# Supporting Information to: "3'-5' crosstalk contributes to transcriptional bursting" 

Massimo Cavallaro, ${ }^{1,2, *}$ Mark D. Walsh, ${ }^{1}$ Matt Jones, ${ }^{1}$ James Teahan, ${ }^{3}$ Simone Tiberi, ${ }^{4}$ Bärbel Finkenstädt, ${ }^{2}$ and Daniel Hebenstreit ${ }^{1, *}$<br>${ }^{1}$ School of Life Sciences, University of Warwick, Coventry, UK<br>${ }^{2}$ Department of Statistics, University of Warwick, Coventry, UK<br>${ }^{3}$ Department of Chemistry, University of Warwick, Coventry, UK<br>${ }^{4}$ Institute of Molecular Life Sciences and Swiss Institute of Bioinformatics, University of Zurich, Zurich, Switzerland

## CONTENTS

S1. Description of data
1
A. Flow-cytometry data 1
B. Control cells 2
C. SmFISH and Nanostring barcoding data

S2. Phenomelogical gene expression models
S3. Measurement and technical error model

S4. Monte Carlo estimation
A. MCMC samplers 8
B. Consensus posteriors 9
C. Goodness of fit 10

S5. mRNA decay rates 10

S6. Cell cycle
S7. Summary of results 12
A. Poisson-beta distribution
B. Negative binomial distribution
C. Poisson distribution

12

S8. Microscopic gene expression model 29
S9. Materials 30
A. Cell lines and cell culture 30
B. Single-molecule RNA fluorescence in situ hybridization30
C. RNA isolation and preparation, and degradation rate estimation
D. Nanostring
E. RNA-seq

References

[^0]
## S1. DESCRIPTION OF DATA

## A. Flow-cytometry data

We obtained flow cytometry data (from the BD LSRFortessa ${ }^{\mathrm{TM}}$ cell analyzer and BD FACSDiva ${ }^{\text {TM }}$ software) for cell lines expressing the genes env and HBB, in both their wild-type (WT) and mutated (mut) versions. For each gene, experimental data were collected in four replicates (8 in total), each containing groups of observations corresponding to cells stimulated with tetracycline (Tet) at concentrations of $5,10,20,40,80$, and $250 \mathrm{ng} / \mathrm{mL}$, respectively.

Each data-set was stored in a .fcs format file and it was imported and pre-processed in R as an object of class flowFrame, which consists of an annotated data-frame class defined in the flowCore R package [1] and designed to deal with flow-cytometry data. Rows in such data frames correspond to single measurements. Each row contains the values of two fluorescence intensities that correspond to staining for mRNA and total DNA and are labeled by R640-670/14-A and UV355-450/50-A, respectively. These readings were compesated for spectral overlap with flowCore. In addition to this, the values of four scattering observations, namely FSC.H, FSC.W, SSC.H, and SSC.W, were recorded. Such observations are thought to be correlated to cell size and granularity. Values for each observation are stored in so-called "arbitrary units" (a.u.) [2].

The first task is to identify records in the data sets corresponding to either cell debris or clumps of cells, which have to be removed from subsequent analysis. We apply the robust model-based clustering approach of Ref. [3], distributed as the flowClust package [4], to identify cell populations in the data. Based on the scattering observations, the points were grouped into 3 clusters, and the set corresponding to single cells is the one with intermediate size and granularity, as suggested by the DNA content (see Fig. S1). For the data sets where the three detected clusters overlap, points where grouped into two clusters, instead. For this second case, inspection shows that the cluster with
lower size and granularity corresponds to single cells. Standard rectangle gates were applied to remove a few outlier points whose UV355-450/50-A reads were lower than 500. Kernel density estimates (KDE) for the populations after this sub-setting are plotted in Fig. 2 (Main Text) and Fig. S2. Technical variation affects the shapes of the distributions only for some HIV replicates, with shoulders at the lower ends sometimes merging with the main mode. The cells corresponding to the data points in the shoulders exhibit normal characteristics (in terms of cell size and DNA content) and thus probably reflect cells without mRNA. The parameter estimates, reported in section S7, appear robust with regards to the absence or presence of the shoulders save for the highest $\mu_{X} \mathrm{~s}$.

## B. Control cells

For each replicate $k$, we consider control cells, where the gene of interest has been deleted (see Main text). Such control cells were subjected to the same staining procedure as the others, which leaves a background of fluorescence probes that are not specifically bound to the mRNA. We argue that such background fluorescence stain are also present in the cells expressing the transgenes and contribute a term $\epsilon_{i}^{(k)}$ to the signal detected by the cell-analyzer channel of label R640-670/14-A for each cell $i$. The histograms of the signal from control cells appear skewed, as illustrated for example in Fig. S3 (left). We chose to fit the Azzalini's skew-normal distribution, that has PDF

$$
\begin{align*}
& f_{\epsilon}\left(y \mid a^{(k)}, \mu_{\epsilon}^{(k)}, \sigma_{\epsilon}^{(k)}\right) \\
& \quad=2 \Phi\left(\left(y-\mu_{\epsilon}^{(k)}\right) \sigma_{\epsilon}^{(k)} a^{(k)}\right) \phi\left(y \mid \mu_{\epsilon}^{(k)}, \sigma_{\epsilon}^{(k)}\right) \tag{1}
\end{align*}
$$

to such data, where $\Phi$ and $\phi$ are the standard normal CDF and normal PDF, respectively, while the mean $\mu_{\epsilon}^{(k)}$, the standard deviation $\sigma_{\epsilon}^{(k)}$, and the skewness parameter $a^{(k)}$ are point estimates from the control data sets. The maximum likelihood estimates for each replicate are reported in Table S1 (see also Figs. S3(left)).

## C. SmFISH and Nanostring barcoding data

Flow-FISH data are supplemented by microscopybased single-molecule FISH counts (which we simply refer to as smFISH) and Nanostring nCounter ${ }^{\circledR}$ Technology bar-coding measurements. These assays are used to choose informative priors for the mean mRNA abundance and, in turn, to calibrate the flow-FISH readouts. Symbols $\bar{x}, s_{x}^{2}$, and $s_{\bar{x}}$ represent
sample mean, sample variance, and standard error of the mean, respectively. Based on these, we chose truncated normal informative priors for the average expression level $\mu_{X} \sim \mathcal{N}\left(\bar{x}, s_{\bar{x}}\right)$, with the constraint $\mu_{X}>0$, for all replicates $k$.

HEK293 cells are not ideal for smFISH, since they tend to overlap when growing, producing dense clusters after dividing. A further problem with smFISH is the limited dynamic range of suitable microscopes. In fact, images tend to be overexposed when recorded during transcriptional bursts at settings that are otherwise optimal for lower transcript numbers and, conversely, optimal settings for transcriptional bursts do not cope well when transcript numbers are low. Therefore we only exploit smFISH results to infer the average expression level, and rely on flow-FISH to study the noise. SmFISH data yield the summary statistics of Table S2 for the mean expression of HBB. Nanostring data (SI Dataset 1) yields the summary statistics of Table S3 for the mean expression of env.


FIG. S1. Clustering of flow-cytometry data. (left) Clusters are projected to the FSC.W-SSC.H plane and plotted with the ellipses that delimit the 0.60 quantiles of fitted $t$-distributions. (right) Inspection of UV355-450/50-A shows signature distributions for DNA content, thus suggesting that the central cluster (in blue color) contains single-cell reads. Duplets have twice the amount of DNA content than single cells.


FIG. S2. KDEs of the flow-FISH single-cell readings corresponding to the abundances of HIV transcripts, from wildtype (blue), mutant (orange), and control (gray) cells, from 4 replicates, $k=1,2,3,4$ (left to right), at the different induction level (Tet concentrations in unit of $\mathrm{ng} / \mathrm{mL}$, shades of colors). Gene expression saturates upon increasing Tet concentration and mutant-cell expressions is lower than the wild-type. Fluorescence is given in arbitrary units (a.u.), $y$-axes are not to scale.


FIG. S3. Control measurements. (left) The MLE skew-normal density (line) of the background data for Tet=40 $\mathrm{ng} / \mathrm{mL}, k=1$, WT, is in good agreement with the empirical histogram. (right) Microscopy FISH count summary of housekeeping gene Akt1 vs induction levels, for both wild-type and mutant cells. Points and error bars are sample means and standard deviations, respectively.

TABLE S1. MLE estimation of the control-cell fluorescence (replicates 2 (HBB) and 4 (HIV) have the same background parameters as measurements were performed the same day with the same control cells).

| gene | k | Tet | $\mu^{(k)}$ | $\sigma^{(k)}$ | $a^{(k)}$ | $s_{\mu}^{(k)}$ | $s_{\sigma}^{(k)}$ |
| :--- | ---: | ---: | :---: | :---: | :---: | :---: | :---: |
| HBB | 1 | 0 | 389.504 | 348.098 | 1.817 | 15.670 | 12.396 |
| HBB | 1 | 250 | 514.828 | 307.460 | 1.632 | 11.667 | 9.027 |
| HBB | 2 | 0 | 625.804 | 458.715 | 1.863 | 21.687 | 17.915 |
| HBB | 2 | 250 | 459.139 | 311.688 | 2.075 | 9.098 | 7.703 |
| HBB | 3 | 0 | 539.613 | 360.195 | 2.046 | 11.838 | 10.270 |
| HBB | 3 | 250 | 493.913 | 337.605 | 2.091 | 10.440 | 9.088 |
| HBB | 4 | 0 | 53.140 | 320.685 | 2.303 | 7.201 | 6.514 |
| HBB | 4 | 250 | 112.667 | 256.183 | 1.748 | 5.507 | 4.497 |
| HIV | 1 | 0 | 565.327 | 390.016 | 1.834 | 8.397 | 6.936 |
| HIV | 1 | 250 | 443.898 | 463.603 | 1.813 | 14.774 | 12.117 |
| HIV | 2 | 0 | -31.395 | 230.794 | 1.108 | 13.129 | 8.356 |
| HIV | 2 | 250 | -23.282 | 312.375 | 2.066 | 8.109 | 7.156 |
| HIV | 3 | 0 | 196.970 | 401.184 | 5.423 | 6.238 | 8.517 |
| HIV | 3 | 250 | 407.813 | 259.586 | 1.445 | 16.865 | 12.125 |
| HIV | 4 | 0 | 625.804 | 458.715 | 1.863 | 21.687 | 17.915 |
| HIV | 4 | 250 | 459.139 | 311.688 | 2.075 | 9.098 | 7.703 |
| H | 0.169 |  |  |  |  |  |  |

TABLE S2. Summary statistics for the mean expression of HBB, obtained from microscopy FISH.

| gene | Tet | $\bar{x}$ | $s_{x}^{2}$ | sample size |
| :---: | :---: | :---: | :---: | :---: |
| WT | 0 | 64.868 | 2984.538 | 585 |
| WT | 5 | 115.450 | 7640.059 | 202 |
| WT | 10 | 175.150 | 18093.547 | 193 |
| WT | 20 | 312.945 | 32327.429 | 347 |
| WT | 40 | 384.111 | 23962.077 | 190 |
| WT | 80 | 414.105 | 31613.582 | 437 |
| WT | 250 | 565.351 | 54765.760 | 342 |
| mut | 0 | 38.953 | 1431.185 | 379 |
| mut | 5 | 41.645 | 717.115 | 279 |
| mut | 10 | 62.995 | 3573.429 | 198 |
| mut | 40 | 90.709 | 3301.445 | 468 |
| mut | 80 | 115.413 | 7096.164 | 179 |
| mut | 250 | 163.547 | 7375.582 | 892 |

TABLE S3. Summary statistics for the mean expression of HIV, obtained from nCounter ${ }^{\circledR}$ data. The standard error of the mean $s_{\bar{x}}^{2}$ is obtained propagating the errors from the nCounter ${ }^{\circledR}$ and the Atk1 smFISH measurements used for normalisation.

| gene | Tet | $\bar{x}$ | $s_{\bar{x}}^{2}$ |
| :---: | :---: | :---: | :---: |
| WT | 0 | 24.723 | 3.247 |
| WT | 5 | 72.975 | 3.247 |
| WT | 10 | 115.872 | 3.247 |
| WT | 20 | 151.462 | 3.247 |
| WT | 40 | 199.433 | 3.247 |
| WT | 80 | 232.644 | 3.247 |
| WT | 250 | 238.178 | 3.247 |
| mut | 0 | 21.875 | 3.247 |
| mut | 5 | 36.842 | 3.247 |
| mut | 10 | 58.108 | 3.247 |
| mut | 20 | 95.460 | 3.247 |
| mut | 40 | 139.979 | 3.247 |
| mut | 80 | 162.874 | 3.247 |
| mut | 250 | 187.287 | 3.247 |

## S2. PHENOMELOGICAL GENE EXPRESSION MODELS

We describe the gene expression in terms of the standard phenomenological two-state model [5]. This model assumes that the gene randomly alternates between an "on" and an "off" state, and that the mRNA is only transcribed, at rate $\tilde{\alpha}$, during the on state. The gene switches from "off" to "on" and from "on" to "off" states after an exponentially distributed random time with mean $1 / k_{\text {on }}$ and $1 / k_{\text {off }}$, respectively. Consequently, the transcriptional bursting is fully characterised by the rates $\tilde{\alpha}, \tilde{k}_{\text {on }}$, and $\tilde{k}_{\text {off }}$. In addition to this, mRNA is degraded at rate $\tilde{d}$. It is convenient to express the rates in units of the inverse of the mean mRNA life-time $\tilde{d}$, i.e.,

$$
\begin{align*}
\tilde{k}_{\text {off }} & =k_{\text {off }} \tilde{d}  \tag{2}\\
\tilde{k}_{\text {on }} & =k_{\text {on }} \tilde{d}  \tag{3}\\
\tilde{\alpha} & =\alpha \tilde{d} \tag{4}
\end{align*}
$$

