

# A robust nonlinear low-dimensional manifold for single cell RNA-seq data

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## Abstract

Modern developments in single cell sequencing technologies enable broad insights into cellular state. Single cell RNA sequencing (scRNA-seq) can be used to explore cell types, states, and developmental trajectories to broaden understanding of cell heterogeneity in tissues and organs. Analysis of these sparse, high-dimensional experimental results requires dimension reduction. Several methods have been developed to estimate low-dimensional embeddings for filtered and normalized single cell data. However, methods have yet to be developed for unfiltered and unnormalized count data. We present a nonlinear latent variable model with robust, heavy-tailed error and adaptive kernel learning to estimate low-dimensional nonlinear structure in scRNA-seq data. Gene expression in a single cell is modeled as a noisy draw from a Gaussian process in high dimensions from low-dimensional latent positions. This model is called the Gaussian process latent variable model (GPLVM). We model residual errors with a heavy-tailed Student's t-distribution to estimate a manifold that is robust to technical and biological noise. We compare our approach to common dimension reduction tools to highlight our model's ability to enable important downstream tasks, including clustering and inferring cell developmental trajectories, on available experimental data. We show that our robust nonlinear manifold is well suited for raw, unfiltered gene counts from high throughput sequencing technologies for visualization and exploration of cell states.

**Keywords:** Manifold Learning, Gaussian Process Latent Variable Model, cell types, Single Cell RNA-Sequencing (scRNA-seq), Robust Statistics

## 1. Background

High-throughput single cell RNA sequencing (scRNA-seq) is a powerful tool for cataloguing cell types and cell states, and investigating changes in expression over cell developmental trajectories. Droplet-based methods encapsulate individual cells with unique barcode tags that are ligated to cellular RNA fragments (Zheng et al., 2017). Sequenced reads are mapped to both a gene and a cell, creating a high-dimensional cell by gene count matrix–

with hundreds to millions of cells and twenty thousand genes per human cell. These cell by gene count matrices contain a substantial proportion of zeros because of low coverage sequencing per cell (i.e., *dropout*), and also contain substantial variance from both technical and biological sources of noise (Buettner et al.). Computational tools for analyzing scRNA-seq results thus require initial dimension reduction to a lower dimensional manifold capturing gene expression patterns for regularization and computational efficiency. Dimension reduction techniques are conducive to noise reduction (Buettner et al.; Eraslan et al., 2018), sub-population identification (Pierson and Yau; Haghverdi et al., 2016), visualization (Amodio et al.; Van Der Maaten and Hinton), pseudotemporal ordering of development stages (Ahmed et al., 2018; Trapnell et al.; Lönnberg et al., 2017), and imputation (Li and Li, 2018). Lower-dimensional mappings also provide convenient visualizations that inform analytic methods and future experiments.

Linear dimension reduction techniques are commonly used as a first step to downstream analyses. Principal component analysis (Hotelling, 1933) (PCA) – the projection of a high dimensional space onto orthogonal bases that capture the directions of greatest variance – is the first step of several scRNA-seq analysis packages such as PAGODA (Fan et al., 2016) and Waterfall (Shin et al.). Zero-Inflated Factor Analysis (Pierson and Yau) (ZIFA) extends the factor analysis (Harman, 1960) paradigm of a linear mapping onto low dimensional latent dimensions to allow dropouts modeled by Bernoulli random variables to account for the excess of zero counts in scRNA-seq data. Independent component analysis (Comon, 1994) (ICA), which assumes non-Gaussian observations, and canonical correlation analysis (Hotelling, 1936) (CCA), which allows for multiple observations, have also been used as dimension reduction techniques for studying cell developmental trajectories (Trapnell et al.) and for experimental batch correction (Butler et al.).

More sophisticated models eschew the linearity assumption to find richer nonlinear structure in the data. The *t*-distributed Stochastic Neighbors Embedding (*t*-SNE) (Van Der Maaten and Hinton) is a popular visualization tool. *t*-SNE computes the similarity between two points in high dimensional space with respect to a Gaussian kernel distance metric, and estimates a lower dimensional mapping with similarity with respect to a Student's *t*-distribution metric that minimizes the Kullback-Leibler divergence between the similarity distributions in high and low dimensions. The Gaussian kernel in *t*-SNE includes a perplexity parameter that controls the decay rate of similarity across the distance between cells. Diffusion maps, used in packages such as Destiny (Angerer et al., 2016), are another tool for nonlinear low dimensional mapping that perform linear decomposition on a kernel similarity matrix of high dimensional observations. SAUCIE (Amodio et al.) implements a variational autoencoder, or a deep neural network that compresses data with the goal of creating an optimal reconstruction from the compressed representation, to execute several single cell tasks. Similarly, scVI (Lopez et al.) uses deep neural networks to create a probabilistic representation in latent space for batch correction, visualization, clustering, and differential expression. One Bayesian probabilistic technique is the Gaussian process latent variable model (Titsias and Lawrence; Lawrence) (GPLVM), which used by scLVM (Buettner et al.) for noise reduction, and GPfates (Lönnberg et al., 2017) and GrandPrix (Ahmed et al., 2018) for pseudotemporal ordering. The GPLVM models observations (i.e., cells) as draws from Gaussian processes representations of lower dimensional latent variables.

