress; forward penalty
52 approach.

53 **1 Introduction**

54 Residual deformations/stresses first discovered in the 1980s (Chuong and Fung 1986;
55 Vaishnav and Vossoughi 1983) have been shown to significantly affect the physiological wall
56 stress distributions (Delfino et al. 1997; Fung 1991; Holzapfel et al. 2000; Humphrey 2002;
57 Matsumoto and Hayashi 1996). To incorporate residual deformations in arteries, traditional
58 forward analysis uses a thick-walled model starting from the stress-free reference configuration.
59 Then deformation relations, constitutive laws and equilibrium equations are utilized to solve the
60 boundary value problem. However, when applying this conventional approach to obtain patient-
61 specific stress fields from the *in vivo* loaded geometries in clinical images, one has to first
62 determine the unknown material parameters and residual deformations, which are required in the
63 thick-walled finite element (FE) models. Some studies have suggested the use of experimentally-
64 determined material and residual deformation parameters (Alastrué et al. 2010; Pierce et al. 2015).
65 However, using residual deformations and material properties that are not patient-specific is a clear
66 limitation.

67 Fortunately, for a specific type of biological membrane structures such as the aorta, the
68 wall stress is nearly insensitive to the variation of material properties. This property is called static
69 determinacy, i.e. the external force (pressure) along the geometry can be used to directly compute
70 the internal tension/stress. This is because the vessel wall can be seen as locally in a plane stress
71 state (Miller and Lu 2013), the solution of the equilibrium is weakly sensitive to the material
72 properties. The aorta is shown to be approximately statically determinate (Joldes et al. 2016; Liu
73 et al. 2017). Thus, its stress distribution can be directly obtained using membrane elements by a
74 forward penalty method (Joldes et al. 2016; Lu and Luo 2016) which enforces an extremely stiff
75 material property as penalty treatment. The computation of the thin-walled stress can be greatly

76 simplified by this forward approach. However, due to the assumption of no bending stiffness in
77 the membrane elements, the self-equilibrium residual deformations are inadmissible to the thin-
78 walled models, which is often stated as a limitation of such models.

79 In this paper, by comparing the predicted stresses from thin-walled models with thick-
80 walled models considering residual deformations, we demonstrate that the transmural mean hoop
81 stress (i.e., averaged stress through the thickness) fields are the same for the two models. Thus, the
82 transmural mean hoop stress can be readily obtained from *in vivo* clinical images using the forward
83 penalty approach without knowing the patient-specific material properties and residual
84 deformations. The remaining sections are organized as follows. In Section 2, the theoretical
85 arguments are described and the validity is shown by analytical examples. Thin-walled and thick-
86 walled FE models with a patient-specific geometry are demonstrated in Section 3. In Section 4,
87 the discussion and conclusions are presented.

88 **2 Theoretical and Analytical Arguments**

89 One prominent example of static determinacy is the use of Laplace law to compute the wall
90 hoop stress by assuming a perfect cylindrical shape of the aorta.

$$91 \quad \sigma_{\theta\theta} = \frac{Pa}{t} \quad (1)$$

92 where $\sigma_{\theta\theta}$ is the hoop stress in the thin-walled tube, P is the pressure, a is the inner radius, and t
93 is the *in vivo* wall thickness. The material properties are not involved in this equation, and stress
94 is directly calculated using the static force equilibrium. Opposite to the middle radius value used
95 in (Horný et al. 2014), we emphasize that inner radius should be used as the blood pressure is
96 applied to the inner surface of the aorta.

97 It is well known that residual stresses alter the mechanical response of arteries, e.g. the
98 pressure-diameter curves (Holzapfel et al. 2000). Nonetheless, from the static determinacy
99 prospective, for the *in vivo* loaded configuration, the equilibrium between the resultant force and
100 the external pressure load should always hold, and thus, the hoop stress resultant (tension) should
101 be insensitive to the material parameters and residual deformations. This implies that no matter
102 how the aorta is internally balanced or residually stressed, the wall tension can always be computed
103 only using the static equilibrium. Therefore, when the wall thickness is given, the simple thin-
104 walled model would be sufficient in determining the transmural mean hoop stress.

105 **2.1 The Opening Angle Method**

106 In this subsection, we use an analytical example to demonstrate that the mean hoop stress is
107 insensitive to the change of opening angles. We assume that the residual stress can be described
108 by the opening angle and that the aorta can be modelled as a perfect tube.

109 Starting from the cut-open, stress-free configuration, summarizing from (Holzapfel et al. 2000)
110 and (Pierce et al. 2015), the total deformation gradient tensor of the tube taking into account the
111 residual stress, \mathbf{F}_{total} , can be obtained as

$$112 \quad \mathbf{F}_{total}(r) = \frac{rk}{\left[A^2 + k\frac{l}{L}(r^2 - a^2)\right]^{\frac{1}{2}}} \mathbf{e}_\theta \otimes \mathbf{E}_\theta + \frac{l}{L} \mathbf{e}_z \otimes \mathbf{E}_Z + \frac{L}{rkl} \left[A^2 + k\frac{l}{L}(r^2 - a^2)\right]^{\frac{1}{2}} \mathbf{e}_r \otimes \mathbf{E}_R \quad (2)$$

113 where $r \in [a, b]$, a and b are the inner and outer radii of the *in vivo* deformed geometry. k ,
114 defined as $k = \frac{2\pi}{2\pi - \alpha}$, is used to describe the opening angle α . A and B are the inner and outer radii
115 of the stress-free geometry. L and l are the axial length of the aorta segment in the stress-free and
116 deformed geometry, respectively. \mathbf{E}_θ , \mathbf{E}_Z and \mathbf{E}_R and \mathbf{e}_θ , \mathbf{e}_z and \mathbf{e}_r are the unit basis vectors for

117 the stress-free and deformed geometry respectively. To make the solution simple, the constitutive
118 relation of the aorta tissue is modelled using the isotropic Neo-Hookean model. The strain energy
119 Ψ is

$$120 \quad \Psi = \frac{1}{2}\mu(I_1 - 3) \quad (3)$$

121 where μ is the shear modulus and I_1 is the first invariant. To solve for the *in vivo* stress when
122 systolic blood pressure ($P = 104\text{mmHg}$) (Martin et al. 2015) is present, we utilize the stress
123 equilibrium equation, which can be expressed in the radial equation

$$124 \quad \frac{d\sigma_{rr}}{dr} = \frac{1}{r}(\sigma_{rr} - \sigma_{\theta\theta}) \quad (4)$$

125 where $\sigma_{\theta\theta}$ and σ_{rr} are the stresses in the circumferential and radial direction respectively. Eqn. (4)
126 can be reduced to $\frac{d\sigma_{rr}}{dr} = \frac{\mu}{r}(\lambda_\theta^2 - \lambda_r^2)$ (Holzapfel and Ogden 2010), with λ_θ and λ_r referring to the
127 stretches in the circumferential and radial directions, respectively. By solving the equilibrium Eqn.
128 (4), together with the traction continuity condition $\sigma_{rr}(a) = -P$, we are able to obtain the radial
129 stress (Holzapfel and Ogden 2010)

$$130 \quad \sigma_{rr}(r) = -P + \mu \left[\frac{k}{\lambda_z} \log\left(\frac{\lambda_r \lambda_z r k}{A}\right) - \frac{1}{k\lambda_z} \log\left(\frac{r}{a}\right) + \frac{1}{2} \left(\lambda_r^2 - \frac{1}{k\lambda_z} \right) \left(\frac{a^2 - r^2}{a^2} \right) \right] \quad (5)$$

131 where λ_z is the stretch in the axial direction. The hoop stress is then calculated using

$$132 \quad \sigma_{\theta\theta}(r) = \sigma_{rr}(r) + \mu(\lambda_\theta^2 - \lambda_r^2) \quad (6)$$

133 The geometry of the aorta in clinical images is always in the *in vivo* deformed state, from which
134 the opening angle is not measurable. To this end, we fixed the inner and outer radii of the *in vivo*
135 deformed geometry, a and b , for all scenarios and vary the opening angle from 0 to 330 degree.