It can be shown that the stationary probability density function (PDF) of the mRNA population $x$ for this model is (see, e.g., Ref. [6])

$$
\begin{equation*}
f_{X}\left(x \mid \alpha, k_{\mathrm{on}}, k_{\mathrm{off}}\right)=\frac{\alpha^{x} \mathrm{e}^{-\alpha} \Gamma\left(k_{\mathrm{on}}+x\right) \Gamma\left(k_{\mathrm{on}}+k_{\mathrm{off}}\right)}{x!\Gamma\left(k_{\mathrm{on}}+k_{\mathrm{off}}+x\right) \Gamma\left(k_{\mathrm{on}}\right)}{ }_{1} F_{1}\left(k_{\mathrm{off}}, k_{\mathrm{on}}+k_{\mathrm{off}}+x ; \alpha\right), \tag{5}
\end{equation*}
$$

where $\Gamma$ is the gamma function and ${ }_{1} F_{1}$ is the confluent hyper-geometric function of the first kind. An alternative representation of the PDF (5) is

$$
\begin{equation*}
f_{X}\left(x \mid \alpha, k_{\mathrm{on}}, k_{\mathrm{off}}\right)=\int_{0}^{1} f_{\mathrm{Poi}}(x \mid \alpha p) f_{\mathrm{Be}}\left(p \mid k_{\mathrm{on}}, k_{\mathrm{off}}\right) \mathrm{d} p \tag{6}
\end{equation*}
$$

where

$$
\begin{align*}
f_{\mathrm{Poi}}(x \mid \alpha) & =\frac{\alpha^{x} \mathrm{e}^{-\alpha}}{x!}  \tag{7}\\
f_{\mathrm{Be}}\left(p \mid k_{\text {on }}, k_{\text {off }}\right) & =p^{k_{\text {on }}-1}(1-p)^{k_{\text {of }}-1} \frac{\Gamma\left(k_{\text {on }}+k_{\text {off }}\right)}{\Gamma\left(k_{\text {off }}\right) \Gamma\left(k_{\text {on }}\right)} \tag{8}
\end{align*}
$$

are density distributions of Poisson and beta random variables (RVs), respectively.
The PDF of equation (6) encodes the following hierarchy

$$
\begin{align*}
X \mid \alpha, P & \sim \operatorname{Poi}(\alpha P)  \tag{9}\\
P \mid k_{\text {on }}, k_{\text {off }} & \sim \operatorname{Beta}\left(k_{\text {on }}, k_{\text {off }}\right) . \tag{10}
\end{align*}
$$

Further details can be found, e.g., in Refs. [6, 7]. It is convenient to reparametrise the Poisson-beta distribution in terms of its mean

$$
\begin{equation*}
\mu_{X}=\alpha \frac{k_{\mathrm{on}}}{k_{\mathrm{off}}+k_{\mathrm{on}}} \tag{11}
\end{equation*}
$$

to get

$$
\begin{gather*}
X \mid \mu_{X}, k_{\mathrm{on}}, k_{\mathrm{off}}, P \sim \operatorname{Poi}\left(\mu_{X} \frac{k_{\mathrm{off}}+k_{\mathrm{on}}}{k_{\mathrm{on}}} P\right)  \tag{12}\\
f_{X}\left(x \mid \alpha, k_{\mathrm{on}}, k_{\mathrm{off}}\right)=: f_{X}^{\prime}\left(x \mid \mu_{X}, k_{\mathrm{on}}, k_{\mathrm{off}}\right) \tag{13}
\end{gather*}
$$

In fact, this allows us to exploit knowledge on $\mu_{X}$ in the form of informative priors of S1 C. The expression for the squared coefficient of variation $\left(\mathrm{CV}^{2}\right)$ can also be written in terms of $\mu_{X}$, i.e.,

$$
\begin{equation*}
\mathrm{CV}_{X}^{2}=\frac{1}{\mu_{X}}+\frac{k_{\mathrm{off}}}{k_{\mathrm{on}}\left(1+k_{\mathrm{off}}+k_{\mathrm{on}}\right)} \tag{14}
\end{equation*}
$$

where the second term on the r.h.s. quantifies the overdispersion of $X$ with respect to a Poisson random variable. Such a functional relation between $\mathrm{CV}_{X}^{2}$ and $\mu_{X}$ has been encountered in gene expression data [812]. The probability $P$ cannot be directly accessed and therefore is a latent (hidden) variable for the model. In fact, for our data, the mRNA number is a latent variable too, being only inferred from the measured fluorescence signals. This can be encoded into a measurement equation, as explained in the next section.

In the limit as $k_{\text {off }} \rightarrow \infty, \alpha \rightarrow \infty$, with the ratio $\alpha / k_{\text {off }}$ held fixed, the population mean and $\mathrm{CV}^{2}$ satisfy

$$
\begin{align*}
\mu_{X} & =\frac{\alpha}{k_{\mathrm{off}}} k_{\mathrm{on}}  \tag{15}\\
\mathrm{CV}_{X}^{2} & =\frac{1}{\mu_{X}}+\frac{1}{k_{\mathrm{on}}} \tag{16}
\end{align*}
$$

respectively, while the distribution of $X$ approaches the negative binomial distribution with PDF

$$
\begin{equation*}
f_{X}^{\prime \prime}\left(x \mid \mu_{X}, k_{\mathrm{on}}\right)=\frac{\Gamma\left(k_{\mathrm{on}}+x\right)}{x!\Gamma\left(k_{\mathrm{on}}\right)}\left(\frac{k_{\mathrm{on}}}{k_{\mathrm{on}}+\mu_{X}}\right)^{k_{\mathrm{on}}}\left(\frac{\mu_{X}}{k_{\mathrm{on}}+\mu_{X}}\right)^{x} . \tag{17}
\end{equation*}
$$

This can be easily proven using the Poisson-gamma mixture formulation of the negative binomial RV $X$, i.e.,

$$
\begin{align*}
X \mid \lambda & \sim \operatorname{Poi}(\lambda)  \tag{18}\\
\lambda \mid k_{\text {on }}, k_{\text {off }} / \alpha & \sim \operatorname{Gamma}\left(k_{\text {on }}, k_{\text {off }} / \alpha\right) \tag{19}
\end{align*}
$$

In fact, the beta distribution scaled by $\alpha>0$ approaches the gamma distribution as $k_{\text {off }} \rightarrow \infty, \alpha \rightarrow \infty$, i.e.,

$$
\begin{equation*}
\frac{1}{\alpha} \frac{\Gamma\left(k_{\text {on }}+k_{\text {off }}\right)}{\Gamma\left(k_{\text {on }}\right) \Gamma\left(k_{\text {off }}\right)}\left(\frac{x}{\alpha}\right)^{k_{\text {on }}-1}\left(1-\frac{x}{\alpha}\right)^{k_{\text {off }}-1} \asymp \frac{\frac{k_{\text {off }} k_{\text {on }}}{\alpha} x^{k_{\text {on }}-1} \mathrm{e}^{-\frac{k_{\text {off }} x}{\alpha} x}}{\Gamma\left(k_{\text {on }}\right)} \tag{20}
\end{equation*}
$$

which follows from known asymptotic relations

$$
\begin{align*}
& \lim _{k_{\text {off }} \rightarrow \infty} \frac{\Gamma\left(k_{\text {off }}+k_{\text {on }}\right)}{\Gamma\left(k_{\text {off }}\right) k_{\text {off }}^{k_{\text {on }}}}=1  \tag{21}\\
& \lim _{\alpha \rightarrow \infty}\left(1-\frac{x}{\alpha}\right)^{\alpha \frac{k_{\text {off }}}{\alpha}}=\mathrm{e}^{-\frac{k_{\text {off }} x}{\alpha} x} \tag{22}
\end{align*}
$$

The ratio $\alpha / k_{\text {off }}$ has a simple interpretation, being the expected number of transcription events during an on phase. In Ref. [13] this ratio has been referred to as the "expected burst size". In the limit as $k_{\text {on }} \rightarrow \infty$ with the mean expression $\mu_{X}$ held fixed, the negative binomial distribution (17) approaches the distribution

$$
\begin{equation*}
f_{X}^{\prime \prime \prime}\left(x \mid \mu_{X}\right):=f_{\mathrm{Poi}}\left(x \mid \mu_{X}\right) \tag{23}
\end{equation*}
$$

## S3. MEASUREMENT AND TECHNICAL ERROR MODEL

The DB FACSDiva ${ }^{\text {TM }}$ software manual [14] specifies that the light intensity from fluorescent dyes is amplified linearly within a wide range (see also, e.g., Refs. [9, 15]). Based on this, we assume that the measured fluorescence $Y_{i}$ of cell $i$ is proportional to the true mRNA abundance $X_{i}$ and therefore can be expressed as in the following "measurement" equation,

$$
\begin{equation*}
Y_{i}^{(k)}=\epsilon_{i}^{(k)}+\kappa^{(k)} X_{i}^{(k)} \tag{24}
\end{equation*}
$$

where $k$ indexes the replicate, $\kappa$ can be thought of as a scale and $\epsilon_{i}$ is the zero of such a scale, also corresponding to the background of unspecific staining and auto-fluorescence of the $i$ th cell [7].

The dispersion of biological data is typically due to both technical errors, caused by the measurement process, and the variability intrinsic to the underlying biology. Separating these two contributions is indeed a central issue in systems biology [16, 17]. In our measurement model, for the variables $X_{i}^{(k)}$ to best accommodate the true biological noise of $Y_{i}^{(k)}$, it is important that $\epsilon_{i}^{(k)}$ and $\kappa^{(k)}$ are specified with sufficient precision and accuracy.


FIG. S4. Flow-FISH data (violin plots [18]) vs mean expression levels obtained from FISH data for the replicate $k=3$, WT HBB gene. Their relation is captured by a linear model with coefficient $\kappa$.

Informative priors for $\epsilon_{i}^{(k)}$ are chosen according to section S1 B, i.e.,

$$
\begin{equation*}
\epsilon_{i}^{(k)} \sim \operatorname{SN}\left(a^{(k)}, \mu_{\epsilon}^{(k)}, \sigma_{\epsilon}^{(k)}\right) \tag{25}
\end{equation*}
$$

where the parameters $a^{(k)}, \mu_{\epsilon}^{(k)}$, and $\sigma_{\epsilon}^{(k)}$ are estimated from the control cells at $250 \mathrm{ng} / \mathrm{mL}$ Tet. The standard errors of the maximum likelihood estimates are neglected. As a consequence, all the single-cell measurements can be thought of as being subjected to the same random background, thus mitigating tractability issues. For a more comprehensive fully-Bayesian hierarchical approach see Ref. [7].

To pin down informative priors for $\kappa^{(k)}$, we perform gamma regression. For each flow-FISH data-set, 500 random cell readings are selected for the main Monte Carlo estimation of section S4. The remaining reads are used as response variables for a gamma regression with identity link. Covariates are mean expression level point estimates from section S 1 C . As an example, this is illustrated in Fig. S4 for $k=3$, WT HBB gene. The GLM estimates of the expected values $\mu_{\kappa}^{\prime}$ along with the standard errors $s_{\kappa}^{\prime}$ are reported in Table S4. Our prior choice is the truncated normal RV

$$
\begin{equation*}
\kappa \sim \mathcal{N}\left(\mu_{\kappa}^{\prime}, s_{\kappa}^{\prime}\right) \tag{26}
\end{equation*}
$$

with the constraint $\kappa>1$, where the mean $\mu_{\kappa}^{\prime}$ and standard deviation $s_{\kappa}^{\prime}$ are obtained from the 16 values in Table S4 according to the laws of total expectation and variance, respectively, i.e.,

$$
\begin{equation*}
\mu_{\kappa}^{\prime}=\overline{\mu_{\kappa}}, \quad s_{\kappa}^{\prime 2}=\overline{s_{\kappa}^{2}}+\overline{\mu_{\kappa}^{2}}-{\overline{\mu_{\kappa}}}^{2} \tag{27}
\end{equation*}
$$

where the bar notation represents averages.
For the remaining parameters we assume

$$
\begin{align*}
k_{\text {on }} & \sim \operatorname{Gamma}\left(\alpha_{k_{\mathrm{on}}}, \beta_{k_{\mathrm{on}}}\right)  \tag{28}\\
k_{\text {off }} & \sim \operatorname{Gamma}\left(\alpha_{k_{\text {off }}}, \beta_{k_{\text {off }}}\right)  \tag{29}\\
\alpha_{k_{\text {on }}} & =\beta_{k_{\text {on }}}=\alpha_{k_{\text {off }}}=\beta_{k_{\text {off }}}=0.001 \tag{30}
\end{align*}
$$

which is a classical choice for vague priors with positive support [19].
Since the replicates are independent, the likelihood of the parameters of the Poisson-beta model, for a data-set of $N$ measurements $y_{1: N}^{(k)}$, is

$$
\begin{equation*}
\mathcal{L}_{k}^{\prime}\left(y_{1: N}^{(k)} \mid \theta^{(k)}, \mu_{X}, k_{\mathrm{on}}, k_{\mathrm{off}}\right)=\prod_{j=1}^{N}\left(\sum_{x} f_{\epsilon}\left(y_{j}^{(k)}-\kappa x \mid a^{(k)}, \mu_{\epsilon}^{(k)}, \sigma_{\epsilon}^{(k)}\right) f_{X}^{\prime}\left(x \mid \mu_{X}, k_{\mathrm{on}}, k_{\mathrm{off}}\right)\right) \tag{31}
\end{equation*}
$$

TABLE S4. Estimated coefficients (means and standard errors of the mean, $\mu_{\kappa}^{(k)}$ and $s_{\kappa}^{(k)}$, respectively) of the gamma GLM.

| k | gene | $\mu_{\kappa}^{(k)}$ | $s_{\kappa}^{(k)}$ |
| :---: | :---: | :---: | :---: |
| 1 | HBB WT | 20.904 | 0.230 |
| 2 | HBB WT | 27.080 | 0.283 |
| 3 | HBB WT | 28.394 | 0.241 |
| 4 | HBB WT | 21.631 | 0.244 |
| 1 | HBB mut | 20.308 | 0.233 |
| 2 | HBB mut | 29.621 | 0.402 |
| 3 | HBB mut | 32.267 | 0.353 |
| 4 | HBB mut | 26.612 | 0.322 |
| 1 | HIV WT | 17.715 | 0.153 |
| 2 | HIV WT | 34.771 | 0.341 |
| 3 | HIV WT | 17.118 | 0.183 |
| 4 | HIV WT | 50.748 | 0.547 |
| 1 | HIV mut | 23.035 | 0.219 |
| 2 | HIV mut | 32.463 | 0.381 |
| 3 | HIV mut | 26.951 | 0.296 |
| 4 | HIV mut | 25.487 | 0.300 |

where $\theta^{(k)}:=\left(\kappa, a^{(k)}, \mu_{\epsilon}^{(k)}, \sigma_{\epsilon}^{(k)}\right)$ is the vector of the parameters that describe the experimental setting. This completes the definition of the first Bayesian model for the observed data. The directed acyclic graph (DAG) of the full posterior of this model is illustrated in Fig. S5(A).

Consistently, the likelihood of the parameters of the negative-binomial model is

$$
\begin{equation*}
\mathcal{L}_{k}^{\prime \prime}\left(y_{1: N}^{(k)} \mid \theta^{(k)}, \mu_{X}, k_{\mathrm{on}}\right)=\prod_{j=1}^{N}\left(\sum_{x} f_{\epsilon}\left(y_{j}^{(k)}-\kappa x \mid a^{(k)}, \mu_{\epsilon}^{(k)}, \sigma_{\epsilon}^{(k)}\right) f_{X}^{\prime \prime}\left(x \mid \mu_{X}, k_{\mathrm{on}}\right)\right) \tag{32}
\end{equation*}
$$

as illustrated in Fig. S5(B). The simplest Poisson model, the likelihood is

$$
\begin{equation*}
\mathcal{L}_{k}^{\prime \prime \prime}\left(y_{1: N}^{(k)} \mid \theta^{(k)}, \mu_{X}\right)=\prod_{j=1}^{N}\left(\sum_{x} f_{\epsilon}\left(y_{j}^{(k)}-\kappa x \mid a^{(k)}, \mu_{\epsilon}^{(k)}, \sigma_{\epsilon}^{(k)}\right) f_{X}^{\prime \prime \prime}\left(x \mid \mu_{X}\right)\right) \tag{33}
\end{equation*}
$$

whose DAG is illustrated in Fig. S5(C).