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While current methods for dimension reduction have been successful with early sequencing experiments and filtered expression data, they are limited in their capacity to accurately represent and inform analyses of raw, high throughput sequencing experiments. Linear methods such as PCA and ZIFA are ill-suited for capturing highly nonlinear biological processes across developmental phase, and many implementations scale poorly with increased sample size. Current non-linear methods are highly sensitive parameter choices, including perplexity for t-SNE, kernel variables for diffusion maps, and network architecture for VAEs. Latent dimensions of t-SNE have no global structure, making embedded positions difficult to interpret and leading to uninformative mappings beyond two dimensions. Downstream analyses of t-SNE results are hindered by an inability to map back to observation space. VAEs, like most neural networks, require tens to hundreds of thousands of cells for accurate estimation, which may not be available in smaller experiments. Current methods, particularly those using the GPLVM, work only with filtered, normalized data and incorporate prior information to facilitate the latent mapping.

Robust statistical methods are a natural solution to modeling noisy, sparse count data. We introduce the t-Distributed Gaussian Process Latent Variable Model (tGPLVM) for learning a low dimensional embedding of unfiltered count data. We introduce three features to the basic GPLVM: 1) a robust Student's t-Distribution noise model; 2) a weighted sum of non-smooth covariance kernel functions with parameters estimate from the data; 3) sparse kernel structure. The heavy tailed Student's t-Distribution improves robustness to outliers, previously demonstrated in Gaussian process regression (Tang et al.; Vanhatalo et al.). Matérn kernels have been successfully used in time series modeling to capture non-smooth trajectories (Ahmed et al., 2018). The sparse kernel structure allows us to effectively reduce the number latent dimensions based on the actual complexity of the data. Our implementation of tGPLVM accepts sparse inputs produced from high-throughput experimental cell by gene count matrices.

We demonstrate tGPLVM's ability to estimate informative manifolds from noisy, raw single cell count matrices and highlight its applicability to multiple downstream tasks. We show improved cell type identification via clustering on the estimated latent space using a data set of cerebral cortex cells labeled with estimated cell type (Pollen et al.). We find that the tGPLVM manifold can learn pseudotemporal ordering from a batch of *Plasmodium*-infected mouse cells sequenced across time post exposure (Lönnberg et al., 2017). Finally, we demonstrate that tGPLVM can be used on unprocessed, unnormalized count data from recent high-throughput sequencing methods (Zheng et al., 2017) and used to explore gene expression across cell states. We implement a scalable inference algorithm that can fit hundreds of thousands to millions of cells.

## Results

The t-distributed Gaussian process latent variable model is a nonlinear latent variable model that captures high dimensional observations in low dimensional nonlinear latent space. Expression of each of  $P$  genes across all cells is modeled as a draw from a multivariate normal distribution with the covariance a function of the low-dimensional, latent positions. The observation error is modeled with a heavy tailed Student's t-Distribution with four degrees of freedom to robustly account for variance in the count data due to technical and

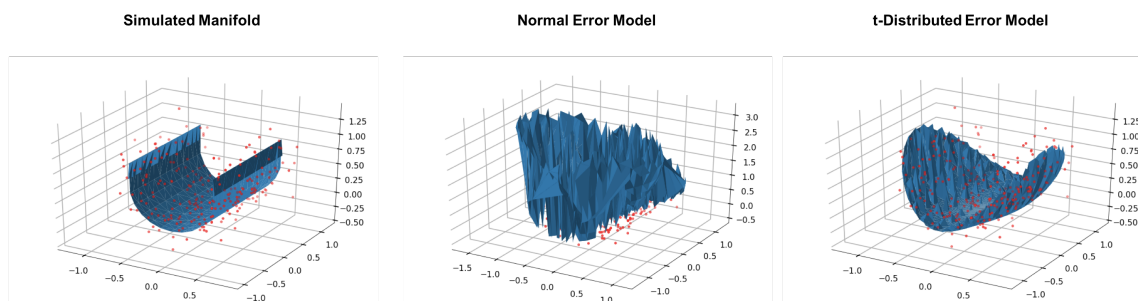


Figure 1: Comparison of error models on the same observations (red) in a multidimensional Gaussian process. Data are simulated from a smooth manifold with  $t$ -distributed error (left). A normally distributed error model (center) overfits to the data and fails to find the manifold structure due to the outliers as compared to the manifold estimated by a GP with  $t$ -distributed error (right).

biological noise relative to a normally distributed error model (O’Hagan, 1979, 1988). Here, we use a weighted sum of Matérn 1/2, Matérn 3/2, Matérn 5/2, and squared exponential kernel functions to model non-smooth manifolds.