136 For a certain opening angle α , the inner and outer radii A and B of the cut-open sectors are

137 solved using the boundary condition $\sigma_{rr}(b) = 0$ and the assumption of incompressibility $b^2 =$
138 $a^2 + \frac{1}{k\lambda_z}(B^2 - A^2)$. Related parameters are listed in Table 1 and values of A and B are shown in
139 Table 2. The transmural mean hoop stress is defined as

$$140 \quad \bar{\sigma}_{\theta\theta} = \frac{1}{b-a} \int_a^b \sigma_{\theta\theta} dr \quad (7)$$

141 The results are shown in Figure 1 (left). The mean hoop stress computed from Eqn. (7) is
142 exactly the same as the thin-walled hoop stress calculated using Eqn. (1). Unsurprisingly, if we
143 rewrite Eqn. (4) as $\sigma_{\theta\theta} = \frac{d}{dr}(r\sigma_{rr})$ and therefore $\bar{\sigma}_{\theta\theta} = \frac{1}{b-a} \int_a^b d(r\sigma_{rr}) = \frac{b\sigma_{rr}(b) - a\sigma_{rr}(a)}{t} = \frac{Pa}{t}$,
144 which is exactly the same formula as the Laplace law. In addition, as shown in Figure 1 (right),
145 the adoption of anisotropic constitutive model (the GOH model (Gasser et al. 2006), described in
146 Section 3.2.1, parameters shown in Table 3) would not affect the static determinacy. The inner
147 hoop stress tends to be reduced while the outer hoop stress is increased when gradually increasing
148 the opening angle. Here, small opening angles may be unusual to observe in experiment (Sokolis
149 2015), they are presented here for illustrative purpose.

150 **2.2 The Layer-Specific Three-Dimensional Residual Stress Model**

151 As a step forward, (Holzapfel and Ogden 2010) proposed a layer-specific three-
152 dimensional residual stress model, in which the residual deformations (stretching and bending) of
153 the three layers (intima, media and adventitia) from (Holzapfel et al. 2007) were encompassed and
154 the residual stresses were calculated using the isotropic Neo-Hookean model. In this section, we
155 first replicate the stress distribution in (Holzapfel and Ogden 2010) by using the original
156 parameters of geometry, material and residual deformations. Next, physiological pressure is

157 applied to the residually-stressed aorta. The result indicates that the transmural mean hoop stress
 158 is independent of residual deformations.

159 The deformation gradient tensors for intima (I), media (M) and adventitia (A) (Pierce et al.
 160 2015) are

$$161 \mathbf{F}_{RS}^{(I)}(r^{(I)}) = \frac{r^{(I)k^{(I)}}}{\left[A^{(I)2} + k^{(I)} \frac{l}{L^{(I)}} (r^{(I)2} - a^{(I)2})\right]^{\frac{1}{2}}} \mathbf{e}_\theta \otimes \mathbf{E}_\theta + \frac{l}{L^{(I)}} \mathbf{e}_z \otimes \mathbf{E}_z + \frac{L^{(I)}}{r^{(I)k^{(I)}l} \left[A^{(I)2} + k^{(I)} \frac{l}{L^{(I)}} (r^{(I)2} - a^{(I)2})\right]^{\frac{1}{2}}} \mathbf{e}_r \otimes \mathbf{E}_R$$

$$163 \mathbf{F}_{RS}^{(M)}(r^{(M)}) = \frac{r^{(M)\beta}}{L^{(M)}} \mathbf{e}_\theta \otimes \mathbf{E}_z + \frac{l^{(M)k^{(M)}}}{\pi \left[A^{(M)2} + \frac{\beta l^{(M)k^{(M)}}}{\pi L^{(M)}} (b^{(M)2} - r^{(M)2})\right]^{\frac{1}{2}}} \mathbf{e}_z \otimes \mathbf{E}_\theta + \frac{\pi L^{(M)}}{r^{(M)\beta l^{(M)k^{(M)}}} \left[A^{(M)2} + \frac{\beta l^{(M)k^{(M)}}}{\pi L^{(M)}} (b^{(M)2} - r^{(M)2})\right]^{\frac{1}{2}}} \mathbf{e}_r \otimes \mathbf{E}_R$$

$$165 \mathbf{F}_{RS}^{(A)}(r^{(A)}) = \frac{\pi r^{(A)}}{L_2^{(A)}} \mathbf{e}_\theta \otimes \mathbf{E}_{X_2} + \frac{l}{L_3^{(A)}} \mathbf{e}_z \otimes \mathbf{E}_{X_3} + \frac{L_2^{(A)} L_3^{(A)}}{\pi r^{(A)} l} \mathbf{e}_r \otimes \mathbf{E}_{X_1} \quad (8)$$

166 The definitions and values of the parameters are referred to (Holzapfel and Ogden 2010). Values
 167 of the related parameters are listed in Table 4.

168 Similar to the procedures for the opening angle method, the hoop stress can be computed
 169 using the equilibrium equation and the boundary conditions. Interested readers are referred to
 170 (Holzapfel and Ogden 2010) for details. A diastolic pressure ($P = 80mmHg$) is applied to the
 171 inner surface of the aorta, and we assume no axial tension caused by *in vivo* loading conditions.
 172 The residual axial stretches have been incorporated in the deformation gradient tensors of each
 173 layer. The transmural mean hoop stress for the three layer composite is defined as

$$174 \bar{\sigma}_{\theta\theta} = \frac{1}{b^{(A)} - a^{(I)}} \sum_{i=I,M,A} \int_{a^{(i)}}^{b^{(i)}} \sigma_{\theta\theta}^{(i)} dr \quad (9)$$

175 As depicted in Figure 2, the mean hoop stress is identical to the thin-walled hoop stress.

176 **3 Finite Element Analyses Incorporating Residual Deformations**

177 In this section, irregularity of patient-specific geometries are taken into account using FE
178 analyses. The validity of the conclusion in Section 2 is examined by a real patient geometry. The
179 forward penalty approach (Section 3.1) is used to estimate the thin-walled membrane stress. For
180 the thick-walled FE models, the generalized pre-stressing algorithm (GPA) (Pierce et al. 2015;
181 Weisbecker et al. 2014) is implemented in ABAQUS (Section 3.2) to predict the *in vivo* stress
182 distribution with both the residual deformations and the pre-stresses incorporated.

183 A CT-derived geometry from the ascending thoracic aortic aneurysm (ATAA) patient
184 (Martin et al. 2015) were used. The inner surface of the aortic wall was divided into 4,950 M3D4
185 membrane elements in ABAQUS, using the automatic algorithm (Liang et al. 2017) previously
186 developed by our group. Mesh sensitivity analysis was performed in our previous work (Martin et
187 al. 2015). Due to partial volume effect, the wall thickness is difficult to infer from CT images,
188 therefore a constant deformed thickness of 1.5 mm was assumed based on (Liang et al. 2017).
189 Sensitivity analyses with respect to the wall thickness were carried out in Section 3.2.2. Next, the
190 membrane mesh was extruded outwardly to create two solid meshes (C3D8 elements) with 8 and
191 9 layers in Section 3.2.2 and Section 3.2.3, respectively.