## S4. MONTE CARLO ESTIMATION

## A. MCMC samplers

Adaptive Metropolis-Hastings samplers to fit the model to the data where implemented using the PyMC library for probabilistic programming [20], version 2.3.7, which has a flexible object oriented syntax and provides tools to handle long traces and perform diagnostics. To improve the convergence speed, the array containing the $N$ elements of a dataset was numerically sorted by value and split into $M=N / 10$ blocks of size 10 . The latent random-
variable arrays $P_{1: N}$ and $X_{1: N}$ were batched too, as the RVs conditioned on the data of a single block are strongly correlated and are conveniently updated during single Metropolis-Hastings steps. Using the symbol $\mathbf{1}_{x}$ for the identity matrix and $\mathcal{N}_{x}$ to represent a multivariate normal RV of dimension $x$, the simulation of posterior samples for the Poisson-beta model proceeds as follows:

- For $i=0,1, \ldots, M$, values of the 10 -value blocks $P_{(i 10+1):(i+1) 10}$ and $\left(X_{i 10+1):(i+1) 10}\right.$ are updated according to a random walk Metropolis with proposals $\mathcal{N}_{10}\left(\mu_{i}^{(P)}, \sigma_{i}^{(P)} \mathbf{1}_{10}\right)$ and $\mathcal{N}_{10}\left(\mu_{i}^{(X)}, \sigma_{i}^{(X)} \mathbf{1}_{10}\right)$ respectively,


FIG. S5. DAG for the Poisson-beta model (A), the negative-binomial model (B), and the naïve Poisson model (C) with measurement equation. Circle nodes represent parameters to be estimated, blank nodes represent set parameters, diamonds correspond to deterministic functions of their parents, and square nodes represent observations.

- The RVs $\mu_{X}, \kappa, k_{\text {on }}$, and $k_{\text {off }}$ are updated as a single block according to the adaptive Metropolis-Hastings method of Ref. [21] with proposal $\mathcal{N}_{4}(\mu, \Sigma)$.

The proposal parameters $\mu_{i}^{(X)}, \sigma_{i}^{(X)}, \mu_{i}^{(P)}, \sigma_{i}^{(P)}$, $(i=1, \ldots, M), \mu$, and $\Sigma$ are chosen adaptively. To improve the adaptation (noting that the posterior for $k_{\text {off }}$ is more disperse than those of $\mu_{X}$, $\kappa$, and $\left.k_{\text {on }}\right), \Sigma$ is initialised to the diagonal matrix $\operatorname{diag}(0.1,0.1,0.1,1)$.

In order to mitigate tractability issues (which is mainly due to the large number of presence of latent variables), the model is only fitted to a randomly sampled subset of $N=500$ data points. For a more modern approach to cope with latent variables see Ref. [7], which also defines a more complex hierarchical model.

The sampler implemented for the negativebinomial model is similar to the one implemented for the Poisson-beta model (with data organised into $M$ ranked batches) but converges and mixes more rapidly, as it does not encode for the latent variables $P_{i}, i=1,2, \ldots, N$. The simulation proceeds as follows:

- For $i=0,1, \ldots, M$, values of $X_{(i 10+1):(i+1) 10}$ are updated according to a random-walk

Metropolis with proposal $\mathcal{N}_{10}\left(\mu_{i}^{(X)}, \sigma_{i}^{(X)} \mathbf{1}_{10}\right)$.

- The random variables $\mu_{X}, \kappa$ and $k_{\text {on }}$ are updated simultaneously according to the adaptive Metropolis-Hastings method with proposal $\mathcal{N}_{3}(\mu, \Sigma)$,
where the quantities $\mathbf{1}_{x}, \mu_{i}^{(X)}, \sigma_{i}^{(X)},(i=1, \ldots, M)$, $\mu$, and $\Sigma$ are defined as in the former case. The Poisson-model sampler is analogous to the negative binomial, except that it does not include the parameter $k_{\text {on }}$. All the samplers were successfully tested with simulated data.


## B. Consensus posteriors

The posterior

$$
\begin{equation*}
p\left(\vartheta \mid y_{1: N}^{(1)}, y_{1: N}^{(2)} y_{1: N}^{(3)}, y_{1: N}^{(4)}\right) \propto \prod_{k=1}^{4}\left[\mathcal{L}_{k}\left(y_{1: N}^{(k)} \mid \vartheta\right)\right] p(\vartheta) \tag{34}
\end{equation*}
$$

represents the consensus belief on the vector of parameters $\vartheta$ among all the replicates $k=1,2,3,4$, with $p(\vartheta)$ being the prior. We approximate such a posterior by means of a consensus Monte Carlo approach, i.e., by running a separate MCMC on each of


FIG. S6. KDE and histogram of the data (blue) (HBB, WT, $80 \mathrm{ng} / \mathrm{mL}$ Tet, $k=1$ ) and of the draws from the corresponding posterior-predictive distribution (orange), according to the negative-binomial model. It is possible to visually appreciate their overlap. To quantify the extent to which the posterior predictive reproduces the true data distribution for all the fit, we studied the Wasserstein distance of the two distribution.
the datasets $y_{1: N}^{(k)}, k=1,2,3,4$, and then averaging individual Monte Carlo draws across, as in Ref. [22]. The draws $\vartheta_{j}^{(k)}, j=1, \ldots, L, k=1,2,3,4$, are combined into the weighted averages

$$
\begin{equation*}
\vartheta_{j}=\frac{\sum_{k} \vartheta_{j}^{(k)} w_{(k)}}{\sum_{k} w_{(k)}} \tag{35}
\end{equation*}
$$

where $w_{(k)}$ is the vector of the reciprocal of the marginal posterior variances. This method has been only justified rigorously for Gaussian posteriors and only yields approximate posteriors, in general. However, it allowed us to distribute the Bayesian analysis across different machines, and therefore was of great utility to aggregate results.

## C. Goodness of fit

To evaluate the goodness of fit (GoF) we estimate the posterior predictive distribution

$$
\begin{equation*}
p\left(\tilde{y}_{1: N}^{(k)} \mid y_{1: N}^{(k)}\right)=\int p\left(\tilde{y}_{1: N}^{(k)} \mid \theta\right) p\left(\theta \mid y_{1: N}^{(k)}\right) \mathrm{d} \theta \tag{36}
\end{equation*}
$$

where $\theta$ is the vector of all parameters, by generating pseudo-data $\tilde{y}_{1: N}^{(k)}$ for the model using the parameters drawn from the posterior $p\left(\theta \mid y_{1: N}^{(k)}\right)$ alongside each MCMC chain. A GoF test follows by measuring to what extent the pseudo-data deviate from $y_{1: N}^{(k)}$. Specifically, we calculate the root mean square
displacement (RMSD) of the data $y_{i}^{(k)}, i=1, \ldots, N$ with respect to the sample mean $\bar{y}_{i}^{(k)}$ of the draws from the marginal posterior predictive, i.e.,

$$
\begin{equation*}
\operatorname{RMSD}\left(\bar{y}_{1: N}^{(k)}, y_{1: N}^{(k)}\right)=\sqrt{\frac{1}{N} \sum_{i=1}^{N}\left(\bar{y}_{i}^{(k)}-y_{i}^{(k)}\right)^{2}} \tag{37}
\end{equation*}
$$

Comparison between the RMSD results for the Poisson-beta and the negative-binomial models is shown in Fig. S7, which suggests that both these models fit the data equally well. Conversely, the RMSDs for the Poisson model are always higher that the RMSDs for the two former models (see Fig. S7) implying that the Poisson model does not fit as well. Further, we computed the Wasserstein distance in distribution between the data and the pseudo-data. The Wasserstein distance between two distributions $u$ and $v$ is defined as

$$
\begin{equation*}
l_{1}(u, v)=\int_{-\infty}^{\infty}|U-V| \tag{38}
\end{equation*}
$$

where $U$ and $V$ are the empirical cumulative distribution functions associated to and $u$ and $v$, respectively. Fig. S7 shows that the distances are always smaller than the $95 \%$ percentile of bootstrapsamples distances from the true data, thus confirming GoF. Bin sizes for the empirical distribution were chosen according to the Freedman-Diaconis rule.

## S5. MRNA DECAY RATES

The draws from the posteriors of the dimensionless rates $k_{\text {on }}, k_{\text {off }}$, and $\alpha$ are converted to number of events per minute $\tilde{k}_{\text {on }}, \tilde{k}_{\text {off }}$, and $\tilde{\alpha}$ by using estimated decay rates of mRNA. For the HBB gene, decay rates were measured in Ref. [24] (and are reported in Table S5). Due to the detection of two different mRNA isoforms, viz., "rt" and "pA", the empirical mRNA distribution can be thought of a Gaussian mixture density with PDF

$$
\begin{equation*}
f(x)=p_{\mathrm{pA}} \phi\left(x \mid \mu_{\mathrm{pA}}, \sigma_{\mathrm{pA}}\right)+p_{\mathrm{rt}} \phi\left(x \mid \mu_{\mathrm{rt}}, \sigma_{\mathrm{rt}}\right) \tag{39}
\end{equation*}
$$

$p_{\mathrm{rt}}+p_{\mathrm{rt}}=1$, with parameters of Table S5. According to equations (2)-(4), the traces from $k_{\text {on }}, k_{\text {off }}$, and $\alpha$ were multiplied by draws from this Gaussian mixture to obtain $\tilde{k}_{\text {on }}, \tilde{k}_{\text {off }}$, and $\tilde{\alpha}$.

We measured total env RNA content (including non-poly-adenylated transcripts) with RT-qPCR, using gene-specific primers (forward primers bind exon 1 and reverse primers bind the 3 'UTR, see section S 9 C ). The decay rates of env transcripts were obtained by fitting a linear model to the logarithm of RT-qPCR measurements of transcripts vs time in


FIG. S7. GoF analysis. (left and centre) RMSDs of the data with respect to the sample means of the (posterior predictive (see eq. (37)) for each dataset. Comparison of Poisson-beta model vs the negative-binomial model (left scatter plot) shows that the two models achieve similar GoF, while the RMSDs obtained from the Poisson model are always the largest (central scatter plot). (right) Wasserstein distances between the empirical histogram of the data and the negative-binomial model posterior predictive ( $x$-axis) is always smaller that the $95 \%$ quantiles of the bootstrapped distances from the true data, which suggests that that the posterior-predictive samples for the negativebinomial model always reproduce the true flow-FISH data (as in, e.g., Fig. S6).

TABLE S5. . Decay rates in number of events per minutes of the two mRNA isoforms for the HBB gene. "pA" and "rt" refer to polyadenylated and read-through isoforms, respectively (it is possible to foresee larger dispersion for the mutant than for the WT).

| SNP | isoform | $\mu$ | $\sigma$ | $p$ |
| :---: | :---: | :---: | :---: | :---: |
| WT | pA | 0.0024 | 0.0002 | 0.72 |
| WT | rt | 0.0067 | 0.0016 | 0.28 |
| mut | pA | 0.0036 | 0.0003 | 0.11 |
| mut | rt | 0.0112 | 0.0023 | 0.89 |

minutes. The inferred mean $\mu_{d}$ and standard error $\sigma_{d}$ of the decay rates are reported in Fig. S8. The draws from the posteriors of dimensionless rates $k_{\text {on }}$, $k_{\text {off }}$, and $\alpha$ were converted to number of events per minute by multiplying by normal draws $\mathcal{N}\left(\mu_{d}, \sigma_{d}\right)$, with parameters listed in Fig. S8.


FIG. S8. Logarithm of RT-qPCR $2^{-\Delta \Delta C_{t}}$ measurements of residual transcripts vs time of the mutant and WT HIV gene transcripts. From linear regression, decay rates in unit of events per minutes are obtained.


FIG. S9. Scatter plot of the UV355-450/50-A vs FSC-A signal for the $40 \mathrm{ng} / \mathrm{mL}$ Tet-induced HBB gene, replicate $k=3$. Cells from the three phases, highlighted with different green-scale colors, were separated using flowClust.

## S6. CELL CYCLE

Staining for DNA concentration allows us to heuristically find cells that are in G1, S, and G2 phases of the cell cycle, see Figs. S1(right) and S9. We considered the dataset with cells treated at concentration of $40 \mathrm{ng} / \mathrm{mL}$ of Tet. We separated the data points corresponding to the G1 phase from those from S and G2 using flowClust [4]. The less dense cluster S-G2 was further separated in two groups (corresponding to the phases S and G2) running the same algorithm again. Results are shown in Fig. S9 for HBB cell line, $k=3$.

Data from phase G1, S, and G2 are referred to as $y_{\mathrm{G} 1}^{(k)}, y_{\mathrm{S}}^{(k)}$, and $y_{\mathrm{G} 2}^{(k)}$, respectively, for each replicate $k$. We refer to their averages (sample standard deviations of mean) as $\bar{y}_{\mathrm{G} 1}, \bar{y}_{\mathrm{S}}$, and $\bar{y}_{\mathrm{G} 2},\left(s_{\bar{y}_{\mathrm{G} 1}}, s_{\bar{y}_{\mathrm{S}}}\right.$, and $s_{\bar{y}_{\mathrm{G} 2}}$ ) respectively. To take into account that the mean gene expression seems to change with the cell phase, we introduce the conversion factors $c^{(k)}=$ $\bar{x} / \bar{y}^{(k)}$ to obtain $\bar{x}_{\mathrm{i}}^{(k)}=c^{(k)} \bar{y}_{\mathrm{i}}$ and $s_{\bar{x}_{\mathrm{i}}}^{(k)}=c^{(k)} s_{\bar{y}_{\mathrm{i}}}$ which in turn are used in the informative priors

$$
\begin{equation*}
\mu_{X_{\mathrm{i}}}^{(k)} \sim \mathcal{N}\left(\bar{x}_{\mathrm{i}}^{(k)}, s_{\bar{x}_{\mathrm{i}}}^{(k)}\right) \tag{40}
\end{equation*}
$$

$\mathrm{i}=\mathrm{G} 1, \mathrm{~S}, \mathrm{G} 2$. Equations (40) take the place of $\mu_{X}$ in the DAG of Fig. S5(B) for the G1, S, and G2 phase, respectively, for each replicate $k$. Fitting the negative-binomial model to 500 samples from each dataset yields the consensus estimates of Fig. 4(CD) (Main text).