### Nonparametric manifold learning improves cell type identification.

First, we evaluated the ability of tGPLVM and commonly used single cell dimension reduction methods to distinguish distinct cell types. tGPLVM, PCA, ZIFA, and t-SNE were used to map cells labeled with their inferred cell type from the Pollen data (Pollen et al.) to latent spaces varying from two to nine dimensions. With more than two latent dimensions, tGPLVM produced clusters that best corresponded to the actual cell type labels of the four methods (Figure 2). Including Matérn kernels in tGPLVM improves cell type separation in the latent space as measured by normalized mutual information and adjusted rand score (Supplementary Figure 1). Inclusion of Matérn kernels also reduces the uncertainty of posterior estimates of the latent embedding as measured by the average scale parameter of the latent position (Supplementary Figure 2). These results suggest that a robust Bayesian nonparametric manifold is superior to current dimension reduction algorithms for identifying and visualizing distinct cell types captured by scRNA-seq experiments.

### Nonparametric manifold learning can reconstruct development time scales without prior information

Next we test the flexibility of tGPLVM to continuous cellular developmental trajectories by fitting latent mappings for a batch of mouse Th1 and Tfh cells sequenced over seven days after infection with *Plasmodium* (Lönnberg et al., 2017). Visually, we find that the latent mapping from tGPLVM represents the temporal relationships accurately, with most cells positioned among cells from the same or adjacent time points. We build a minimum spanning tree on the latent mappings to infer developmental trajectories. For a two dimensional mapping, only tGPLVM accurately spans the first time point (day 0) to the final time point

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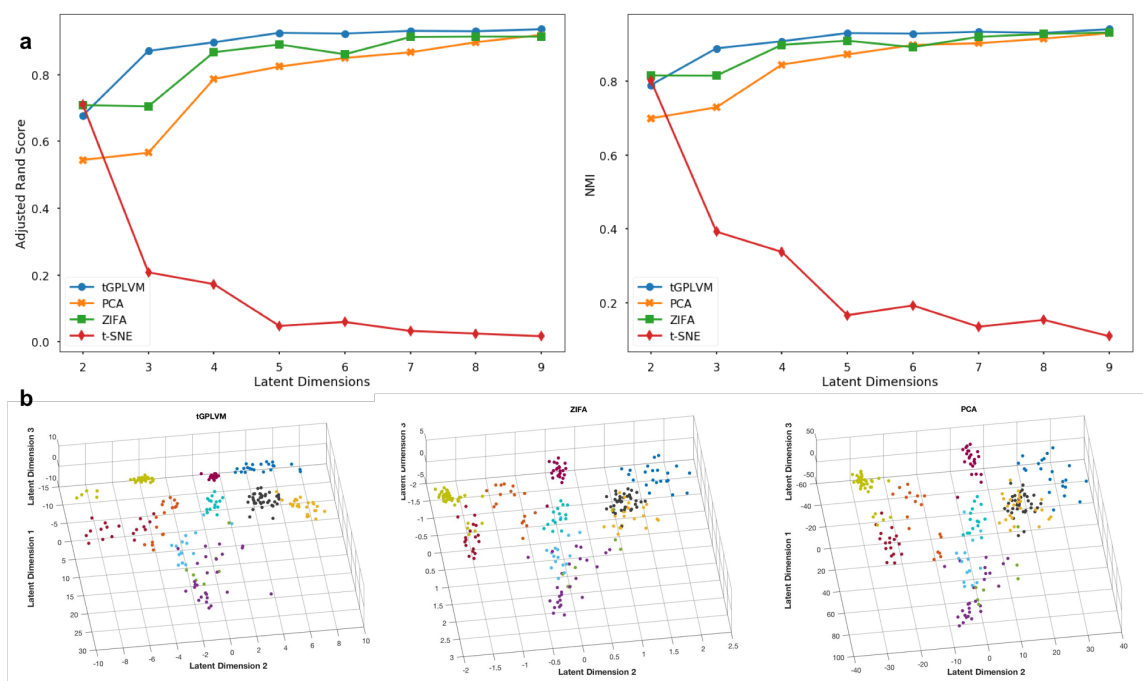


Figure 2: Comparison of manifold learning methods on 11 neural and blood cell populations (Pollen et al.). (a) Average ARS (left) and NMI (right) of ten K-means cluster labels versus available cell type labels with respect to the number of latent dimensions. (b) Three dimensional latent mappings from tGPLVM, PCA, and ZIFA colored by inferred cell type label. t-SNE (not pictured) collapses in three dimensions.