192 **3.1 A Thin-walled Model using the Forward Penalty Approach**

193 The prediction of the *in vivo* stress of the aortic wall has been relied on the recovery of the unloaded
194 state and the incorporation of residual deformations, which requires the use of iterative techniques
195 (Alastrué et al. 2010). A simple and effective forward penalty approach (Joldes et al. 2016) (Lu

196 and Luo 2016) has been recently proposed to predict the *in vivo* membrane stress without knowing
197 the material properties. In statically determinant structures, the stress is independent of the material
198 properties, it would be legitimate to assume an extremely stiff property, so that the
199 deformation/change of shape from the unloaded configuration to the loaded configuration is
200 infinitesimal/negligible. This allows us to use the *in vivo* configuration as the unloaded
201 configuration because the deformation is infinitesimal. In the forward method, an artificially stiff
202 material property (i.e. $\mu = 10^7 Pa$, Neo-Hookean model) is assigned to the aortic wall, realizing a
203 penalty treatment to enforce a nearly rigid condition (Lu and Luo 2016). When the *in vivo* pressure
204 is applied to the *in vivo*, image-derived geometry, the deformation would be infinitesimal due to
205 the high stiffness of the material. The correct *in vivo* membrane stress field is readily obtained in
206 this forward analysis due to the fact that the aortic wall is approximately statically determinate.
207 This approach was shown as effective as iterative approach (Lu and Luo 2016).

208 Similar to the reason for the use of the inner radius in the Laplace equation (Eqn. (1)), we
209 emphasize that the inner surface of the aortic wall should be used in the thin-walled model when
210 applying the forward approach.

211 **3.2 Thick-walled Models Incorporating Residual Deformations**

212 **3.2.1 Method to incorporate Residual Deformations to Thick-walled** 213 **Models**

214 The aortic tissue is described by the Gasser-Ogden-Holzapfel (GOH) model (Gasser et al.
215 2006)

$$216 \quad \Psi = C_{10}(\bar{\mathbf{I}}_1 - 3) + \frac{k_1}{2k_2} \sum_{i=1}^2 \left[\exp \left\{ k_2 \left[\kappa \bar{\mathbf{I}}_1 + (1 - 3\kappa) \bar{\mathbf{I}}_{4(6)} - 1 \right]^2 \right\} - 1 \right] + \frac{1}{D} \left[\frac{J^2 - 1}{2} - \ln J \right] \quad (10)$$

217 where C_{10} , k_1 , k_2 and κ are material parameters, θ defines the fiber directions, please refer to
218 (Abaqus 2014; Gasser et al. 2006) for detailed definitions. The parameter D enforces the nearly
219 incompressibility and is fixed to be 1×10^{-5} .

220 The GPA (Pierce et al. 2015; Weisbecker et al. 2014) is utilized to incorporate the residual
221 deformation. The total deformation gradient \mathbf{F}_t is stored as a history variable for each integration
222 point. \mathbf{F}_t is updated based on the incremental deformation gradient $\Delta\mathbf{F}$ resulting from the
223 prescribed load and boundary conditions.

$$224 \quad \mathbf{F}_{t+1} = \Delta\mathbf{F}\mathbf{F}_t \quad (11)$$

225 The incremental deformation gradient of the residual stress $\Delta\mathbf{F}_{RS}$ is first iteratively applied to the
226 image-derived geometry and stored in \mathbf{F}_t . Next, the incremental deformation gradient of the pre-
227 stress $\Delta\mathbf{F}_{PS}$ resulting from the *in vivo* blood pressure is incrementally applied and stored in \mathbf{F}_t .
228 Thus, deformation gradient tensors associated with the residual stress \mathbf{F}_{RS} and the pre-stress \mathbf{F}_{PS}
229 are accounted sequentially. The GPA is implemented in the ABAQUS user subroutine UMAT.
230 The implementation was validated by comparing the analytical and FE results as in (Pierce et al.
231 2015).

232 **3.2.2 Thick-walled Models with Various Opening Angles**

233 The thick-walled solid elements were utilized in this section to encompass the opening
234 angle. Various values of the opening angle were incorporated through the GPA. Small opening
235 angles may be unusual to observe in experiments, they are shown here for illustration purposes.
236 The aorta was modelled as a single layer wall. This assumption may be relevant to abdominal
237 aneurysmal tissue since collagen structure becomes nearly homogenous across the entire wall

238 (Gasser et al. 2012). For ascending aortic aneurysms, collagen organization may be different in
239 different layers (Sassani et al. 2015). The GOH model (Eqn (10)) was used as the constitutive law,
240 and the material parameters (shown in Table 3) were determined from fitting the biaxial data from
241 (Martin et al. 2015) of the particular patient.

242 Mean absolute percentage error (MAPE) was used to compare the transmural mean hoop
243 stress (Eqn.(7)) of the thin-walled and thick-walled models:

$$244 \quad MAPE = \frac{1}{N} \sum_{i=1}^N \left| \frac{\bar{\sigma}_{\theta\theta,i}^{(thin)} - \bar{\sigma}_{\theta\theta,i}^{(thick)}}{\bar{\sigma}_{\theta\theta,i}^{(thin)}} \right| \quad (12)$$

245 where $\bar{\sigma}_{\theta\theta,i}^{(thin)}$ and $\bar{\sigma}_{\theta\theta,i}^{(thick)}$ are the transmural mean hoop stress predicted by the thin-
246 walled and thick-walled models respectively. i is an element index for the thin-walled model and
247 N is the number of elements.

248 To study the sensitivity of the MAPE of the transmural mean stress with respect to the
249 thickness, three representative thickness values (1mm, 2mm and 3mm) were chosen with $\alpha =$
250 120° . The results are summarized in Table 5. Note that this opening angle value is chosen because
251 the corresponding stress distribution is close to homogenized state in the FE simulation, and this
252 value may not be consistent with the average value obtained from experiment (Sokolis 2015). We
253 also notice that opening angle values are widely distributed according to (Sokolis 2015), 120
254 degree can be considered as a feasible value.

255 In order to quantify the transmural variation, we define a signed transmural percentage
256 error (STPE), corresponding to the i th thin-walled membrane element, as

$$257 \quad STPE(i) = sign[\sigma_{\theta\theta,i}^{(thick)}(a_i) - \sigma_{\theta\theta,i}^{(thick)}(b_i)] \frac{1}{b_i - a_i} \int_{a_i}^{b_i} \left| \frac{\sigma_{\theta\theta,i}^{(thick)}(r) - \bar{\sigma}_{\theta\theta,i}^{(thick)}}{\bar{\sigma}_{\theta\theta,i}^{(thick)}} \right| dr \quad (13)$$

258 where a_i and b_i represent the inner and outer radii respectively, and r is the radius. The sign is
259 given based on the difference between the inner and outer wall hoop stress. If the inner wall stress
260 is greater than the outer, the STPE is positive, otherwise the STPE is negative.

261 The results are shown in Figure 3, the transmural mean hoop stress fields are almost
262 identical for various opening angles and the forward penalty approach. More detailed views of ring
263 cuts at the same location are shown in Figure 4. With increased opening angle, the mean signed
264 transmural percentage error (MSTPE) changes from positive to negative.