We now assume that the cell cycle is extrinsic contributor to the total transgene mRNA variability, with the remaining variability sources thought of as

TABLE S6. Intrinsic noise, extrinsic noise and total noise from each replicate of wild-type HBB and HIV genes, $40 \mathrm{ng} / \mathrm{mL}$ Tet. Extrinsic noise has the lowest contribution to the total noise.

| gene | k | intr. noise | extr. noise | tot. noise |
| :---: | :---: | :---: | :---: | :---: |
| HBB | 1 | 0.566 | 0.077 | 0.643 |
| HBB | 2 | 0.411 | 0.104 | 0.515 |
| HBB | 3 | 0.365 | 0.092 | 0.457 |
| HBB | 4 | 0.364 | 0.078 | 0.443 |
| HIV | 1 | 0.308 | 0.092 | 0.401 |
| HIV | 2 | 0.325 | 0.093 | 0.418 |
| HIV | 3 | 0.451 | 0.088 | 0.539 |
| HIV | 4 | 0.406 | 0.102 | 0.508 |

being intrinsic. As in Ref. [8], using the symbol $\langle\cdot\rangle_{\mathrm{I}}$ for the average over the intrinsic variables, with the cell phase held fixed, and $\langle\cdot\rangle_{\mathrm{E}}$ for the average over the different cell phases, the law of total variance allows us to write, for the mRNA abundance $X$,

$$
\begin{equation*}
\mathrm{CV}_{X}^{2}=\frac{\left\langle\left\langle X^{2}\right\rangle_{\mathrm{I}}-\langle X\rangle_{\mathrm{I}}^{2}\right\rangle_{\mathrm{E}}}{\left\langle\langle X\rangle_{\mathrm{I}}\right\rangle_{\mathrm{E}}^{2}}+\frac{\left\langle\langle X\rangle_{\mathrm{I}}^{2}\right\rangle_{\mathrm{E}}-\left\langle\langle X\rangle_{\mathrm{I}}\right\rangle_{\mathrm{E}}^{2}}{\left\langle\langle X\rangle_{\mathrm{I}}\right\rangle_{\mathrm{E}}^{2}}, \tag{41}
\end{equation*}
$$

where the first term on the r.h.s. is the intrinsic noise, while the second term is the extrinsic noise. Computing the two terms gives the intrinsic and extrinsic noise levels of Table S6 and Fig. 4(B) (Main text), which show that the cell cycle always contributed only a minor term to the total noise.

## S7. SUMMARY OF RESULTS

## A. Poisson-beta distribution

Fig. S10 shows two results obtained from fitting the Poisson-beta model. The traces of the posterior chains from each replicate were combined according to the consensus Monte Carlo procedure (see section S 4 B ) to obtain a representation of the consensus belief of Fig. 3 (Main text). It is worth noting that the credible intervals for $\tilde{k}_{\text {off }}$ and $\tilde{\alpha}$ are very wide, while the MCMC draws of $k_{\text {off }}$ and $\alpha$ appear strongly cross-correlated (see, e.g., Fig. S11), where the drawn samples form an angle $\operatorname{arccot}\left(\alpha / k_{\text {off }}\right)$ with the abscissae axis.

The $90 \%$ highest posterior density credible intervals (HPD CIs) and medians of the estimated parameters $\kappa, \mu_{X}, k_{\text {on }}, k_{\text {off }}$, and $\alpha / k_{\text {off }}$ are reported in Tables S7-S11 (HBB gene) and Tables S12-S16 (HIV gene).


FIG. S10. Estimates for parameters $\tilde{k}_{\text {on }}, \tilde{k}_{\text {off }}, \mu_{X}$, and $\alpha / k_{\text {off }}$ of the Poisson- beta model, from wild-type (blue) and mutant (orange) cell data, for all induction levels, shades of colors correspond to replicates. Points are medians, error bars comprise $90 \%$ HPD CIs. HBB-gene results show results consistent across all the replicates (panel A). The HIB-gene results are reported in panels B. Increasing expression levels, three of the HIV replicates show a drop-off in the average burst size and an increase in the burst, see also Fig. S2. The consensus estimates are reported in Fig. 3, Main text.


FIG. S11. Cross-correlation between the MCMC draws for the dimensionless parameters $k_{\text {off }}$ and $\alpha$.

TABLE S7. $\kappa$ of HBB gene

|  | SNP |  | low.HPD | median | upp.HPD |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | WT | 5 | 9.559 | 11.257 | 13.031 |
| 1 | WT | 10 | 22.384 | 25.504 | 28.796 |
| 1 | WT | 20 | 18.866 | 20.662 | 22.473 |
| 1 | WT | 40 | 20.378 | 22.284 | 24.023 |
| 1 | WT | 80 | 25.675 | 27.393 | 29.289 |
| 1 | WT | 250 | 17.021 | 18.443 | 19.965 |
| 1 | mut | 5 | 32.504 | 35.358 | 38.254 |
| 1 | mut | 10 | 16.315 | 18.734 | 21.498 |
| 1 | mut | 40 | 23.342 | 25.416 | 27.553 |
| 1 | mut | 80 | 27.117 | 30.304 | 33.601 |
| 1 | mut | 250 | 17.047 | 18.166 | 19.282 |
| 2 | WT | 5 | 23.567 | 26.676 | 30.352 |
| 2 | WT | 10 | 22.676 | 24.647 | 28.696 |
| 2 | WT | 20 | 21.508 | 23.394 | 25.370 |
| 2 | WT | 40 | 25.727 | 27.715 | 29.846 |
| 2 |  | 80 | 32.172 | 34.216 | 35.902 |
| 2 | WT | 250 | 24.276 | 25.938 | 27.678 |
| 2 | mut | 5 | 26.707 | 29.680 | 32.950 |
| 2 | mut | 10 | 31.601 | 36.919 | 41.979 |
| 2 | mut | 40 | 41.755 | 44.750 | 48.189 |
| 2 | mut | 80 | 27.094 | 30.426 | 33.609 |
| 2 | mut | 250 | 22.821 | 24.260 | 25.754 |
| 3 | WT | 5 | 25.938 | 29.050 | 32.440 |
| 3 | WT | 10 | 32.783 | 32.859 | 38.434 |
| 3 | WT | 20 | 30.479 | 32.865 | 35.207 |
| 3 | WT | 40 | 30.875 | 32.961 | 35.374 |
| 3 | WT | 80 | 25.001 | 26.456 | 28.116 |
| 3 | WT | 250 | 20.897 | 21.763 | 23.000 |
| 3 | mut | 5 | 24.799 | 27.457 | 30.501 |
| 3 | mut | 10 | 42.772 | 47.404 | 51.766 |
| 3 | mut | 40 | 35.653 | 38.381 | 41.145 |
| 3 | mut | 80 | 31.024 | 34.699 | 38.285 |
| 3 | mut | 250 | 28.080 | 29.963 | 31.857 |
| 4 | WT | 5 | 18.060 | 20.483 | 23.184 |
| 4 | WT | 10 | 17.540 | 19.780 | 22.412 |
| 4 | WT | 20 | 23.540 | 23.855 | 24.425 |
| 4 | WT | 40 | 30.582 | 32.931 | 35.604 |
| 4 |  | 80 | 29.660 | 31.397 | 33.534 |
| 4 | WT | 250 | 23.480 | 25.104 | 26.733 |
| 4 | mut | 5 | 14.313 | 16.032 | 17.731 |
| 4 | mut | 10 | 23.044 | 26.306 | 30.166 |
|  | mut | 40 | 32.880 | 35.456 | 38.080 |
|  | mut | 80 | 28.650 | 31.858 | 35.336 |
|  | mut | 250 | 26.352 | 27.831 | 29.268 |

TABLE S8. $\mu_{\mathrm{x}}$ HBB

| k | SNP | tet | low.HPD | median | upp.HPD |
| :---: | :--- | :---: | :---: | :---: | :---: |
| 1 | WT | 5 | 105.402 | 116.589 | 128.100 |
| 1 | WT | 10 | 156.821 | 173.998 | 191.793 |
| 1 | WT | 20 | 292.726 | 311.989 | 329.181 |
| 1 | WT | 40 | 360.207 | 382.007 | 402.900 |
| 1 | WT | 80 | 398.889 | 414.140 | 429.588 |
| 1 | WT | 250 | 539.820 | 564.476 | 589.263 |
| 1 | mut | 5 | 39.239 | 42.300 | 45.212 |
| 1 | mut | 10 | 53.895 | 61.338 | 69.030 |
| 1 | mut | 40 | 85.582 | 90.517 | 95.846 |
| 1 | mut | 80 | 105.474 | 115.611 | 127.808 |
| 1 | mut | 250 | 157.627 | 163.197 | 168.546 |
| 2 | WT | 5 | 103.760 | 116.053 | 126.933 |
| 2 | WT | 10 | 161.211 | 184.961 | 190.736 |
| 2 | WT | 20 | 295.510 | 313.086 | 329.837 |
| 2 | WT | 40 | 362.581 | 384.120 | 404.709 |
| 2 | WT | 80 | 401.521 | 423.421 | 432.840 |
| 2 | WT | 250 | 542.177 | 565.898 | 589.598 |
| 2 | mut | 5 | 39.110 | 42.207 | 44.976 |
| 2 | mut | 10 | 56.058 | 64.338 | 72.119 |
| 2 | mut | 40 | 86.483 | 91.288 | 96.615 |
| 2 | mut | 80 | 104.644 | 115.765 | 127.515 |
| 2 | mut | 250 | 158.168 | 163.425 | 168.913 |
| 3 | WT | 5 | 105.803 | 116.504 | 127.453 |
| 3 | WT | 10 | 170.108 | 195.097 | 195.097 |
| 3 | WT | 20 | 297.477 | 314.601 | 331.874 |
| 3 | WT | 40 | 365.211 | 385.872 | 405.793 |
| 3 | WT | 80 | 399.257 | 413.887 | 429.090 |
| 3 | WT | 250 | 544.380 | 565.718 | 583.439 |
| 3 | mut | 5 | 39.047 | 41.876 | 44.904 |
| 3 | mut | 10 | 64.976 | 71.285 | 77.862 |
| 3 | mut | 40 | 86.882 | 91.723 | 96.974 |
| 3 | mut | 80 | 106.385 | 118.060 | 128.903 |
| 3 | mut | 250 | 157.720 | 163.555 | 168.630 |
| 4 | WT | 5 | 103.952 | 114.542 | 126.282 |
| 4 | Wut | 10 | 155.066 | 173.247 | 191.033 |
| 4 | mut | 80 | 1060 | 158.561 | 163.656 | 168.8629

TABLE S9. $\tilde{k}_{\text {on }}$ HBB gene

| k | SNP tet | low.HPD | median | upp.HPD |
| :---: | :---: | :---: | :---: | :---: |
| 1 | WT 5 | 0.001 | 0.001 | 0.005 |
| 1 | WT 10 | 0.002 | 0.003 | 0.011 |
| 1 | WT 20 | 0.002 | 0.003 | 0.012 |
| 1 | WT 40 | 0.003 | 0.004 | 0.016 |
| 1 | WT 80 | 0.003 | 0.005 | 0.016 |
| 1 | WT 250 | 0.002 | 0.003 | 0.011 |
| 1 | mut 5 | 0.012 | 0.042 | 0.072 |
| 1 | mut 10 | 0.005 | 0.018 | 0.031 |
| 1 | mut 40 | 0.005 | 0.019 | 0.032 |
| 1 | mut 80 | 0.006 | 0.023 | 0.038 |
| 1 | mut 250 | 0.007 | 0.026 | 0.042 |
| 2 | WT 5 | 0.002 | 0.003 | 0.011 |
| 2 | WT 10 | 0.002 | 0.003 | 0.012 |
| 2 | WT 20 | 0.003 | 0.004 | 0.014 |
| 2 | WT 40 | 0.003 | 0.004 | 0.015 |
| 2 | WT 80 | 0.003 | 0.005 | 0.016 |
| 2 | WT 250 | 0.003 | 0.004 | 0.013 |
| 2 | mut 5 | 0.003 | 0.011 | 0.018 |
| 2 | mut 10 | 0.004 | 0.014 | 0.024 |
| 2 | mut 40 | 0.005 | 0.022 | 0.037 |
| 2 | mut 80 | 0.006 | 0.019 | 0.032 |
| 2 | mut 250 | 0.006 | 0.024 | 0.039 |
| 3 | WT 5 | 0.003 | 0.004 | 0.013 |
| 3 | WT 10 | 0.002 | 0.003 | 0.012 |
| 3 | WT 20 | 0.004 | 0.006 | 0.021 |
| 3 | WT 40 | 0.004 | 0.006 | 0.021 |
| 3 | WT 80 | 0.005 | 0.007 | 0.025 |
| 3 | WT 250 | 0.006 | 0.008 | 0.031 |
| 3 | mut 5 | 0.003 | 0.013 | 0.021 |
| 3 | mut 10 | 0.007 | 0.026 | 0.042 |
| 3 | mut 40 | 0.007 | 0.026 | 0.044 |
| 3 | mut 80 | 0.006 | 0.021 | 0.035 |
| 3 | mut 250 | 0.005 | 0.018 | 0.031 |
| 4 | WT 5 | 0.002 | 0.003 | 0.011 |
| 4 | WT 10 | 0.002 | 0.004 | 0.013 |
| 4 | WT 20 | 0.003 | 0.003 | 0.012 |
| 4 | WT 40 | 0.003 | 0.004 | 0.014 |
| 4 | WT 80 | 0.004 | 0.005 | 0.019 |
| 4 | WT 250 | 0.004 | 0.006 | 0.020 |
| 4 | mut 5 | 0.003 | 0.013 | 0.021 |
| 4 | mut 10 | 0.005 | 0.018 | 0.031 |
| 4 | mut 40 | 0.006 | 0.024 | 0.040 |
| 4 | mut 80 | 0.009 | 0.034 | 0.057 |
| 4 | mut 250 | 0.010 | 0.038 | 0.063 |