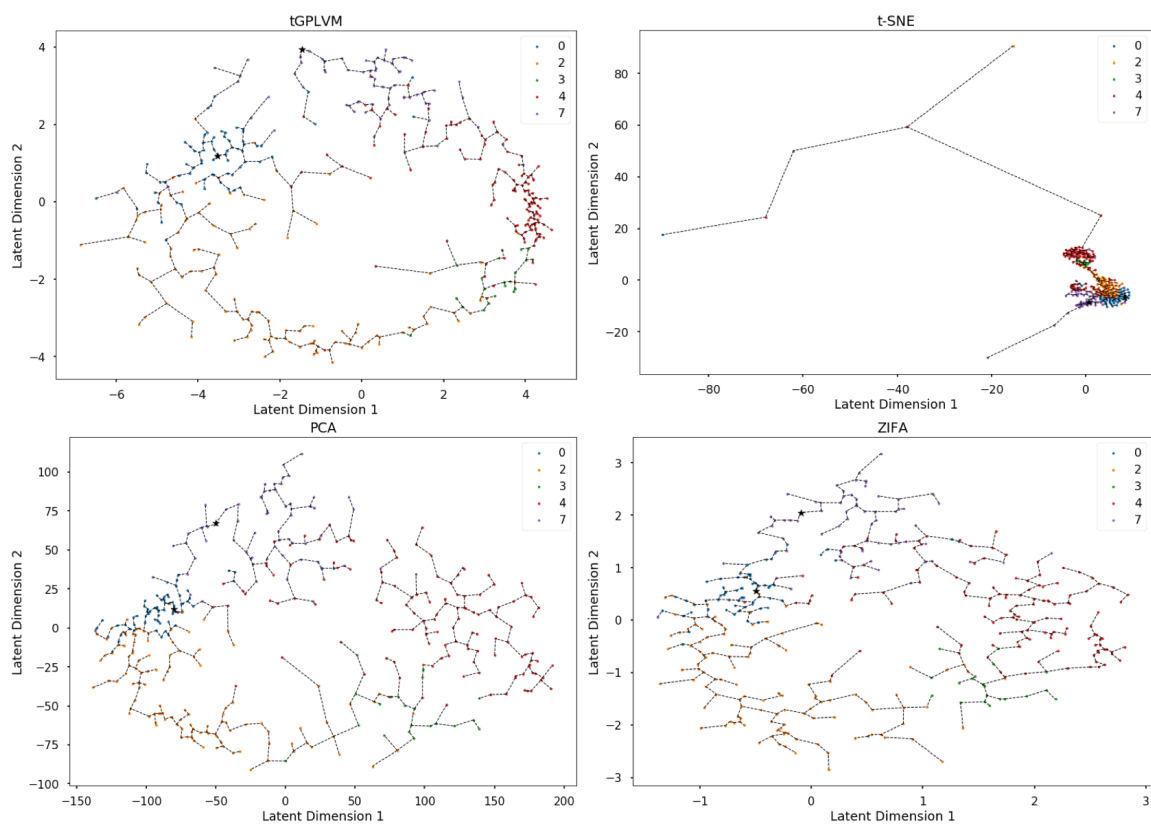


Figure 3: Comparison of manifold learning Methods on *Plasmodium*-infected Th1 and Tfh cells (Lönnerberg et al., 2017): Plot of two dimensional latent mapping from tGPLVM, PCA, ZIFA, and t-SNE. Labels indicate days after infection prior to sequencing. Dotted lines represent connections along the minimum spanning tree.

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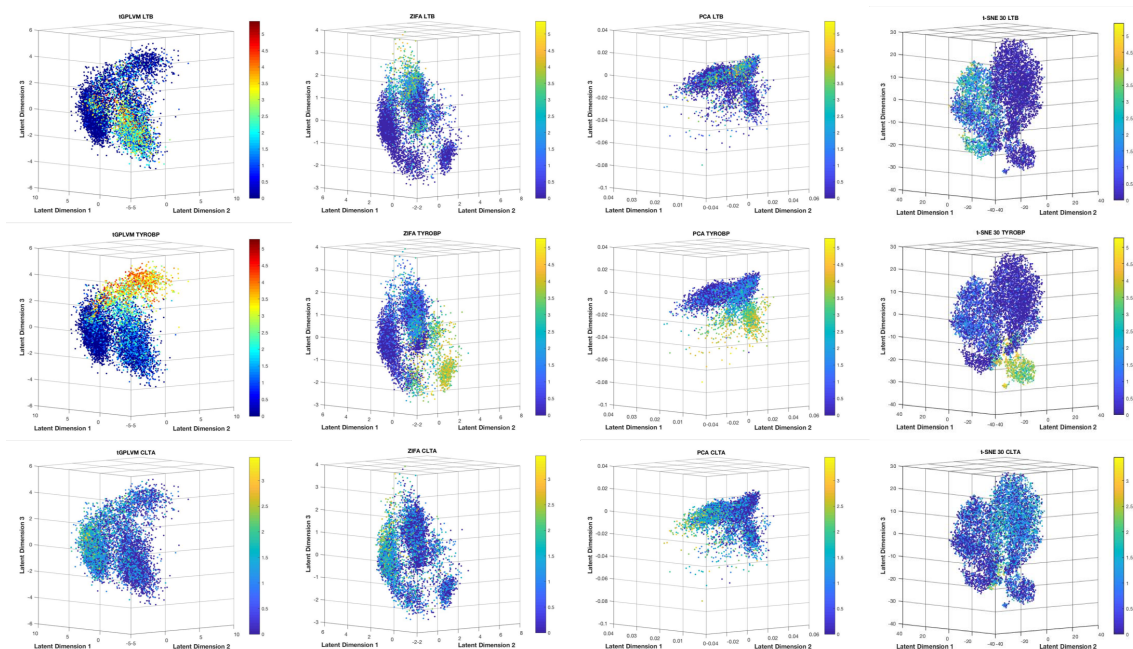


Figure 4: Manifold learning methods on unprocessed CD34+ PBMCs (Zheng et al., 2017) counts. tGPLVM shows the best separation of expression patterns based on cell state marker genes. Color bars indicate  $\log_2(1 + Y)$ , where  $Y$  represents total counts per gene per cell.