265 The probability density functions (PDFs) of the STPE are plotted in Figure 5. The PDFs
266 are fitted using the Gaussian distribution. It can be observed that the PDF shifts leftward with the
267 increase of the opening angle.

268 **3.2.3 A Thick-walled Model with Layer-Specific Three-Dimensional** 269 **Residual Deformation**

270 In this section, the deformation gradient tensors $F_{RS}^{(i)}$ ($i = I, M, A$) of Section 2.2 was
271 incorporated in a FE simulation using the GPA. The ratio of intima, media and adventitia (18.53%,
272 45.56%, 35.91%) and the layer-specific GOH parameters (shown in Table 6) were taken from the
273 median experimental value for human thoracic aortas in (Weisbecker et al. 2012). Layer-specific
274 material parameter data for ATAA is also available in (Sassani et al. 2015; Sokolis et al. 2012).
275 The geometrical parameters determining the residual deformation of abdominal aorta from
276 (Holzapfel and Ogden 2010), same as Section 2.2 (Table 4), were directly used for the ATAA
277 patient. (Sokolis 2015) documented layer-specific residual stretch and opening angle data for
278 ATAA. Unfortunately, it is not compatible with the current three-dimensional residual stress model

279 (Holzapfel and Ogden 2010). Specifically, (Holzapfel and Ogden 2010) considered different
280 geometries of reference configurations for different layers and would need more complicated
281 experimental setups.

282 Regardless of the discrepancy of the stress in the thickness direction (Figure 6, first row),
283 the transmural mean stress field predicted by the forward penalty approach and the GPA are, again,
284 almost identical, with a MAPE of 3.98% (Figure 6, second row). Since the details of transmural
285 distribution of hoop stress is not clearly shown in the first row of Figure 6, we use Figure 7 to show
286 hoop stress distributions in a ring predicted by method described in Section 3.1 (forward,
287 membrane), Section 3.2.2 (opening angle $\alpha = 180^\circ$) and Section 3.2.3 (layer-specific 3D residual
288 deformation), respectively.

289 **4 Discussions and Conclusions**

290 One of the biggest obstacles in the field of biomechanical analysis of the aorta is the
291 difficulty in obtaining both the patient-specific material properties and the patient-specific residual
292 deformations from *in vivo* clinical images. This paper offers an appealing prospect that the mean
293 hoop stress (or hoop wall tension) of the aortic wall can be computed without knowing the
294 mechanical properties and the residual deformations of the aortic tissue. Computation of patient-
295 specific mean hoop stress can be greatly simplified by using membrane model and the forward
296 penalty approach, which may be clinically valuable. In some wall strength tests (Ferrara et al. 2016;
297 Pham et al. 2013), the intact wall is tested without separation of each individual layer, which
298 corresponds to the averaged wall strength across the wall thickness, consistent with the membrane
299 assumption. The mean hoop stress may be used together with the experimentally-obtained strength

300 to calculate an approximation of rupture risk such as the rupture potential index (RPI) (Vande
301 Geest et al. 2006).

302 Because of the difference in constituents and thus mechanical properties, the hoop stress
303 distribution may not be uniform in multi-layer models. The iterative approaches such as the GPA,
304 may yield detailed results with through-thickness and layer-specific stress distributions using
305 multilayered thick-walled models. Therefore, it would be natural to combine layer-specific wall
306 stress distribution with available layer-specific wall strength data (Sokolis et al. 2012) for a more
307 detailed rupture/dissection analysis. Nonetheless, residual deformations are shown to be highly
308 patient-specific and axial location-dependent (Sokolis 2015). Elastic properties also exhibit
309 regional (Iliopoulos et al. 2009; Sassani et al. 2015) and intra-patient (Martin et al. 2015) variations.
310 Thus, such complex patient- and layer-specific residual deformation and elastic property fields
311 need to be noninvasively estimated for an accurate modeling prediction of clinical events (e.g.
312 rupture). Currently, it is impossible to estimate the layer-specific and heterogeneous material and
313 residual deformation parameters simultaneously from *in vivo* clinical images. We admit that hoop
314 stress within each layer may be more useful than mean hoop stress for predicting some clinical
315 adverse events such as aortic dissection. However, the mean hoop stress is clinically valuable too
316 because it is patient-specific, which does not depend on material parameters and residual
317 deformations.

318 The inclusion of residual deformation often reduces the hoop stress gradient, and thus tends
319 to homogenize the hoop stress distribution in the *in vivo* deformed configuration (Chaudhry et al.
320 1997; Chuong and Fung 1986; Fung 1991; Holzapfel et al. 2000; Humphrey 2002; Raghavan et al.
321 2004). This makes the thin-walled hoop stress, or the mean hoop stress more physiologically
322 relevant in the sense that it represents the ideal homogenized wall stress in single layer models.

323 Homogenized stress state is an assumption for some growth models, e.g., (Polzer et al. 2013), and
324 the method proposed in (Schröder and Brinkhues 2014) is based on smoothing the stress gradient.
325 In this study, the incorporation of opening angles also tends to homogenize the hoop stress
326 distribution as shown in Figure 1. In Figure 3, the MSTPE is close to 0 when 120~180 degree
327 opening angle is incorporated. However, this value seems to be lower than the average value
328 obtained from experiment (Sokolis 2015). This might be due to the assumption of uniform material
329 properties and uniform thickness in the computational model, which could impact the transmural
330 stress distribution. We also notice that a wide range of opening angle is documented in (Sokolis
331 2015), 120~180 degree opening angle can be considered feasible.

332 The transmural mean axial/longitudinal stress of the aorta may be statically determinant
333 when the longitudinal force is known. The ascending aorta also has *in vivo* longitudinal
334 deformations/stretching due to the heart movements during cardiac cycles. Such boundary condition
335 is very complex and it can be difficult to model in a FE simulation. In the present study, a simplified
336 boundary condition was used: the boundary nodes were only allowed to move in the radial
337 directions. We have tried different boundary conditions such as prescribing the longitudinal forces,
338 but encountered convergence problems in the FE simulations. The *in vivo* longitudinal boundary
339 conditions would significantly impact the longitudinal stress field, which warrants further studies
340 in the future.

341 In conclusion, due to static determinacy, the transmural mean hoop stress in the *in vivo*
342 configuration of the aorta is independent of mechanical properties and residual deformations. The
343 forward penalty method, which enforces a rigid condition as the penalty treatment, can greatly
344 simplify the computation of the mean hoop stress for patient-specific geometries.

345 **5 Acknowledgements**

346 This study is supported in part by NIH grants HL104080 and HL127570. Liang is
347 supported by an American Heart Association postdoctoral fellowship 16POST30210003.