TABLE S10. $\tilde{k}_{\text {off }}$ HBB gene

| k | SNP tet | low.HPD | median | upp.HPD |
| :---: | :---: | :---: | :---: | :---: |
| 1 | WT 5 | 0.125 | 0.765 | 3.017 |
| 1 | WT 10 | 0.072 | 0.380 | 1.567 |
| 1 | WT 20 | 0.012 | 0.039 | 0.154 |
| 1 | WT 40 | 0.011 | 0.024 | 0.094 |
| 1 | WT 80 | 0.016 | 0.040 | 0.161 |
| 1 | WT 250 | 0.009 | 0.015 | 0.056 |
| 1 | mut 5 | 0.073 | 1.278 | 4.126 |
| 1 | mut 10 | 0.050 | 0.832 | 3.991 |
| 1 | mut 40 | 0.013 | 0.130 | 0.474 |
| 1 | mut 80 | 0.031 | 0.375 | 2.323 |
| 1 | mut 250 | 0.042 | 0.622 | 3.073 |
| 2 | WT 5 | 0.083 | 0.468 | 1.868 |
| 2 | WT 10 | 0.004 | 0.103 | 0.843 |
| 2 | WT 20 | 0.011 | 0.052 | 0.334 |
| 2 | WT 40 | 0.007 | 0.013 | 0.048 |
| 2 | WT 80 | 0.010 | 0.016 | 0.057 |
| 2 | WT 250 | 0.005 | 0.008 | 0.031 |
| 2 | mut 5 | 0.024 | 0.496 | 2.425 |
| 2 | mut 10 | 0.006 | 0.077 | 0.442 |
| 2 | mut 40 | 0.011 | 0.050 | 0.095 |
| 2 | mut 80 | 0.010 | 0.040 | 0.073 |
| 2 | mut 250 | 0.012 | 0.127 | 0.909 |
| 3 | WT 5 | 0.050 | 0.399 | 1.601 |
| 3 | WT 10 | 0.013 | 0.030 | 0.343 |
| 3 | WT 20 | 0.011 | 0.035 | 0.156 |
| 3 | WT 40 | 0.010 | 0.022 | 0.104 |
| 3 | WT 80 | 0.013 | 0.055 | 0.230 |
| 3 | WT 250 | 0.027 | 0.097 | 0.495 |
| 3 | mut 5 | 0.036 | 0.807 | 3.286 |
| 3 | mut 10 | 0.033 | 0.440 | 2.258 |
| 3 | mut 40 | 0.013 | 0.091 | 0.224 |
| 3 | mut 80 | 0.011 | 0.044 | 0.084 |
| 3 | mut 250 | 0.014 | 0.067 | 0.137 |
| 4 | WT 5 | 0.180 | 0.813 | 3.140 |
| 4 | WT 10 | 0.095 | 0.443 | 1.732 |
| 4 | WT 20 | 0.013 | 0.019 | 0.101 |
| 4 | WT 40 | 0.009 | 0.019 | 0.072 |
| 4 | WT 80 | 0.020 | 0.068 | 0.265 |
| 4 | WT 250 | 0.019 | 0.060 | 0.254 |
| 4 | mut 5 | 0.122 | 1.271 | 3.884 |
| 4 | mut 10 | 0.072 | 1.031 | 3.537 |
| 4 | mut 40 | 0.020 | 0.149 | 0.768 |
| 4 | mut 80 | 0.049 | 0.431 | 1.596 |
| 4 | mut 250 | 0.037 | 0.844 | 3.754 |

TABLE S11. $\alpha / k_{\text {off }}$ HBB

| k | SNP tet | low.HPD | median | D |
| :---: | :---: | :---: | :---: | :---: |
| 1 | WT 5 | 180.624 | 213.178 | 250.384 |
| 1 | WT 10 | 125.805 | 147.598 | 168.798 |
| 1 | WT 20 | 216.330 | 261.287 | 314.761 |
| 1 | WT 40 | 212.117 | 270.985 | 329.730 |
| 1 | WT 80 | 214.432 | 257.880 | 304.762 |
| 1 | WT 250 | 477.217 | 564.684 | 648.593 |
| 1 | ut | 9.080 | 10.960 | 13.174 |
| 1 | ut 10 | 28.529 | 36.058 | 43.356 |
| 1 | ut 40 | 44.662 | 57.687 | 71.459 |
| 1 | ut 80 | 43.999 | 55.447 | 69.005 |
| 1 | mut 250 | 59.522 | 70.628 | 84.777 |
| 2 | WT | 82.844 | 96.710 | 111.647 |
| 2 | WT 10 | 118.521 | 142.328 | 186.000 |
| 2 | WT 20 | 180.061 | 222.460 | 269.372 |
| 2 | WT 40 | 256.229 | 310.385 | 368.848 |
| 2 | WT 80 | 238.431 | 309.659 | 327.796 |
| 2 | WT 250 | 481.478 | 561.546 | 649.773 |
| 2 | mut | 35.331 | 42.066 | 49.954 |
| 2 | mut 10 | 38.750 | 55.886 | 71.052 |
| 2 | mut 40 | 47.767 | 63.226 | 77.583 |
| 2 | mut 80 | 72.672 | 93.342 | 115.105 |
| 2 | mut 250 | 61.357 | 86.761 | 108.005 |
| 3 | WT | 69.549 | 81.039 | 94.196 |
| 3 | WT 10 | 114.861 | 170.317 | 170.356 |
| 3 | WT 20 | 124.441 | 164.157 | 206.865 |
| 3 | WT 40 | 153.175 | 216.801 | 264.885 |
| 3 | WT 80 | 141.653 | 180.708 | 226.265 |
| 3 | WT 250 | 149.771 | 189.476 | 213.603 |
| 3 | mut | 29.455 | 35.547 | 42.080 |
| 3 | mut 10 | 25.170 | 31.255 | 38.079 |
| 3 | mut 40 | 34.828 | 47.958 | 60.944 |
| 3 | mut 80 | 65.273 | 87.569 | 108.009 |
| 3 | mut 250 | 95.857 | 119.091 | 143.867 |
| 4 | WT 5 | 84.599 | 97.546 | 113.826 |
| 4 | WT 10 | 104.599 | 123.292 | 142.147 |
| 4 | WT 20 | 215.483 | 270.814 | 270.814 |
| 4 | WT 40 | 249.124 | 305.273 | 366.964 |
| 4 | WT 80 | 182.471 | 217.225 | 258.564 |
| 4 | WT 250 | 234.660 | 281.937 | 339.927 |
| 4 | mut | 28.552 | 34.146 | 40.265 |
| 4 | mut 10 | 30.453 | 37.481 | 44.636 |
| 4 | mut 40 | 34.310 | 46.529 | 58.219 |
| 4 | mut 80 | 30.063 | 38.528 | 47.509 |
| 4 | mut 250 | 40.276 | 47.439 | 56.269 |

TABLE S12. $\kappa$ HIV

| k SNP | tet | low.HPD | median | upp.HPD |
| :---: | :---: | :---: | :---: | :---: |
| 1 WT | 5 | 8.727 | 9.937 | 11.325 |
| 1 WT | 10 | 10.993 | 12.288 | 13.592 |
| 1 WT | 20 | 16.932 | 18.302 | 19.718 |
| 1 WT | 40 | 18.385 | 19.903 | 20.011 |
| 1 WT | 80 | 16.735 | 17.896 | 18.957 |
| 1 WT | 250 | 24.550 | 25.974 | 27.432 |
| 1 mut | 5 | 16.621 | 19.959 | 23.859 |
| 1 mut | 10 | 21.241 | 23.881 | 27.318 |
| 1 mut | 20 | 22.935 | 25.057 | 27.444 |
| 1 mut | 40 | 19.246 | 20.688 | 22.178 |
| 1 mut | 80 | 26.489 | 28.430 | 30.467 |
| 1 mut | 250 | 24.061 | 25.942 | 27.678 |
| 2 WT | 5 | 15.392 | 17.512 | 19.808 |
| 2 WT | 10 | 25.774 | 28.301 | 30.527 |
| 2 WT | 20 | 36.093 | 38.649 | 40.924 |
| 2 WT | 40 | 34.004 | 35.801 | 37.798 |
| 2 WT | 80 | 30.599 | 31.966 | 33.382 |
| 2 WT | 250 | 37.518 | 39.354 | 41.293 |
| 2 mut | 5 | 21.794 | 25.773 | 29.947 |
| 2 mut | 10 | 43.278 | 47.496 | 52.058 |
| 2 mut | 20 | 21.203 | 23.231 | 25.227 |
| 2 mut | 40 | 30.657 | 32.984 | 35.508 |
| 2 mut | 80 | 19.609 | 21.327 | 22.957 |
| 2 mut | 250 | 16.493 | 17.664 | 18.954 |
| 3 WT | 5 | 10.456 | 12.000 | 13.644 |
| 3 WT | 10 | 9.830 | 10.997 | 12.398 |
| 3 WT | 20 | 29.612 | 31.544 | 33.564 |
| 3 WT | 40 | 15.692 | 16.768 | 17.845 |
| 3 WT | 80 | 16.705 | 17.712 | 18.798 |
| 3 WT | 250 | 19.341 | 20.192 | 21.265 |
| 3 mut | 5 | 33.804 | 38.444 | 42.521 |
| 3 mut | 10 | 29.922 | 32.648 | 36.088 |
| 3 mut | 20 | 25.741 | 28.254 | 30.406 |
| 3 mut | 40 | 21.824 | 23.662 | 25.236 |
| 3 mut | 80 | 21.917 | 23.597 | 25.190 |
| 3 mut | 250 | 16.718 | 17.810 | 18.906 |
| 4 WT | 5 | 34.743 | 38.418 | 42.457 |
| 4 WT | 10 | 37.419 | 40.703 | 43.863 |
| 4 WT | 20 | 42.950 | 46.185 | 49.408 |
| 4 WT | 40 | 48.250 | 51.426 | 54.571 |
| 4 WT | 80 | 54.240 | 57.360 | 60.494 |
| 4 WT | 250 | 53.328 | 56.927 | 60.132 |
| 4 mut | 5 | 25.447 | 29.942 | 34.647 |
| 4 mut | 10 | 34.322 | 38.062 | 42.392 |
| 4 mut | 20 | 28.296 | 30.815 | 33.340 |
| 4 mut | 40 | 26.199 | 28.116 | 29.970 |
| 4 mut | 80 | 20.818 | 22.328 | 23.790 |
| 4 mut | 250 | 20.302 | 21.626 | 22.973 |

TABLE S13. $\mu_{\mathrm{x}}$ HIV gene

| k | SNP | tet | low.HPD | me | upp.HPD |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | WT | 5 | 67.005 | 72.838 | 79.536 |
| 1 | WT | 10 | 109.810 | 115.686 | 121.952 |
| 1 | WT | 20 | 144.574 | 151.166 | 157.431 |
| 1 | WT | 40 | 194.485 | 198.005 | 203.512 |
| 1 | WT | 80 | 226.244 | 232.415 | 238.770 |
| 1 | WT | 250 | 231.469 | 238.032 | 244.048 |
| 1 | ut | 5 | 31.686 | 37.672 | 43.353 |
| 1 | mut | 10 | 52.227 | 58.303 | 64.193 |
| 1 | ut | 20 | 89.312 | 95.185 | 101.282 |
| 1 | mut | 40 | 133.357 | 139.395 | 145.681 |
| 1 | mut | 80 | 156.590 | 162.956 | 169.112 |
| 1 | mut | 250 | 180.919 | 187.275 | 193.192 |
| 2 | WT | 5 | 66.834 | 73.001 | 78.969 |
| 2 | WT | 10 | 110.155 | 116.385 | 122.115 |
| 2 | WT | 20 | 146.758 | 152.367 | 157.963 |
| 2 | WT | 40 | 193.705 | 200.206 | 205.751 |
| 2 | WT | 80 | 226.944 | 232.737 | 238.957 |
| 2 | WT | 250 | 233.022 | 238.787 | 244.544 |
| 2 | mut | 5 | 33.431 | 38.818 | 44.292 |
| 2 | ut | 10 | 58.085 | 63.467 | 68.566 |
| 2 | mut | 20 | 88.921 | 95.219 | 101.320 |
| 2 | mut | 40 | 133.678 | 140.521 | 146.283 |
| 2 | mut | 80 | 155.778 | 162.631 | 168.680 |
| 2 | mut | 250 | 180.361 | 186.879 | 193.168 |
| 3 | WT | 5 | 67.657 | 73.147 | 79.668 |
| 3 | WT | 10 | 110.114 | 115.655 | 121.652 |
| 3 | WT | 20 | 145.713 | 152.053 | 157.986 |
| 3 | WT | 40 | 192.742 | 199.274 | 205.308 |
| 3 | WT | 80 | 226.620 | 233.022 | 238.193 |
| 3 | WT | 250 | 232.351 | 238.150 | 244.483 |
| 3 | mut | 5 | 36.811 | 40.994 | 45.435 |
| 3 | mut | 10 | 55.191 | 60.551 | 64.555 |
| 3 | mut | 20 | 89.503 | 95.670 | 101.158 |
| 3 | ut | 40 | 133.490 | 139.695 | 145.130 |
| 3 | mut | 80 | 156.822 | 162.594 | 169.538 |
| 3 | mut | 250 | 180.569 | 186.922 | 192.906 |
| 4 | WT | 5 | 69.528 | 75.142 | 81.061 |
| 4 | WT | 10 | 111.093 | 117.156 | 122.570 |
| 4 | WT | 20 | 146.982 | 153.429 | 159.127 |
| 4 | WT | 40 | 195.212 | 201.345 | 207.570 |
| 4 | WT | 80 | 229.161 | 235.061 | 241.236 |
| 4 |  | 250 | 234.182 | 240.462 | 246.342 |
| 4 | mut | 5 | 34.228 | 39.432 | 44.903 |
| 4 | mut | 10 | 54.750 | 60.567 | 66.025 |
| 4 | mut | 20 | 89.372 | 95.749 | 101.674 |
| 4 | mut | 40 | 133.614 | 139.938 | 146.017 |
| 4 | mut | 80 | 156.317 | 162.691 | 168.913 |
| 4 | mut | 250 | 180.968 | 186.969 | 193.373 |