(day 7). PCA, ZIFA, and alternate GPLVM models with different error or kernel choices find endpoints of the tree in days 2 or 4. t-SNE is able to separate cells based on time but does not accurately reconstruct the ordering and is clearly sensitive to outliers (Figure 3). This suggests that tGPLVM is a superior dimension reduction technique for identifying developmental pathways in unlabeled settings.

### Nonparametric manifold learning improves visualization of raw count data and captures cell state

Next, we tested tGPLVM's performance on unfiltered count data. Models were fit on  $\sim 10,000$  CD34+ peripheral blood mononuclear cells (PBMCs) sequenced on a high throughput parallel 10x system (Zheng et al., 2017). Each model was able to find three distinct regions based on expression patterns (Figure 4). PCA is dominated by total counts, with cells with more reads moving further away in latent space, and more frequent cell types dominate the space (Engelhardt and Stephens, 2010). CD34 is a marker for hematopoietic stem cells (Sidney et al., 2014), which differentiate into myeloid and lymphoid cells. From tGPLVM, we can observe this separation from different expression patterns in progenitor cells across dimensions. Dimension three correlates with myeloid cells, demonstrated visually by marker *TYROBP* (Tomasello and Vivier, 2005) (Pearson's  $r = 0.647$ ; Figure 4), in addition to correlations with macrophage-associated genes (Donato et al.; Xia et al.,

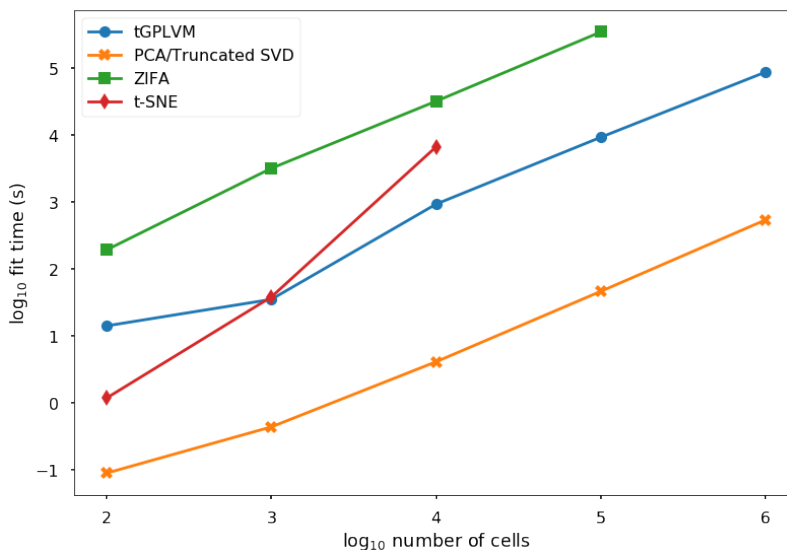


Figure 5: Time to fit a two dimensional embedding vs. sample size on a 16 VCPU, 224 GB memory high performance computing system.

2018) *S100A4* (Pearson’s  $r = 0.623$ ) and *S100A6* (Pearson’s  $r = 0.665$ ). Dimension two correlates to lymphoid cells, visualized by marker *LTB* (Browning et al., 1993) (Pearson’s  $r = 0.306$ ; Figure 4), and further supported by correlation with lymphocyte specific protein-1 *LSP1* (Pearson’s  $r = 0.481$ ). Dimension one corresponds to general cellular functions, with strong correlation with mitochondrial activity genes *COX5A* (Pearson’s  $r = 0.587$ ) and *STOML2* (Pearson’s  $r = 0.474$ ), and shown with *CLTA*, an endocytosis-mediating gene (Stelzer et al., 2016) (Pearson’s  $r = 0.461$ ; Figure 4). These distinct expression patterns reflect the broadly different immune cellular functions into which hematopoietic stem cells may develop. Gradients of expression levels projected on tGPLVM embeddings can be used to further interrogate changes in cell states from different experiments and across these manifolds.

### tGPLVM scales to a million cells

Finally, we evaluate the ability of tGPLVM and related methods to fit embeddings for unfiltered, unnormalized, high throughput scRNA-seq data. Models with two latent dimensions were fit on subsamples from 100 to 1 million cells from the 10x 1 million mouse brain cell data (Zheng et al., 2017). tGPLVM and PCA are the only methods that can fit one million cells in a computationally tractable way (Figure 5). ZIFA is slower than tGPLVM by an order of magnitude consistently across sample sizes. Since ZIFA requires a dense input, its input cell count matrix is limited in our framework to approximately 100,000 cells. While t-SNE’s implementation can input a sparse matrix format, it does not converge beyond  $10^4$  samples.