348 **6 Conflict of Interest**

349 The authors declare that they have no conflict of interest.

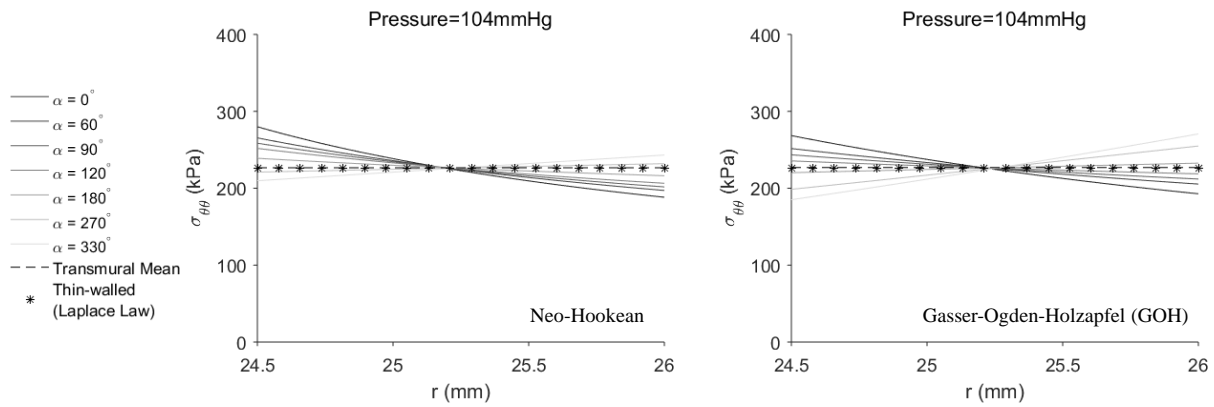
350 **7 References**

- 351 Abaqus (2014) Abaqus 6.14 Documentation.
- 352 Alastrué V, Garía A, Peña E, Rodríguez JF, Martínez MA, Doblaré M (2010) Numerical framework for
353 patient-specific computational modelling of vascular tissue International Journal for Numerical
354 Methods in Biomedical Engineering 26:35-51 doi:10.1002/cnm.1234
- 355 Chaudhry HR, Bukiet B, Davis A, Ritter AB, Findley T (1997) Residual stresses in oscillating thoracic
356 arteries reduce circumferential stresses and stress gradients Journal of Biomechanics 30:57-62
357 doi:[http://dx.doi.org/10.1016/S0021-9290\(97\)81292-4](http://dx.doi.org/10.1016/S0021-9290(97)81292-4)
- 358 Chuong CJ, Fung YC (1986) Residual Stress in Arteries. In: Schmid-Schönbein GW, Woo SLY, Zweifach BW
359 (eds) Frontiers in Biomechanics. Springer New York, New York, NY, pp 117-129.
360 doi:10.1007/978-1-4612-4866-8_9
- 361 Delfino A, Stergiopoulos N, Moore JE, Meister JJ (1997) Residual strain effects on the stress field in a thick
362 wall finite element model of the human carotid bifurcation Journal of Biomechanics 30:777-786
363 doi:[http://dx.doi.org/10.1016/S0021-9290\(97\)00025-0](http://dx.doi.org/10.1016/S0021-9290(97)00025-0)
- 364 Ferrara A, Morganti S, Totaro P, Mazzola A, Auricchio F (2016) Human dilated ascending aorta:
365 Mechanical characterization via uniaxial tensile tests Journal of the Mechanical Behavior of
366 Biomedical Materials 53:257-271 doi:<http://dx.doi.org/10.1016/j.jmbbm.2015.08.021>
- 367 Fung YC (1991) What are the residual stresses doing in our blood vessels? Annals of Biomedical
368 Engineering 19:237-249 doi:10.1007/bf02584301
- 369 Gasser TC, Gallinetti S, Xing X, Forsell C, Swedenborg J, Roy J (2012) Spatial orientation of collagen fibers
370 in the abdominal aortic aneurysm's wall and its relation to wall mechanics Acta Biomaterialia
371 8:3091-3103 doi:<http://dx.doi.org/10.1016/j.actbio.2012.04.044>
- 372 Gasser TC, Ogden RW, Holzapfel GA (2006) Hyperelastic modelling of arterial layers with distributed
373 collagen fibre orientations Journal of The Royal Society Interface 3:15-35
374 doi:10.1098/rsif.2005.0073
- 375 Guo X, Kassab GS (2003) Variation of mechanical properties along the length of the aorta in C57bl/6
376 mice American Journal of Physiology - Heart and Circulatory Physiology 285:H2614-H2622
377 doi:10.1152/ajpheart.00567.2003
- 378 Holzapfel GA, Gasser TC, Ogden RW (2000) A New Constitutive Framework for Arterial Wall Mechanics
379 and a Comparative Study of Material Models Journal of elasticity and the physical science of
380 solids 61:1-48 doi:10.1023/a:1010835316564
- 381 Holzapfel GA, Ogden RW (2010) Modelling the layer-specific three-dimensional residual stresses in
382 arteries, with an application to the human aorta Journal of The Royal Society Interface 7:787-
383 799 doi:10.1098/rsif.2009.0357

- 384 Holzapfel GA, Sommer G, Auer M, Regitnig P, Ogden RW (2007) Layer-Specific 3D Residual Deformations
385 of Human Aortas with Non-Atherosclerotic Intimal Thickening *Annals of Biomedical Engineering*
386 35:530-545 doi:10.1007/s10439-006-9252-z
- 387 Horný L, Netušil M, Voňavková T (2014) Axial prestretch and circumferential distensibility in
388 biomechanics of abdominal aorta *Biomechanics and Modeling in Mechanobiology* 13:783-799
389 doi:10.1007/s10237-013-0534-8
- 390 Humphrey JD (2002) *Cardiovascular solid mechanics. Cells, tissues, and organs* doi:10.1016/S0021-
391 9290(03)00032-0
- 392 Iliopoulos DC et al. (2009) Regional and directional variations in the mechanical properties of ascending
393 thoracic aortic aneurysms *Medical Engineering & Physics* 31:1-9
394 doi:<http://dx.doi.org/10.1016/j.medengphy.2008.03.002>
- 395 Joldes GR, Miller K, Wittek A, Doyle B (2016) A simple, effective and clinically applicable method to
396 compute abdominal aortic aneurysm wall stress *Journal of the Mechanical Behavior of*
397 *Biomedical Materials* 58:139-148 doi:<https://doi.org/10.1016/j.jmbbm.2015.07.029>
- 398 Liang L, Liu M, Martin C, Elefteriades JA, Sun W (2017) A machine learning approach to investigate the
399 relationship between shape features and numerically predicted risk of ascending aortic
400 aneurysm *Biomechanics and Modeling in Mechanobiology* doi:10.1007/s10237-017-0903-9
- 401 Liu M, Liang L, Sun W (2017) A new inverse method for estimation of in vivo mechanical properties of
402 the aortic wall *Journal of the Mechanical Behavior of Biomedical Materials* 72:148-158
403 doi:<http://dx.doi.org/10.1016/j.jmbbm.2017.05.001>
- 404 Lu J, Luo Y (2016) Solving membrane stress on deformed configuration using inverse elastostatic and
405 forward penalty methods *Computer Methods in Applied Mechanics and Engineering* 308:134-
406 150 doi:<http://dx.doi.org/10.1016/j.cma.2016.05.017>
- 407 Martin C, Sun W, Elefteriades J (2015) Patient-specific finite element analysis of ascending aorta
408 aneurysms *American Journal of Physiology - Heart and Circulatory Physiology* 308:H1306-H1316
409 doi:10.1152/ajpheart.00908.2014
- 410 Matsumoto T, Hayashi K (1996) Stress and Strain Distribution in Hypertensive and Normotensive Rat
411 Aorta Considering Residual Strain *Journal of Biomechanical Engineering* 118:62-73
412 doi:10.1115/1.2795947
- 413 Miller K, Lu J (2013) On the prospect of patient-specific biomechanics without patient-specific properties
414 of tissues *Journal of the Mechanical Behavior of Biomedical Materials* 27:154-166
415 doi:<http://dx.doi.org/10.1016/j.jmbbm.2013.01.013>
- 416 Pham T, Martin C, Elefteriades J, Sun W (2013) Biomechanical characterization of ascending aortic
417 aneurysm with concomitant bicuspid aortic valve and bovine aortic arch *Acta Biomaterialia*
418 9:7927-7936 doi:<http://dx.doi.org/10.1016/j.actbio.2013.04.021>
- 419 Pierce DM et al. (2015) A method for incorporating three-dimensional residual stretches/stresses into
420 patient-specific finite element simulations of arteries *Journal of the Mechanical Behavior of*
421 *Biomedical Materials* 47:147-164 doi:<http://dx.doi.org/10.1016/j.jmbbm.2015.03.024>
- 422 Polzer S, Bursa J, Gasser TC, Staffa R, Vlachovsky R (2013) A Numerical Implementation to Predict
423 Residual Strains from the Homogeneous Stress Hypothesis with Application to Abdominal Aortic
424 Aneurysms *Annals of Biomedical Engineering* 41:1516-1527 doi:10.1007/s10439-013-0749-y
- 425 Raghavan ML, Trivedi S, Nagaraj A, McPherson DD, Chandran KB (2004) Three-Dimensional Finite
426 Element Analysis of Residual Stress in Arteries *Annals of Biomedical Engineering* 32:257-263
427 doi:10.1023/b:abme.0000012745.05794.32
- 428 Sassani SG, Tsangaris S, Sokolis DP (2015) Layer- and region-specific material characterization of
429 ascending thoracic aortic aneurysms by microstructure-based models *Journal of Biomechanics*
430 48:3757-3765 doi:10.1016/j.jbiomech.2015.08.028