TABLE S14. $\tilde{k}_{\text {on }}$ HIV gene

| k | SNP tet | low.HPD | median | upp.HPD |
| :---: | :---: | :---: | :---: | :---: |
| 1 | WT 5 | 0.003 | 0.004 | 0.005 |
| 1 | WT 10 | 0.003 | 0.004 | 0.005 |
| 1 | WT 20 | 0.006 | 0.009 | 0.011 |
| 1 | WT 40 | 0.009 | 0.012 | 0.016 |
| 1 | WT 80 | 0.006 | 0.008 | 0.010 |
| 1 | WT 250 | 0.009 | 0.012 | 0.015 |
| 1 | mut 5 | 0.002 | 0.003 | 0.004 |
| 1 | mut 10 | 0.003 | 0.004 | 0.005 |
| 1 | mut 20 | 0.004 | 0.005 | 0.007 |
| 1 | mut 40 | 0.007 | 0.009 | 0.011 |
| 1 | mut 80 | 0.004 | 0.006 | 0.007 |
| 1 | mut 250 | 0.004 | 0.005 | 0.007 |
| 2 | WT 5 | 0.002 | 0.003 | 0.004 |
| 2 | WT 10 | 0.004 | 0.005 | 0.007 |
| 2 | WT 20 | 0.006 | 0.008 | 0.011 |
| 2 | WT 40 | 0.008 | 0.012 | 0.015 |
| 2 | WT 80 | 0.013 | 0.018 | 0.023 |
| 2 | WT 250 | 0.010 | 0.014 | 0.019 |
| 2 | mut 5 | 0.002 | 0.003 | 0.004 |
| 2 | mut 10 | 0.004 | 0.005 | 0.006 |
| 2 | mut 20 | 0.004 | 0.005 | 0.006 |
| 2 | mut 40 | 0.004 | 0.005 | 0.007 |
| 2 | mut 80 | 0.003 | 0.004 | 0.005 |
| 2 | mut 250 | 0.004 | 0.005 | 0.006 |
| 3 | WT 5 | 0.002 | 0.003 | 0.004 |
| 3 | WT 10 | 0.002 | 0.003 | 0.004 |
| 3 | WT 20 | 0.009 | 0.012 | 0.015 |
| 3 | WT 40 | 0.007 | 0.010 | 0.012 |
| 3 | WT 80 | 0.008 | 0.011 | 0.014 |
| 3 | WT 250 | 0.008 | 0.011 | 0.014 |
| 3 | mut 5 | 0.003 | 0.004 | 0.005 |
| 3 | mut 10 | 0.003 | 0.004 | 0.005 |
| 3 | mut 20 | 0.005 | 0.006 | 0.008 |
| 3 | mut 40 | 0.005 | 0.006 | 0.008 |
| 3 | mut 80 | 0.004 | 0.005 | 0.007 |
| 3 | mut 250 | 0.006 | 0.008 | 0.010 |
| 4 | WT 5 | 0.004 | 0.006 | 0.007 |
| 4 | WT 10 | 0.004 | 0.006 | 0.007 |
| 4 | WT 20 | 0.004 | 0.006 | 0.008 |
| 4 | WT 40 | 0.005 | 0.007 | 0.009 |
| 4 | WT 80 | 0.006 | 0.008 | 0.011 |
| 4 | WT 250 | 0.004 | 0.006 | 0.008 |
| 4 | mut 5 | 0.003 | 0.003 | 0.004 |
| 4 | mut 10 | 0.004 | 0.005 | 0.006 |
| 4 | mut 20 | 0.005 | 0.006 | 0.008 |
| 4 | mut 40 | 0.005 | 0.007 | 0.009 |
| 4 | mut 80 | 0.004 | 0.005 | 0.007 |
| 4 | mut 250 | 0.006 | 0.008 | 0.010 |

TABLE S15. $\tilde{k}_{\text {off }}$ HIV gene

| k | SNP | tet | low.HPD | median | upp.HPD |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | WT | 5 | 0.144 | 0.881 | 2.155 |
| 1 | WT | 10 | 0.084 | 0.562 | 1.685 |
| 1 | WT | 20 | 0.229 | 0.744 | 1.857 |
| 1 | WT | 40 | 0.038 | 0.078 | 0.654 |
| 1 | WT | 80 | 0.015 | 0.031 | 0.060 |
| 1 | WT | 250 | 0.135 | 0.590 | 1.298 |
| 1 | mut | 5 | 0.073 | 0.454 | 1.354 |
| 1 | mut | 10 | 0.051 | 0.368 | 1.166 |
| 1 | mut | 20 | 0.018 | 0.140 | 0.616 |
| 1 | mut | 40 | 0.123 | 0.478 | 1.361 |
| 1 | mut | 80 | 0.014 | 0.024 | 0.042 |
| 1 | mut | 250 | 0.016 | 0.044 | 0.189 |
| 2 | WT | 5 | 0.103 | 0.570 | 1.574 |
| 2 | WT | 10 | 0.052 | 0.401 | 1.406 |
| 2 | WT | 20 | 0.023 | 0.151 | 0.667 |
| 2 | WT | 40 | 0.045 | 0.191 | 0.656 |
| 2 | WT | 80 | 0.047 | 0.204 | 0.605 |
| 2 | WT | 250 | 0.023 | 0.097 | 0.353 |
| 2 | mut | 5 | 0.022 | 0.201 | 0.769 |
| 2 | mut | 10 | 0.013 | 0.037 | 0.169 |
| 2 | mut | 20 | 0.007 | 0.011 | 0.017 |
| 2 | mut | 40 | 0.012 | 0.019 | 0.031 |
| 2 | mut | 80 | 0.009 | 0.014 | 0.021 |
| 2 | mut | 250 | 0.012 | 0.020 | 0.031 |
| 3 | WT | 5 | 0.068 | 0.611 | 2.029 |
| 3 | WT | 10 | 0.038 | 0.556 | 1.761 |
| 3 | WT | 20 | 0.096 | 0.582 | 1.690 |
| 3 | WT | 40 | 0.088 | 0.348 | 0.988 |
| 3 | WT | 80 | 0.037 | 0.374 | 1.164 |
| 3 | WT | 250 | 0.059 | 0.117 | 0.226 |
| 3 | mut | 5 | 0.015 | 0.120 | 0.623 |
| 3 | mut | 10 | 0.011 | 0.030 | 0.233 |
| 3 | mut | 20 | 0.015 | 0.088 | 0.720 |
| 3 | mut | 40 | 0.024 | 0.146 | 0.626 |
| 3 | mut | 80 | 0.010 | 0.017 | 0.026 |
| 3 | mut | 250 | 0.016 | 0.036 | 0.122 |
| 4 | WT | 5 | 0.103 | 0.649 | 1.825 |
| 4 | WT | 10 | 0.020 | 0.195 | 0.755 |
| 4 | WT | 20 | 0.014 | 0.038 | 0.156 |
| 4 | WT | 40 | 0.011 | 0.017 | 0.026 |
| 4 | WT | 80 | 0.017 | 0.036 | 0.091 |
| 4 | WT | 250 | 0.010 | 0.015 | 0.021 |
| 4 | mut | 5 | 0.037 | 0.288 | 0.992 |
| 4 | mut | 10 | 0.010 | 0.020 | 0.041 |
| 4 | mut | 20 | 0.011 | 0.020 | 0.035 |
| 4 | mut | 40 | 0.013 | 0.024 | 0.042 |
| 4 | mut | 80 | 0.007 | 0.010 | 0.014 |
| 4 | mut | 250 | 0.017 | 0.037 | 0.078 |

TABLE S16. $\alpha / k_{\text {off }}$ HIV

| k | SNP tet | low.HPD | media | upp.HPD |
| :---: | :---: | :---: | :---: | :---: |
| 1 | WT 5 | 67.863 | 83.165 | 98.742 |
| 1 | WT 10 | 110.233 | 128.710 | 148.311 |
| 1 | WT 20 | 66.024 | 75.388 | 86.230 |
| 1 | WT 40 | 62.055 | 86.423 | 86.468 |
| 1 | WT 80 | 125.449 | 163.587 | 197.794 |
| 1 | WT 250 | 78.058 | 89.165 | 100.338 |
| 1 | mut | 31.371 | 40.575 | 50.588 |
| 1 | mut 10 | 39.406 | 48.621 | 58.372 |
| 1 | mut 20 | 53.780 | 65.401 | 80.227 |
| 1 | mut 40 | 50.390 | 58.418 | 66.771 |
| 1 | mut 80 | 102.514 | 128.589 | 150.897 |
| 1 | mut 250 | 107.239 | 136.430 | 165.491 |
| 2 | WT 5 | 78.383 | 92.406 | 106.888 |
| 2 | WT 10 | 85.438 | 97.416 | 111.716 |
| 2 | WT 20 | 69.807 | 83.687 | 101.449 |
| 2 | WT 40 | 65.879 | 78.702 | 95.247 |
| 2 | WT 80 | 49.412 | 61.013 | 75.818 |
| 2 | WT 250 | 65.047 | 84.639 | 112.098 |
| 2 | mut 5 | 34.809 | 44.291 | 54.536 |
| 2 | mut 10 | 38.469 | 51.197 | 64.148 |
| 2 | mut 20 | 82.526 | 102.341 | 124.937 |
| 2 | mut 40 | 98.967 | 120.615 | 143.267 |
| 2 | mut 80 | 152.628 | 182.466 | 213.368 |
| 2 | mut 250 | 135.805 | 164.695 | 193.826 |
| 3 | WT 5 | 95.847 | 112.554 | 131.718 |
| 3 | WT 10 | 152.746 | 177.896 | 201.429 |
| 3 | WT 20 | 50.485 | 58.166 | 66.866 |
| 3 | WT 40 | 80.785 | 93.509 | 106.082 |
| 3 | WT 80 | 87.801 | 99.460 | 133.676 |
| 3 | WT 250 | 88.618 | 103.986 | 120.259 |
| 3 | mut 5 | 28.936 | 36.907 | 43.947 |
| 3 | mut 10 | 46.523 | 63.475 | 79.732 |
| 3 | mut 20 | 44.259 | 56.383 | 76.276 |
| 3 | mut 40 | 67.822 | 80.040 | 95.357 |
| 3 | mut 80 | 114.493 | 141.460 | 169.534 |
| 3 | mut 250 | 77.043 | 104.075 | 130.875 |
| 4 | WT 5 | 50.970 | 59.826 | 68.217 |
| 4 | WT 10 | 77.752 | 95.427 | 118.147 |
| 4 | WT 20 | 99.153 | 128.960 | 158.025 |
| 4 | WT 40 | 147.939 | 179.362 | 211.985 |
| 4 | WT 80 | 113.452 | 149.386 | 180.708 |
| 4 | WT 250 | 205.845 | 243.531 | 280.996 |
| 4 | mut 5 | 33.166 | 41.173 | 49.774 |
| 4 | mut 10 | 40.803 | 53.581 | 68.012 |
| 4 | mut 20 | 53.655 | 70.015 | 87.576 |
| 4 | mut 40 | 71.864 | 90.996 | 110.250 |
| 4 | mut 80 | 132.897 | 162.495 | 192.626 |
| 4 | mut 250 | 82.023 | 104.932 | 129.862 |

## B. Negative binomial distribution

In contrast to the Poisson-beta model, the negative-binomial model directly encodes the ratio $\alpha / k_{\text {off }}$ as a single parameter, which is inferred with rather narrow credible intervals. Parsimony suggests that the negative-binomial model is a reasonable choice for the genes considered here, as it encodes for the most relevant kinetic parameters, viz., the average burst size $\alpha / k_{\text {off }}$ and the burst frequency $k_{\text {on }}$. Fig. S12 shows the parameters estimated from the negative-binomial model. The traces of the posteriors chains from each replicates were combined according to the consensus Monte Carlo procedure (see section S 4 B ) to obtain a representation of the consensus belief in Fig. S13.

The $90 \%$ HPD CIs and medians of the estimated parameters $\kappa, \mu_{X}, k_{\text {on }}$, and $\alpha / k_{\text {off }}$ are reported in Tables S17-S20 (HBB gene) and Tables S21-S24 (HIV gene).

TABLE S17. $\kappa$ HBB

| k | SNP | tet | low.HPD | median | upp.HPD |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | WT | 5 | 9.675 | 11.307 | 13.213 |
| 1 | WT | 10 | 22.041 | 25.232 | 28.377 |
| 1 | WT | 20 | 18.950 | 20.813 | 22.843 |
| 1 | WT | 40 | 20.381 | 22.383 | 24.447 |
| 1 | WT | 80 | 25.673 | 27.661 | 29.771 |
| 1 | WT | 250 | 17.300 | 18.830 | 20.649 |
| 1 | mut | 5 | 32.550 | 35.329 | 38.490 |
| 1 | mut | 10 | 16.072 | 18.688 | 21.615 |
| 1 | mut | 40 | 23.375 | 25.503 | 27.566 |
| 1 | mut | 80 | 27.251 | 30.309 | 34.188 |
| 1 | mut | 250 | 16.967 | 18.178 | 19.357 |
| 2 | WT | 5 | 23.578 | 26.572 | 29.961 |
| 2 | WT | 10 | 23.453 | 26.572 | 29.997 |
| 2 | WT | 20 | 21.562 | 23.610 | 25.909 |
| 2 | WT | 40 | 25.687 | 28.089 | 30.456 |
| 2 | WT | 80 | 31.358 | 33.931 | 36.457 |
| 2 | WT | 250 | 24.861 | 26.446 | 28.603 |
| 2 | mut | 5 | 26.626 | 29.697 | 33.019 |
| 2 | mut | 10 | 31.988 | 36.180 | 41.474 |
| 2 | mut | 40 | 40.925 | 44.272 | 47.412 |
| 2 | mut | 80 | 27.407 | 30.832 | 34.251 |
| 2 | mut | 250 | 22.856 | 24.409 | 26.037 |
| 3 | WT | 5 | 25.676 | 29.020 | 32.801 |
| 3 | WT | 10 | 32.162 | 35.936 | 39.914 |
| 3 | WT | 20 | 30.366 | 32.843 | 35.501 |
| 3 | WT | 40 | 30.841 | 33.200 | 35.696 |
| 3 | WT | 80 | 24.637 | 26.419 | 28.015 |
| 3 | WT | 250 | 20.441 | 21.780 | 23.367 |
| 3 | mut | 5 | 24.569 | 27.336 | 30.290 |
| 3 | mut | 10 | 42.637 | 47.605 | 52.771 |
| 3 | mut | 40 | 35.782 | 38.444 | 41.379 |
| 3 | mut | 80 | 31.078 | 35.045 | 38.636 |
| 3 | mut | 250 | 28.041 | 30.084 | 32.136 |
| 4 | WT | 5 | 17.636 | 20.198 | 23.004 |
| 4 | WT | 10 | 17.386 | 19.747 | 22.473 |
| 4 | WT | 20 | 21.240 | 23.109 | 25.149 |
| 4 | WT | 40 | 29.894 | 32.568 | 35.174 |
| 4 | WT | 80 | 29.411 | 31.375 | 33.856 |
| 4 | WT | 250 | 23.425 | 25.036 | 26.811 |
| 4 | mut | 5 | 14.236 | 15.954 | 17.716 |
| 4 | mut | 10 | 22.721 | 26.243 | 30.138 |
| 4 | mut | 40 | 32.983 | 35.546 | 38.257 |
| 4 | mut | 80 | 28.575 | 31.849 | 35.621 |
| 4 | mut | 250 | 26.358 | 27.852 | 29.440 |



FIG. S12. Estimates for parameters $\tilde{k}_{\text {on }}, \mu_{X}$, and $\alpha / k_{\text {off }}$ of the negative-binomial model, from wild-type (blue) and mutant (orange) cell data, for all induction levels, shades of colors corresponds to replicates. Points are medians, error bars comprise $90 \%$ HPD CIs. HBB-gene results show consistent results across all the replicates (panel A). The HIV-gene results are reported in panels B. Results are consistent with the Poisson-beta model estimates (Fig. S10). Increasing expression levels, three replicates show a drop-off in the average burst size and an increase in the burst frequency, see also Fig. S2.


FIG. S13. Consensus estimates of the parameters $\tilde{k}_{\text {on }}, \mu_{X}$, and $\alpha / k_{\text {off }}$ from the negative-binomial model, from wild-type (blue) and mutant (orange) cell data, for all induction levels. Points are medians, error bars comprise $90 \%$ HPD CIs. HBB-gene results are in panel A, HIV-gene results are reported in panel B.