To check that the embedding has biological significance, we again used Pearson’s correlation to identify genes whose expression is correlated with latent dimensions. We find



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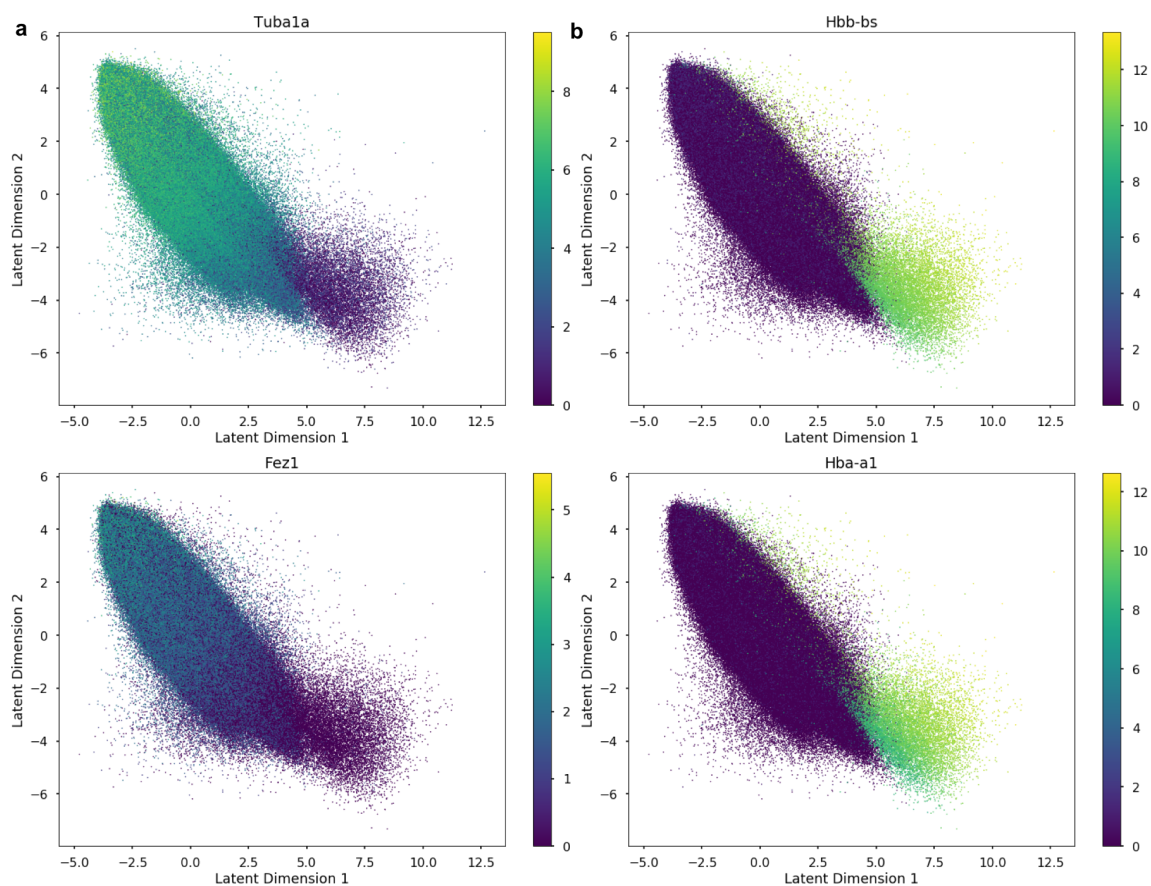


Figure 6: Gene expression patterns across tGPLVM manifold in 1 million mouse brain cells (Zheng et al., 2017). Latent dimension one separates circulatory and blood genes. Latent dimension two is correlated with neural genes. Color bars indicate  $\log_2(1 + Y)$ , where  $Y$  represents total counts per gene per cell.

that latent dimension one corresponds to increased expression of genes associated with the circulatory system and hemoglobin, such as *HBB-BS* (Pearson's  $r = 0.320$ ) and *HBA-A1* (Pearson's  $r = 0.316$ ; Figure 6b). Dimension two correlates with genes such as *TUBA1A* (Pearson's  $r = 0.474$ ) and *FEZ1* (Pearson's  $r = 0.427$ ) that are associated with neural cells (Figure 6a). The ability of tGPLVM to scale to high-throughput data and capture global structure from unnormalized count matrices makes it a powerful method for analyzing future single cell experiments.

## Discussion

We present a Bayesian nonparametric model for robust nonlinear manifold estimation in scRNA-seq settings. tGPLVM captures transcriptional signals in single cell data using a robust Student's  $t$ -distribution noise model and integrating adaptive kernel structure in settings with no *a priori* information about clusters or sequencing order. Our results show that tGPLVM is flexible to cell type, cell development, and cell perturbation experiments and can learn informative mappings from filtered and processed data as well as unfiltered raw count data. tGPLVM scales to the size of a million cells as produced by the latest single cell sequencing systems. Despite the sparsity, these data are complex and require several factors to capture variation; we did not use the ARD kernel parameters to remove dimensions for any of our experiments. However, the embedding dimensions in our experiments were able to capture informative representations of these complex data, and as the number of latent dimensions increased some would eventually be removed for being redundant in capturing these complex transcriptional profiles. We expect that the estimated latent mappings can be used for more sophisticated, nonparametric approaches for a variety of single cell tasks from normalization and imputation to cell type identification. We also hope that this robust manifold estimation can be used for other types of data with noisy outliers and sparse features.