- 431 Schröder J, Brinkhues S (2014) A novel scheme for the approximation of residual stresses in arterial walls
432 Archive of Applied Mechanics 84:881-898 doi:10.1007/s00419-014-0838-x
- 433 Sokolis DP (2015) Effects of aneurysm on the directional, regional, and layer distribution of residual
434 strains in ascending thoracic aorta Journal of the Mechanical Behavior of Biomedical Materials
435 46:229-243 doi:<http://dx.doi.org/10.1016/j.jmbbm.2015.01.024>
- 436 Sokolis DP, Kritharis EP, Iliopoulos DC (2012) Effect of layer heterogeneity on the biomechanical
437 properties of ascending thoracic aortic aneurysms Medical & Biological Engineering &
438 Computing 50:1227-1237 doi:10.1007/s11517-012-0949-x
- 439 Vaishnav RN, Vossoughi J (1983) Residual stress and strain in aortic segments Journal of Biomechanics
440 20:235-237 doi:10.1016/0021-9290(87)90290-9
- 441 Vande Geest JP, Di Martino ES, Bohra A, Makaroun MS, Vorp DA (2006) A Biomechanics-Based Rupture
442 Potential Index for Abdominal Aortic Aneurysm Risk Assessment Annals of the New York
443 Academy of Sciences 1085:11-21 doi:10.1196/annals.1383.046
- 444 Weisbecker H, Pierce DM, Holzapfel GA (2014) A generalized prestressing algorithm for finite element
445 simulations of preloaded geometries with application to the aorta International Journal for
446 Numerical Methods in Biomedical Engineering 30:857-872 doi:10.1002/cnm.2632
- 447 Weisbecker H, Pierce DM, Regitnig P, Holzapfel GA (2012) Layer-specific damage experiments and
448 modeling of human thoracic and abdominal aortas with non-atherosclerotic intimal thickening
449 Journal of the Mechanical Behavior of Biomedical Materials 12:93-106
450 doi:<http://dx.doi.org/10.1016/j.jmbbm.2012.03.012>
- 451
- 452

453



454

455 Figure 1 the transmural mean, thin-walled and thick-walled hoop stresses across the wall thickness.

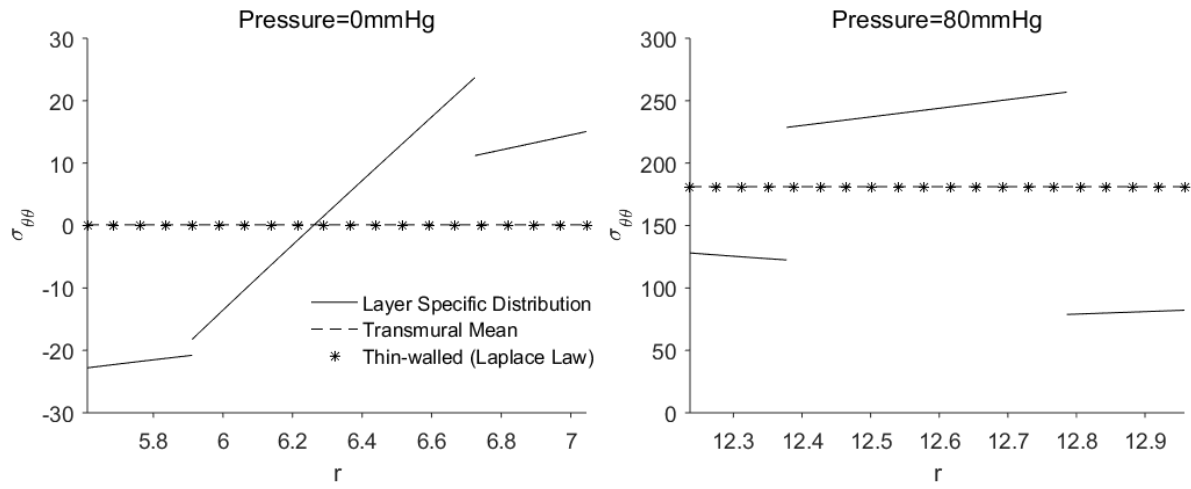
456 In the left figure, thick-walled hoop stresses were computed using Neo-Hookean model, while in

457 the right figure, GOH model was used. Transmural mean hoop stress remains the same for all

458 scenarios, thus only one line is plotted.