TABLE S18. $\mu_{\mathrm{X}} \mathrm{HBB}$

| k | SNP tet | low.HPD | median | upp.HPD |
| :---: | :---: | :---: | :---: | :---: |
| 1 | WT 5 | 103.193 | 115.687 | 126.875 |
| 1 | WT 10 | 157.904 | 175.269 | 194.327 |
| 1 | WT 20 | 293.749 | 311.584 | 331.666 |
| 1 | WT 40 | 357.077 | 382.068 | 402.882 |
| 1 | WT 80 | 397.085 | 413.625 | 430.396 |
| 1 | WT 250 | 537.710 | 562.996 | 589.504 |
| 1 | mut | 39.255 | 42.325 | 45.219 |
| 1 | mut 10 | 53.368 | 61.634 | 69.735 |
| 1 | mut 40 | 85.470 | 90.459 | 95.633 |
| 1 | mut 80 | 103.857 | 115.778 | 127.620 |
| 1 | mut 250 | 157.575 | 163.136 | 168.842 |
| 2 | WT | 104.884 | 115.671 | 127.339 |
| 2 | WT 10 | 156.119 | 174.189 | 192.233 |
| 2 | WT 20 | 292.999 | 311.784 | 331.770 |
| 2 | WT 40 | 360.835 | 382.742 | 404.849 |
| 2 | WT 80 | 397.159 | 414.805 | 430.893 |
| 2 | WT 250 | 542.191 | 564.530 | 590.432 |
| 2 | mut 5 | 39.031 | 42.105 | 45.158 |
| 2 | mut 10 | 57.835 | 65.531 | 72.791 |
| 2 | mut 40 | 86.987 | 92.086 | 96.961 |
| 2 | mut 80 | 103.723 | 115.390 | 127.323 |
| 2 | mut 250 | 157.508 | 163.312 | 168.777 |
| 3 | WT | 104.159 | 116.177 | 127.657 |
| 3 | WT 10 | 163.259 | 179.811 | 197.030 |
| 3 | WT 20 | 296.072 | 314.846 | 332.531 |
| 3 | WT 40 | 363.753 | 384.298 | 406.525 |
| 3 | WT 80 | 397.951 | 414.531 | 430.464 |
| 3 | WT 250 | 538.464 | 564.609 | 588.360 |
| 3 | mut 5 | 38.955 | 41.838 | 44.995 |
| 3 | mut 10 | 63.927 | 70.558 | 77.724 |
| 3 | mut 40 | 86.541 | 91.645 | 96.597 |
| 3 | mut 80 | 105.958 | 117.686 | 129.102 |
| 3 | mut 250 | 157.977 | 163.500 | 169.119 |
| 4 | WT 5 | 103.541 | 114.909 | 127.623 |
| 4 | WT 10 | 153.130 | 172.692 | 190.326 |
| 4 | WT 20 | 292.635 | 312.606 | 330.812 |
| 4 | WT 40 | 365.063 | 386.377 | 407.528 |
| 4 | WT 80 | 399.614 | 415.485 | 432.185 |
| 4 | WT 250 | 540.558 | 564.417 | 589.218 |
| 4 | mut 5 | 38.556 | 41.557 | 44.534 |
| 4 | mut 10 | 55.032 | 63.112 | 71.160 |
| 4 | mut 40 | 85.960 | 91.252 | 96.281 |
| 4 | mut 80 | 104.157 | 116.484 | 127.851 |
| 4 | mut 250 | 157.935 | 163.479 | 169.027 |

TABLE S19. $\tilde{k}_{\text {on }}$ HBB

| k | SNP tet | low.HPD | median | upp.HPD |
| :---: | :---: | :---: | :---: | :---: |
| 1 | WT 5 | 0.001 | 0.001 | 0.005 |
| 1 | WT 10 | 0.002 | 0.003 | 0.011 |
| 1 | WT 20 | 0.003 | 0.004 | 0.013 |
| 1 | WT 40 | 0.004 | 0.005 | 0.018 |
| 1 | WT 80 | 0.004 | 0.005 | 0.019 |
| 1 | WT 250 | 0.002 | 0.003 | 0.013 |
| 1 | mut 5 | 0.012 | 0.045 | 0.074 |
| 1 | mut 10 | 0.005 | 0.019 | 0.032 |
| 1 | mut 40 | 0.006 | 0.021 | 0.035 |
| 1 | mut 80 | 0.007 | 0.025 | 0.041 |
| 1 | mut 250 | 0.007 | 0.027 | 0.044 |
| 2 | WT 5 | 0.002 | 0.003 | 0.011 |
| 2 | WT 10 | 0.003 | 0.004 | 0.013 |
| 2 | WT 20 | 0.003 | 0.004 | 0.015 |
| 2 | WT 40 | 0.004 | 0.005 | 0.019 |
| 2 | WT 80 | 0.004 | 0.006 | 0.022 |
| 2 | WT 250 | 0.003 | 0.005 | 0.018 |
| 2 | mut 5 | 0.003 | 0.011 | 0.019 |
| 2 | mut 10 | 0.005 | 0.017 | 0.027 |
| 2 | mut 40 | 0.008 | 0.031 | 0.051 |
| 2 | mut 80 | 0.007 | 0.028 | 0.045 |
| 2 | mut 250 | 0.007 | 0.028 | 0.046 |
| 3 | WT 5 | 0.003 | 0.004 | 0.014 |
| 3 | WT 10 | 0.003 | 0.004 | 0.015 |
| 3 | WT 20 | 0.005 | 0.007 | 0.025 |
| 3 | WT 40 | 0.005 | 0.007 | 0.026 |
| 3 | WT 80 | 0.005 | 0.008 | 0.027 |
| 3 | WT 250 | 0.006 | 0.009 | 0.034 |
| 3 | mut 5 | 0.003 | 0.013 | 0.021 |
| 3 | mut 10 | 0.008 | 0.027 | 0.045 |
| 3 | mut 40 | 0.009 | 0.033 | 0.054 |
| 3 | mut 80 | 0.009 | 0.030 | 0.050 |
| 3 | mut 250 | 0.006 | 0.023 | 0.038 |
| 4 | WT 5 | 0.002 | 0.003 | 0.011 |
| 4 | WT 10 | 0.003 | 0.004 | 0.013 |
| 4 | WT 20 | 0.003 | 0.004 | 0.015 |
| 4 | WT 40 | 0.003 | 0.005 | 0.017 |
| 4 | WT 80 | 0.004 | 0.006 | 0.021 |
| 4 | WT 250 | 0.004 | 0.006 | 0.022 |
| 4 | mut 5 | 0.003 | 0.013 | 0.023 |
| 4 | mut 10 | 0.005 | 0.019 | 0.031 |
| 4 | mut 40 | 0.008 | 0.028 | 0.045 |
| 4 | mut 80 | 0.010 | 0.037 | 0.063 |
| 4 | mut 250 | 0.011 | 0.041 | 0.065 |

TABLE S20. $\alpha / k_{\text {on }}$ HBB

| k | SNP tet | low.HPD | median | upp.HPD |
| :---: | :---: | :---: | :---: | :---: |
| 1 | WT | 175.298 | 206.600 | 246.085 |
| 1 | WT 10 | 120.820 | 144.691 | 167.516 |
| 1 | WT 20 | 190.560 | 217.633 | 249.246 |
| 1 | WT 40 | 170.125 | 194.471 | 222.404 |
| 1 | WT 80 | 181.291 | 206.076 | 232.972 |
| 1 | WT 250 | 354.322 | 406.867 | 459.164 |
| 1 | mut | 8.579 | 10.098 | 12.020 |
| 1 | mut 10 | 26.797 | 33.634 | 40.524 |
| 1 | mut 40 | 38.121 | 44.326 | 50.962 |
| 1 | mut 80 | 41.023 | 48.685 | 57.037 |
| 1 | mut 250 | 56.503 | 64.628 | 73.724 |
| 2 | WT | 79.762 | 93.656 | 108.321 |
| 2 | WT 10 | 104.729 | 123.788 | 143.569 |
| 2 | WT 20 | 161.758 | 186.721 | 211.985 |
| 2 | WT 40 | 160.712 | 183.674 | 208.313 |
| 2 | WT 80 | 156.316 | 177.567 | 201.871 |
| 2 | WT 250 | 258.811 | 295.198 | 334.765 |
| 2 | mut | 33.600 | 39.522 | 45.973 |
| 2 | mut 10 | 34.079 | 40.868 | 48.606 |
| 2 | mut 40 | 27.354 | 31.636 | 36.378 |
| 2 | mut 80 | 36.745 | 44.057 | 51.733 |
| 2 | mut 250 | 54.699 | 62.845 | 71.400 |
| 3 | WT | 66.198 | 78.414 | 90.605 |
| 3 | WT 10 | 97.887 | 114.499 | 133.520 |
| 3 | WT 20 | 102.297 | 117.574 | 133.503 |
| 3 | WT 40 | 117.655 | 135.267 | 152.972 |
| 3 | WT 80 | 120.396 | 137.453 | 155.178 |
| 3 | WT 250 | 134.325 | 154.819 | 174.205 |
| 3 | mut 5 | 28.241 | 33.741 | 39.190 |
| 3 | mut 10 | 22.808 | 27.192 | 31.662 |
| 3 | mut 40 | 25.141 | 29.293 | 33.804 |
| 3 | mut 80 | 34.292 | 41.021 | 47.862 |
| 3 | mut 250 | 66.549 | 75.912 | 86.434 |
| 4 | WT 5 | 81.257 | 95.302 | 110.891 |
| 4 | WT 10 | 100.651 | 119.384 | 138.147 |
| 4 | WT 20 | 169.562 | 193.318 | 219.519 |
| 4 | WT 40 | 187.460 | 214.568 | 244.764 |
| 4 | WT 80 | 160.411 | 183.310 | 206.170 |
| 4 | WT 250 | 204.957 | 232.671 | 264.311 |
| 4 | mut | 27.219 | 32.786 | 38.579 |
| 4 | mut 10 | 29.960 | 35.848 | 43.176 |
| 4 | mut 40 | 30.133 | 34.940 | 39.916 |
| 4 | mut 80 | 27.685 | 33.007 | 38.815 |
| 4 | mut 250 | 37.564 | 42.970 | 49.161 |

TABLE S21. $\kappa$ HIV

| k | SNP tet | low.HPD | median | pp.HPD |
| :---: | :---: | :---: | :---: | :---: |
| 1 | WT 5 | 8.683 | 9.925 | 11.355 |
| 1 | WT 10 | 10.830 | 12.201 | 13.502 |
| 1 | WT 20 | 16.946 | 18.291 | 19.696 |
| 1 | WT 40 | 17.995 | 19.136 | 20.300 |
| 1 | WT 80 | 16.814 | 17.998 | 19.187 |
| 1 | WT 250 | 24.456 | 25.898 | 27.413 |
| 1 | mut 5 | 16.450 | 19.977 | 23.853 |
| 1 | mut 10 | 20.826 | 23.864 | 27.041 |
| 1 | mut 20 | 22.626 | 24.912 | 27.281 |
| 1 | mut 40 | 19.162 | 20.677 | 22.160 |
| 1 | mut 80 | 26.548 | 28.642 | 30.929 |
| 1 | mut 250 | 24.056 | 26.022 | 27.857 |
| 2 | WT 5 | 15.406 | 17.501 | 19.880 |
| 2 | WT 10 | 25.483 | 28.210 | 30.905 |
| 2 | WT 20 | 36.121 | 38.563 | 41.320 |
| 2 | WT 40 | 33.654 | 35.716 | 37.949 |
| 2 | WT 80 | 30.499 | 32.001 | 33.633 |
| 2 | WT 250 | 37.389 | 39.279 | 41.400 |
| 2 | mut 5 | 21.886 | 25.819 | 30.735 |
| 2 | mut 10 | 42.808 | 47.476 | 52.389 |
| 2 | mut 20 | 21.292 | 23.474 | 25.662 |
| 2 | mut 40 | 30.748 | 33.257 | 35.874 |
| 2 | mut 80 | 19.858 | 21.585 | 23.433 |
| 2 | mut 250 | 16.597 | 17.858 | 19.331 |
| 3 | WT 5 | 10.305 | 11.911 | 13.704 |
| 3 | WT 10 | 9.759 | 10.994 | 12.394 |
| 3 | WT 20 | 15.478 | 16.882 | 18.327 |
| 3 | WT 40 | 15.642 | 16.744 | 17.818 |
| 3 | WT 80 | 16.849 | 17.904 | 19.029 |
| 3 | WT 250 | 19.059 | 20.252 | 21.472 |
| 3 | mut 5 | 33.159 | 38.139 | 43.447 |
| 3 | mut 10 | 28.905 | 32.666 | 36.895 |
| 3 | mut 20 | 25.677 | 28.163 | 30.685 |
| 3 | mut 40 | 21.877 | 23.641 | 25.428 |
| 3 | mut 80 | 22.105 | 23.864 | 25.581 |
| 3 | mut 250 | 16.736 | 17.863 | 19.027 |
| 4 | WT 5 | 34.330 | 38.261 | 42.130 |
| 4 | WT 10 | 37.272 | 40.394 | 44.002 |
| 4 | WT 20 | 43.020 | 46.210 | 50.115 |
| 4 | WT 40 | 48.515 | 51.534 | 54.970 |
| 4 | WT 80 | 54.224 | 57.441 | 60.923 |
| 4 | WT 250 | 54.376 | 57.496 | 61.232 |
| 4 | mut 5 | 25.398 | 29.929 | 34.621 |
| 4 | mut 10 | 34.261 | 38.357 | 42.825 |
| 4 | mut 20 | 28.323 | 31.007 | 33.655 |
| 4 | mut 40 | 26.348 | 28.262 | 30.242 |
| 4 | mut 80 | 21.116 | 22.690 | 24.363 |
| 4 | mut 250 | 20.337 | 21.627 | 23.114 |