1 **Methods**

2 **The t-Distribution Gaussian process latent variable model.**

3 The tGPLVM assumes that samples in high dimensional space are noisy observations of  
 4 a Gaussian process of lower dimensional latent features. Let  $Y \in \mathbb{R}^{N \times P}$  represent  $N$   
 5 observations in a high dimensional space of dimension  $P$ , and let  $X \in \mathbb{R}^{N \times Q}$  represent the  
 6 same observations in a lower dimensional space  $Q \ll P$ . Each sample  $x_n$  in  $\{n \in 1, 2, \dots, N\}$   
 7 is assumed to be drawn from a  $Q$  dimensional multivariate normal distribution with identity  
 8 variance:

$$x_i \sim \mathcal{N}_Q(0, I_Q).$$

9 Noiseless observations of each of the  $P$  high dimensional features across  $N$  samples,  $f_p(X)$ ,  
 10 are draws from a zero-mean Gaussian process of  $x$  across a weighted sum of  $M$  kernels:

$$\begin{aligned} f_p(X) &\sim \mathcal{N}_n(0, K_{NN}) \\ k(x, x') &= \sum_{m=1}^M k_m(x, x'), \end{aligned}$$

11 where  $K_{NN}$  represents the  $N \times N$  covariance matrix defined by  $k(x, x')$ . In the traditional  
 12 GPLVM, observations  $y_{n,p}$  are noisy realization of a Normal distribution with mean  $f_{n,p}$   
 13 and variance  $\tau^2$ :

$$y_{n,p} | f_{n,p}(X), \tau^2 \sim \mathcal{N}(f_{n,p}, \tau^2)$$

14 For tGPLVM, each observation  $y_{n,p}$  is drawn from a heavy-tailed Student's t-distribution  
 15 with a set degrees of freedom  $\nu$  and feature-specific variance  $\tau_p^2$  (Tang et al.):

$$\begin{aligned} y_{n,p} | f_{n,p}(X), \tau_p^2, \nu &\sim \text{StudentT}(f_{n,p}, \nu, \tau_p^2) \\ &= \frac{\Gamma((\nu + 1)/2)}{\Gamma(\nu/2)\sqrt{\nu\pi}\tau_p} \left(1 + \frac{(y_{n,p} - f_{n,p})^2}{\nu\tau_p^2}\right)^{-(\nu+1)/2}, \end{aligned}$$

16 where we use  $f_{n,p}$  to represent the  $n$ th component of the  $N$  dimensional vector  $f_p(X)$ . We  
 17 set  $\nu = 4$  based on previous work with the supervised Gaussian process with t-distributed  
 18 error (Tang et al.; Vanhatalo et al.).

19 The kernel we use is a flexible sum of an automatic relevance determination (ARD)  
 20 squared exponential kernel and three different Matérn ARD kernels each with hyperparam-  
 21 eters scale  $\sigma_k$  and length scales  $\ell_{k,q}$ . Each ARD dimension-specific length scale,  $l_{k,q}$  indicates  
 22 the distance of that latent dimension over which points are similar. Letting  $r$  represent the  
 23 length scale-weighted distance in latent space, the kernels are defined as:

$$\begin{aligned} r &= \sum_{q=1}^Q \frac{(x_q - x'_q)}{\ell_{m,q}} \\ k_1(x, x') &= k_{SE}(x, x') = \sigma_1^2 \exp\left\{-\frac{1}{2}r^2\right\} \end{aligned}$$

$$\begin{aligned}k_2(x, x') &= k_{Mat1/2}(x, x') = \sigma_2^2 \exp\{-r\} \\k_3(x, x') &= k_{Mat3/2}(x, x') = \sigma_3^2(1 + \sqrt{3}r) \exp\{-\sqrt{3}r\} \\k_4(x, x') &= k_{Mat5/2}(x, x') = \sigma_4^2(1 + \sqrt{3}r + \frac{5}{3}r^2) \exp\{-\sqrt{5}r\}.\end{aligned}$$

24 We use Black Box Variational Inference (Ranganath et al.) to estimate the posterior dis-  
25 tribution for tGPLVM. We adapt the variational distributions from prior work (Damianou  
26 et al., 2016). Inference is implemented in Python using Edward (Tran et al., 2016, 2017).  
27 To scale to large data sets, minibatches of both cells and genes are used to approximate  
28 gradients at each update. Genes (i.e., features) are sampled in proportion to the percent-  
29 age of cells in which they are expressed to efficiently approximate the covariance matrix  
30 calculated during inference, inspired by previous random matrix algorithms to approximate  
31 high dimensional matrix multiplication (Drineas et al.). Cells (i.e., samples) are sampled  
32 uniformly in every batch. Inference was performed on Microsoft Azure High Performance  
33 Computing cores.