459

460

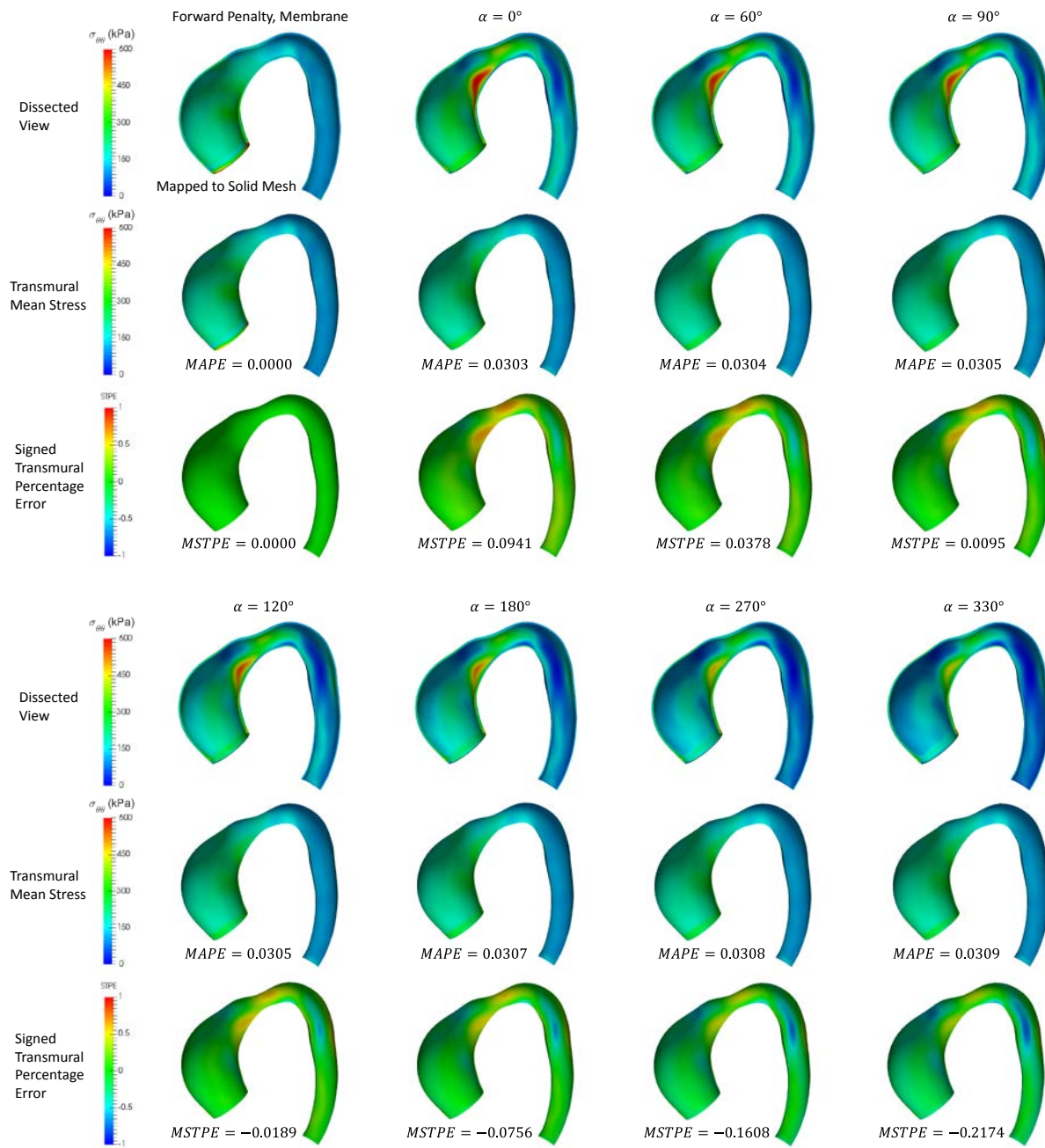


461

462 Figure 2 the transmural mean, thin-walled and layer-specific hoop stress distributions in the three
463 layer composite wall when 0 and 80 mmHg pressures are applied.

464

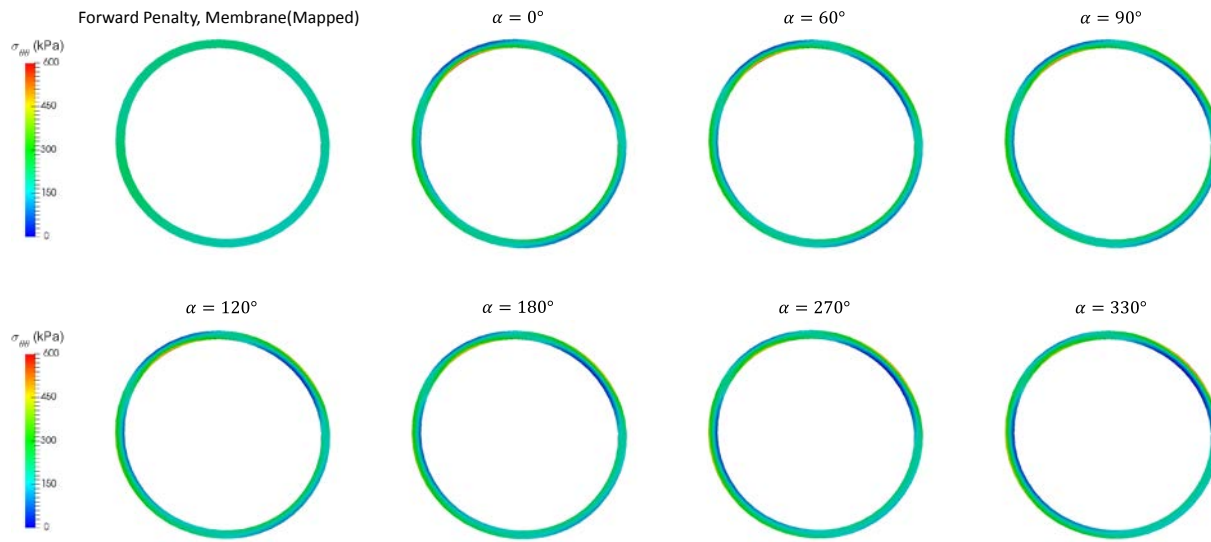
465



466

467 Figure 3 predicted results using the forward penalty approach and the GPA approach with different
468 opening angles: (1) the hoop stress distribution in the dissected view (row 1 and row 4), (2) the
469 transmural mean hoop stress (row 2 and row 5), and (3) the signed transmural percentage error
470 (row 3 and row 6).

471



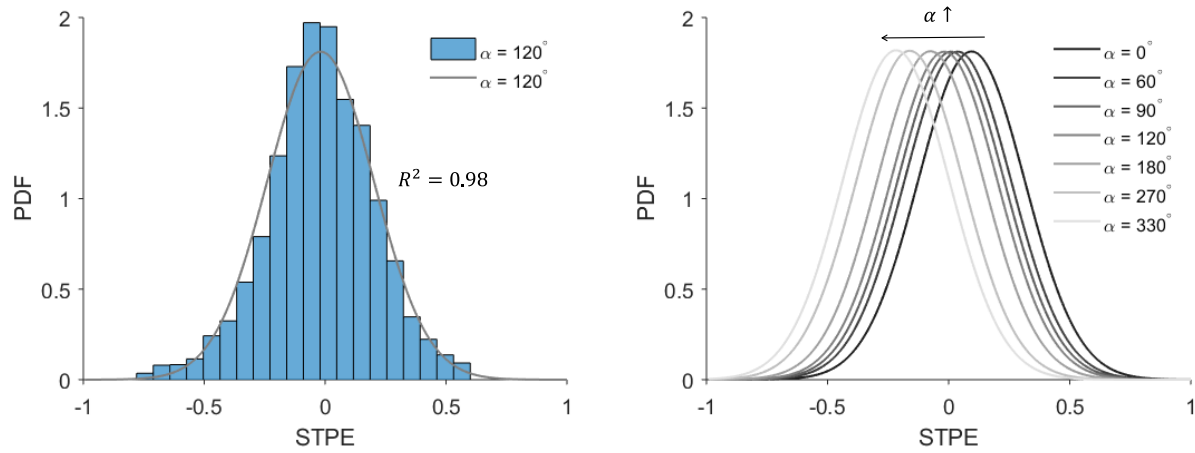
472

473 Figure 4 hoop stress in aortic rings using the forward penalty approach and the iterative approach

474 (GPA) with different opening angles.

475

476



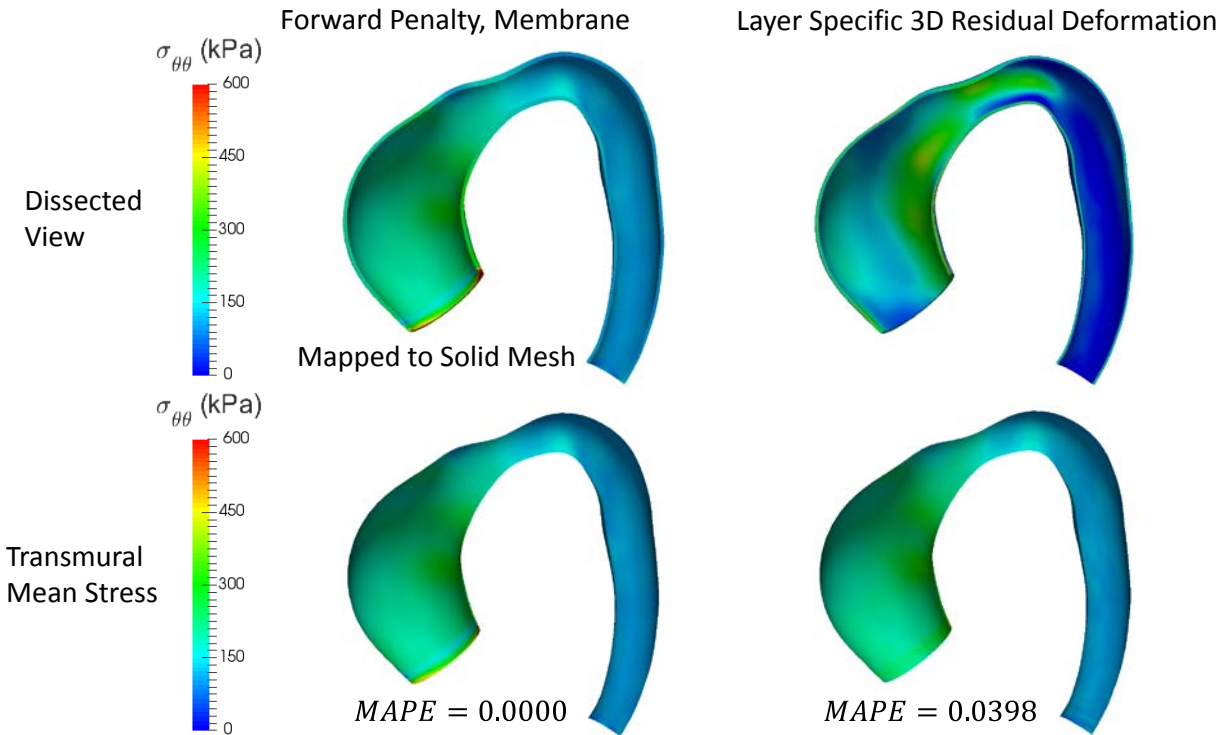
478

479 Figure 5 the probability density function (PDF) of the STPE is shown in the histogram and fitted

480 using the Gaussian distribution (left) and fitted PDFs correspond to different opening angles (right).

481

481

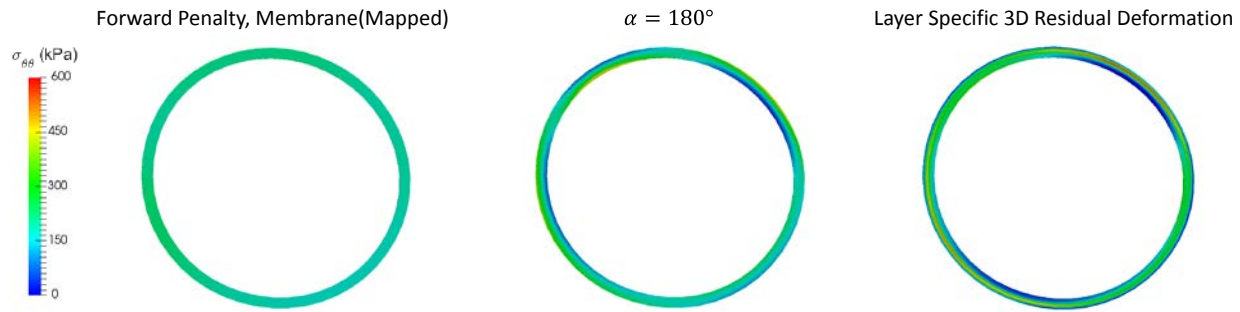


482

483 Figure 6 predicted results using the forward penalty approach and the iterative approach (GPA)
484 with layer-specific three-dimensional residual deformations: (1) the hoop stress distribution in the
485 dissected view (row 1), (2) the transmural mean hoop stress (row 2).

486

487



488

489 Figure 7 the hoop stress distribution in the aortic rings using the forward approach, the opening

490 angle method ($\alpha = 180^\circ$) and the layer-specific 3D residual deformation.

491

492

inner radius a (mm)	outer radius b (mm)	residual axial stretch l/L	systolic pressure (mmHg)	shear modulus μ (kPa)
24.5 ^a	26 ^b	1.2 ^c	104 ^a	67.68 ^a

493 ^a from (Martin et al. 2015), a and systolic pressure are from clinical recorded data, μ was fitted
494 using biaxial experiment of patient “BAV17” with coefficient of determination of 0.8656; ^b based
495 on mean value of deformed wall thickness in (Liang et al. 2017); ^c approximated from (Guo and
496 Kassab 2003) which refers to the residual axial stretch. We assume there is no axial tension caused
497 by *in vivo* loading conditions

498 Table 1 the parameters used in the opening angle method.

499

Neo-Hookean Model							
$\alpha(^{\circ})$	0	60	90	120	180	270	330
$A(mm)$	11.55	14.20	15.96	18.16	24.77	51.23	157.10
$B(mm)$	14.98	17.62	19.39	21.59	28.20	54.67	160.54
Gasser-Ogden-Holzappel (GOH) Model							
$\alpha(^{\circ})$	0	60	90	120	180	270	330
$A(mm)$	18.25	22.12	24.70	27.93	37.63	76.43	231.66
$B(mm)$	20.59	24.46	27.04	30.26	39.97	78.77	234.00

500 Table 2 the inner and outer radii A and B of the stress-free configurations corresponding to
501 various opening angles α .

502

$C_{10}(kPa)$	$k_1(kPa)$	k_2	κ	$\theta(^{\circ})$
27.91	512.56	0.00	0.31	90.00

503 Table 3 GOH material parameters of the patient “BAV17” extracted from (Martin et al. 2015).

504 Coefficient of determination of the curve fitting is 0.9551.

505

Intima	Media	Adventitia
$A^{(I)} = 7.50mm$	$A^{(M)} = 8.41mm$	$L_1^{(A)} = 0.21mm$
$B^{(I)} = 7.76mm$	$B^{(M)} = 8.99mm$	$L_2^{(A)} = 18.35mm$
$L^{(I)} = 2.58mm$	$L^{(M)} = 2.52mm$	$L_3^{(A)} = 2.29mm$
$k^{(I)} = 1.19$	$k^{(M)} = 2.79$	$b^{(A)} = 7.05mm$
$a^{(I)} = 5.61mm$	$\mu^{(M)} = 31.4kPa$	$\mu^{(A)} = 17.3kPa$
$\mu^{(I)} = 39.8kPa$		

506 Table 4 material and residual deformation parameters from (Holzapfel and Ogden 2010;
507 Holzapfel et al. 2007). In addition, $l = 2.48mm$, $b^{(I)} = a^{(M)}$ and $b^{(M)} = a^{(A)}$ can be calculated
508 according to (Holzapfel and Ogden 2010).

509

Wall thickness (mm)	1.0	2.0	3.0
MAPE	0.0191	0.0355	0.0458

510 Table 5 sensitivity of MAPE w.r.t. the thickness.

511

512

	C_{10} (kPa)	k_1 (kPa)	k_2	κ	θ (°)
Intima	17	4340	13.32	0.20	46.5
Media	14	140	11.90	0.21	38.4
Adventitia	10	390	6.79	0.23	52.3

513 Table 6 layer-specific GOH material parameters from (Weisbecker et al. 2012).

514

515

516

517

518

519

520

521

522

523

524