#### 34 **Single cell RNA-seq data**

35 We chose four data sets to evaluate tGPLVM’s applicability to identifying cell type, state,  
36 and developmental trajectory and scalability to experiments with large numbers of cells.  
37 The Pollen data (Pollen et al.), which were used to evaluate clustering, consist of 11 distinct  
38 mouse neural and blood cell populations across 249 cells sequenced on a Fluidigm C1 sys-  
39 tems. Pollen is a dense matrix because of high read depth, with about 80% non-zero values.  
40 The counts are log normalized as  $\log_{10}(1 + Y)$ . Inference of development trajectories was  
41 evaluated on the data used to develop the method GPfates from Lonnberg (Lönnerberg et al.,  
42 2017). Lonnberg (Lönnerberg et al., 2017) sequenced 408 T helper cells cells over 7 days  
43 after *Plasmodium* infection on a Fluidigm C1 system. The Lonnberg data are provided  
44 as TPM measurements. The data are sparse and normalized by  $\log_2(1 + Y)$ . Cell state  
45 was explored on batch of CD34+ peripheral blood mononuclear cells (Zheng et al., 2017)  
46 (PBMCs). About 10,000 cells were captured with 10x Cell Ranger sequencing technologies.  
47 These sparse data were also normalized as  $\log_2(1 + Y)$ . Finally, to ensure scalability to the  
48 most recent experimental data sets, we fit the model to 1 million mice brain cells sequenced  
49 on a 10x Cell Ranger (Zheng et al., 2017). We normalized the mice brain cells as  $\log_2(1 + Y)$

#### 50 IDENTIFYING CELL TYPES WITH K-MEANS CLUSTERING

51 Clustering for cell type identification was evaluated on the Pollen (Pollen et al.) data.  
52 tGPLVM and comparison methods were used to fit latent mappings between 2 and 9 di-  
53 mensions. To perform clustering for each of the estimated latent manifolds, we used k-means  
54 clustering with the number of clusters equal to the number of different cell type or cell state  
55 labels in the existing data. Clustering with k-means was repeated 10 times on the mean  
56 of the posterior of the latent position and evaluated against true labels using normalized  
57 mutual information (NMI) and adjusted rand score (ARS). Mutual information measures  
58 the amount of information contained in one random variable, the true labels, by another  
59 random variable, the inferred labels. NMI normalizes mutual information by the geometric  
60 mean of the entropy of both labels to a scale of zero - no mutual information - to one - the

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61 same distribution (Strehl and Ghosh, 2003). ARS is a measure of the proportion of shared  
62 members between pairs of true and estimated clusters (Hubert and Arabie). Zero inflated  
63 factor analysis (ZIFA) (Pierson and Yau), t-SNE (Van Der Maaten and Hinton) (perplex-  
64 ity set to default 30), and PCA (Hotelling, 1933) were tested as comparison methods. To  
65 evaluate the robust adaptations of the tGPLVM model, we fit tGPLVM with only an SE  
66 kernel or SE and Matérn 1/2 kernel as well as tGPLVM with normally distributed error.

### 67 TRAJECTORY BUILDING WITH MINIMUM SPANNING TREES

68 tGPLVM was used to fit a two dimensional latent mappings for the Lonnberg (Lönnberg  
69 et al., 2017) developmental data. The minimum spanning tree was found on Euclidean  
70 distance matrix of the posterior means of the low dimensional embedding and compared  
71 to sequencing time to verify correct ordering. The same analysis was performed with  
72 ZIFA (Pierson and Yau), t-SNE (Van Der Maaten and Hinton) (perplexity set to default  
73 30), and PCA (Hotelling, 1933).

### 74 VISUALIZATION OF SPARSE, RAW COUNT MATRICES

75 tGPLVM was used to fit a three-dimensional mapping for the two 10x data sets, CD34+  
76 cells and mice brain cells. Pearson correlation between latent position posterior mean and  
77 expression counts was used to identify genes associated with latent dimensions. ZIFA (Pier-  
78 son and Yau), t-SNE (Van Der Maaten and Hinton) (perplexity set to default 30), and  
79 PCA (truncated SVD (Halko et al.)) were also fit to the CD34+ cell data to compare  
80 computational time.

### 81 **Scaling inference to high-throughput experiments**

82 Computational times were recorded for samples of 100, 1,000, 10,000, 100,000 and 1,000,000  
83 cells from the 10x 1 million mouse brains cells (Zheng et al., 2017) to a latent embedding  
84 using tGPLVM and comparison methods. Experiments were run on a Standard H16m (16  
85 VCPUs, 224 GB memory) Azure high performance computing Unix system. tGPLVM was  
86 run for 100 passes through the data. Each minibatch contained the minimum of 2500 or  
87 the number of cells and 250 genes. ZIFA, t-SNE, and PCA (using truncated SVD) were fit  
88 until convergence or failure.

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92 The authors declare that they have no competing financial interests.

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94 Software is available at <https://github.com/architverma1/tGPLVM>

95 **Author Contributions**

96 A.V. and B.E.E designed experiments, analyzed experiments, and wrote paper. A.V. wrote  
97 implementation and ran experiments.

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