TABLE S22. $\mu_{\mathrm{X}}$ HIV

| k | SNP tet | low.HPD | median | upp.HPD |
| :---: | :---: | :---: | :---: | :---: |
| 1 | WT | 66.749 | 72.714 | 79.573 |
| 1 | WT 10 | 109.215 | 115.559 | 121.995 |
| 1 | WT 20 | 144.952 | 151.119 | 157.583 |
| 1 | WT 40 | 192.656 | 199.155 | 205.249 |
| 1 | WT 80 | 225.999 | 232.337 | 238.487 |
| 1 | WT 250 | 232.122 | 238.168 | 244.560 |
| 1 | mut | 31.397 | 37.570 | 43.353 |
| 1 | mut 10 | 52.305 | 58.144 | 64.224 |
| 1 | mut 20 | 89.505 | 95.336 | 101.545 |
| 1 | mut 40 | 133.432 | 139.663 | 146.087 |
| 1 | mut 80 | 156.547 | 162.818 | 169.259 |
| 1 | mut 250 | 180.854 | 187.190 | 193.263 |
| 2 | WT | 66.749 | 72.966 | 79.146 |
| 2 | WT 10 | 109.748 | 115.981 | 122.719 |
| 2 | WT 20 | 145.901 | 152.450 | 158.287 |
| 2 | WT 40 | 193.656 | 199.980 | 206.311 |
| 2 | WT 80 | 226.495 | 232.703 | 239.197 |
| 2 | WT 250 | 232.607 | 238.913 | 244.831 |
| 2 | mut 5 | 32.501 | 38.487 | 44.321 |
| 2 | mut 10 | 58.006 | 63.477 | 68.941 |
| 2 | mut 20 | 88.766 | 95.055 | 101.497 |
| 2 | mut 40 | 133.898 | 140.390 | 146.663 |
| 2 | mut 80 | 156.238 | 162.667 | 168.835 |
| 2 | mut 250 | 180.541 | 186.876 | 193.151 |
| 3 | WT 5 | 66.720 | 73.187 | 79.646 |
| 3 | WT 10 | 109.464 | 116.001 | 122.177 |
| 3 | WT 20 | 144.933 | 151.139 | 157.537 |
| 3 | WT 40 | 192.762 | 199.160 | 205.284 |
| 3 | WT 80 | 225.811 | 232.428 | 238.252 |
| 3 | WT 250 | 231.709 | 237.918 | 244.421 |
| 3 | mut | 36.242 | 41.159 | 46.565 |
| 3 | mut 10 | 53.945 | 59.653 | 65.693 |
| 3 | mut 20 | 89.646 | 95.617 | 102.024 |
| 3 | mut 40 | 133.566 | 139.776 | 146.113 |
| 3 | mut 80 | 156.555 | 162.557 | 168.787 |
| 3 | mut 250 | 180.705 | 186.932 | 193.327 |
| 4 | WT 5 | 69.298 | 75.131 | 81.160 |
| 4 | WT 10 | 111.288 | 117.514 | 123.639 |
| 4 | WT 20 | 146.956 | 153.289 | 159.350 |
| 4 | WT 40 | 194.908 | 201.289 | 207.621 |
| 4 | WT 80 | 228.382 | 234.962 | 240.972 |
| 4 | WT 250 | 234.384 | 240.198 | 246.468 |
| 4 | mut 5 | 34.044 | 39.263 | 45.106 |
| 4 | mut 10 | 54.421 | 60.168 | 65.966 |
| 4 | mut 20 | 89.558 | 95.636 | 101.791 |
| 4 | mut 40 | 133.322 | 139.929 | 146.100 |
| 4 | mut 80 | 156.024 | 162.534 | 168.737 |
| 4 | mut 250 | 180.686 | 187.024 | 193.207 |

TABLE S23. $\tilde{k}_{\text {on }}$ HIV

| k | SNP tet | low.HPD | median | upp.HPD |
| :---: | :---: | :---: | :---: | :---: |
| 1 | WT 5 | 0.003 | 0.004 | 0.005 |
| 1 | WT 10 | 0.003 | 0.004 | 0.005 |
| 1 | WT 20 | 0.007 | 0.009 | 0.012 |
| 1 | WT 40 | 0.010 | 0.014 | 0.018 |
| 1 | WT 80 | 0.007 | 0.010 | 0.012 |
| 1 | WT 250 | 0.009 | 0.012 | 0.016 |
| 1 | mut 5 | 0.003 | 0.003 | 0.004 |
| 1 | mut 10 | 0.003 | 0.004 | 0.006 |
| 1 | mut 20 | 0.004 | 0.006 | 0.007 |
| 1 | mut 40 | 0.007 | 0.009 | 0.011 |
| 1 | mut 80 | 0.005 | 0.007 | 0.008 |
| 1 | mut 250 | 0.005 | 0.006 | 0.008 |
| 2 | WT 5 | 0.003 | 0.004 | 0.005 |
| 2 | WT 10 | 0.004 | 0.005 | 0.007 |
| 2 | WT 20 | 0.007 | 0.009 | 0.012 |
| 2 | WT 40 | 0.009 | 0.013 | 0.016 |
| 2 | WT 80 | 0.015 | 0.020 | 0.026 |
| 2 | WT 250 | 0.011 | 0.016 | 0.021 |
| 2 | mut 5 | 0.002 | 0.003 | 0.004 |
| 2 | mut 10 | 0.004 | 0.006 | 0.007 |
| 2 | mut 20 | 0.005 | 0.007 | 0.008 |
| 2 | mut 40 | 0.005 | 0.006 | 0.008 |
| 2 | mut 80 | 0.004 | 0.005 | 0.006 |
| 2 | mut 250 | 0.005 | 0.006 | 0.008 |
| 3 | WT 5 | 0.002 | 0.003 | 0.004 |
| 3 | WT 10 | 0.002 | 0.003 | 0.004 |
| 3 | WT 20 | 0.005 | 0.007 | 0.008 |
| 3 | WT 40 | 0.007 | 0.010 | 0.013 |
| 3 | WT 80 | 0.008 | 0.011 | 0.014 |
| 3 | WT 250 | 0.009 | 0.012 | 0.016 |
| 3 | mut 5 | 0.003 | 0.004 | 0.006 |
| 3 | mut 10 | 0.003 | 0.004 | 0.005 |
| 3 | mut 20 | 0.005 | 0.007 | 0.009 |
| 3 | mut 40 | 0.005 | 0.007 | 0.009 |
| 3 | mut 80 | 0.005 | 0.007 | 0.009 |
| 3 | mut 250 | 0.007 | 0.009 | 0.011 |
| 4 | WT 5 | 0.004 | 0.006 | 0.007 |
| 4 | WT 10 | 0.004 | 0.006 | 0.007 |
| 4 | WT 20 | 0.005 | 0.007 | 0.009 |
| 4 | WT 40 | 0.007 | 0.009 | 0.012 |
| 4 | WT 80 | 0.007 | 0.010 | 0.013 |
| 4 | WT 250 | 0.006 | 0.008 | 0.010 |
| 4 | mut 5 | 0.003 | 0.004 | 0.004 |
| 4 | mut 10 | 0.005 | 0.006 | 0.008 |
| 4 | mut 20 | 0.006 | 0.008 | 0.010 |
| 4 | mut 40 | 0.007 | 0.009 | 0.011 |
| 4 | mut 80 | 0.006 | 0.008 | 0.010 |
| 4 | mut 250 | 0.007 | 0.009 | 0.011 |

TABLE S24. $\alpha / k_{\text {on }}$ HIV

| k | SNP tet | low.HPD | median | upp.HPD |
| :---: | :---: | :---: | :---: | :---: |
| 1 | WT 5 | 65.123 | 80.224 | 96.055 |
| 1 | WT 10 | 105.447 | 124.804 | 144.056 |
| 1 | WT 20 | 62.196 | 72.693 | 82.359 |
| 1 | WT 40 | 55.029 | 62.855 | 71.783 |
| 1 | WT 80 | 91.968 | 105.675 | 120.244 |
| 1 | WT 250 | 73.419 | 84.125 | 95.301 |
| 1 | ut | 30.097 | 38.894 | 48.835 |
| 1 | ut 10 | 38.247 | 46.466 | 55.041 |
| 1 | ut 20 | 50.656 | 59.480 | 68.448 |
| 1 | ut 40 | 48.290 | 55.669 | 63.965 |
| 1 | ut 80 | 75.487 | 87.423 | 99.612 |
| 1 | mut 250 | 95.084 | 108.486 | 123.429 |
| 2 | WT | 76.105 | 89.902 | 104.946 |
| 2 | WT 10 | 81.737 | 93.017 | 105.950 |
| 2 | WT 20 | 65.181 | 74.363 | 83.731 |
| 2 | WT 40 | 61.362 | 69.436 | 78.737 |
| 2 | WT 80 | 44.884 | 51.005 | 57.922 |
| 2 | WT 250 | 56.796 | 64.669 | 73.245 |
| 2 | mut | 33.159 | 41.584 | 50.823 |
| 2 | mut 10 | 33.516 | 39.641 | 45.896 |
| 2 | mut 20 | 43.426 | 51.414 | 59.328 |
| 2 | mut 40 | 67.491 | 76.908 | 87.220 |
| 2 | mut 80 | 101.936 | 115.771 | 132.951 |
| 2 | mut 250 | 94.257 | 109.003 | 123.450 |
| 3 | WT 5 | 92.776 | 109.735 | 130.017 |
| 3 | WT 10 | 147.773 | 172.554 | 197.158 |
| 3 | WT 20 | 88.084 | 100.818 | 114.809 |
| 3 | WT 40 | 77.288 | 87.747 | 99.551 |
| 3 | WT 80 | 80.400 | 91.796 | 103.421 |
| 3 | WT 250 | 76.617 | 86.925 | 98.877 |
| 3 | ut | 27.222 | 32.985 | 39.572 |
| 3 | mut 10 | 40.746 | 48.876 | 57.111 |
| 3 | mut 20 | 41.247 | 48.004 | 54.827 |
| 3 | mut 40 | 63.647 | 72.555 | 83.277 |
| 3 | ut 80 | 72.747 | 84.254 | 95.213 |
| 3 | mut 250 | 63.329 | 72.322 | 82.431 |
| 4 | WT 5 | 49.732 | 58.001 | 66.871 |
| 4 | WT 10 | 77.064 | 88.972 | 100.832 |
| 4 | WT 20 | 85.802 | 97.901 | 110.523 |
| 4 | WT 40 | 83.077 | 94.987 | 107.236 |
| 4 | WT 80 | 87.612 | 99.737 | 112.167 |
| 4 | WT 250 | 114.176 | 129.687 | 146.438 |
| 4 | mut | 32.124 | 39.552 | 47.867 |
| 4 | mut 10 | 29.015 | 34.206 | 40.406 |
| 4 | mut 20 | 35.600 | 41.423 | 47.690 |
| 4 | mut 40 | 48.680 | 56.041 | 64.317 |
| 4 | mut 80 | 64.269 | 73.738 | 84.134 |
| 4 | mut 250 | 63.794 | 73.262 | 83.058 |

TABLE S25. $\kappa$ HBB

| k | SNP | tet | low.HPD | median | upp.HPD |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | WT | 5 | 260.821 | 267.314 | 273.981 |
| 2 | WT | 10 | 270.677 | 277.100 | 284.186 |
| 2 | WT | 20 | 298.204 | 304.307 | 311.348 |
| 2 | WT | 40 | 305.802 | 312.765 | 319.762 |
| 2 | WT | 80 | 321.045 | 327.146 | 334.050 |
| 2 | WT | 250 | 338.221 | 345.124 | 349.512 |
| 2 | mut | 5 | 165.515 | 172.460 | 179.105 |
| 2 | mut | 10 | 192.112 | 198.697 | 206.055 |
| 2 | mut | 40 | 183.042 | 189.718 | 196.109 |
| 2 | mut | 80 | 185.722 | 192.343 | 199.121 |
| 2 | mut | 250 | 178.993 | 185.852 | 192.476 |

TABLE S26. $\mu_{\mathrm{X}}$ HBB

| k | SNP | tet | low.HPD | median | upp.HPD |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | WT | 5 | 11.310 | 11.728 | 12.148 |
| 2 | WT | 10 | 16.261 | 16.814 | 17.355 |
| 2 | WT | 20 | 23.646 | 24.285 | 25.005 |
| 2 | WT | 40 | 33.593 | 34.477 | 35.487 |
| 2 | WT | 80 | 42.511 | 43.591 | 44.589 |
| 2 | WT | 250 | 42.719 | 43.602 | 44.549 |
| 2 | mut | 5 | 7.300 | 7.704 | 8.128 |
| 2 | mut | 10 | 11.689 | 12.224 | 12.753 |
| 2 | mut | 40 | 21.350 | 22.256 | 23.111 |
| 2 | mut | 80 | 17.868 | 18.619 | 19.384 |
| 2 | mut | 250 | 21.277 | 22.220 | 23.121 |

## C. Poisson distribution

The Poisson model encodes only one biological parameter, viz., the average gene expression level $\mu_{X}$. We fitted this model to data from one of the replicates as a benchmark. The $90 \%$ HPD CIs and medians of the estimated parameters $\kappa$ and $\mu_{X}$, are reported in Tables S25-S26 (HBB gene) and Tables S27-S28 (HIV gene). It is worth noting that, compared to the prior derived in section S3 and both the estimates from the Poisson-beta and negativebinomial models of Tables S7, S12, S17, and S21, the $\kappa$ is overestimated. In fact, high values of $\kappa$ compensate for the small dispersion encoded in a Poisson random variable. Jointly with the fact that the Poisson model shows lower GoF than the two general models (subsection S4C and figure S7), we conclude that the expression of the genes HIV and HBB is relative to a Poisson random variable and a flexible gene expression model for $X_{i}^{(k)}$, such as the Poisson-beta or the negative-binomial models, is necessary to exploit the measurement equation (24).

TABLE S27. $\mathrm{CV}_{X}^{2}$ HBB

| k | SNP | tet | low.HPD | median | upp.HPD |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | WT | 5 | 0.082 | 0.085 | 0.088 |
| 2 | WT | 10 | 0.058 | 0.059 | 0.061 |
| 2 | WT | 20 | 0.040 | 0.041 | 0.042 |
| 2 | WT | 40 | 0.028 | 0.029 | 0.030 |
| 2 | WT | 80 | 0.022 | 0.023 | 0.024 |
| 2 | WT | 250 | 0.022 | 0.023 | 0.023 |
| 2 | mut | 5 | 0.123 | 0.130 | 0.137 |
| 2 | mut | 10 | 0.078 | 0.082 | 0.085 |
| 2 | mut | 40 | 0.043 | 0.045 | 0.047 |
| 2 | mut | 80 | 0.052 | 0.054 | 0.056 |
| 2 | mut | 250 | 0.043 | 0.045 | 0.047 |

TABLE S28. $\kappa$ HIV

| k | SNP | tet | low.HPD | median | upp.HPD |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 4 | WT | 5 | 245.093 | 251.782 | 258.632 |
| 4 | WT | 10 | 276.948 | 283.448 | 290.234 |
| 4 | WT | 20 | 291.102 | 297.690 | 304.497 |
| 4 | WT | 40 | 285.262 | 291.524 | 298.090 |
| 4 | WT | 80 | 300.678 | 306.854 | 313.563 |
| 4 | WT | 250 | 315.404 | 321.897 | 328.357 |
| 4 | mut | 5 | 172.173 | 179.215 | 186.314 |
| 4 | mut | 10 | 183.339 | 189.696 | 196.491 |
| 4 | mut | 20 | 178.477 | 185.104 | 191.728 |
| 4 | mut | 40 | 189.708 | 196.348 | 203.244 |
| 4 | mut | 80 | 183.321 | 189.823 | 196.490 |
| 4 | mut | 250 | 185.416 | 192.345 | 198.760 |

TABLE S29. $\mu_{\mathrm{X}}$ HIV

| k | SNP | tet | low.HPD | median | upp.HPD |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 4 | WT | 5 | 11.520 | 11.960 | 12.434 |
| 4 | WT | 10 | 17.091 | 17.645 | 18.199 |
| 4 | WT | 20 | 24.572 | 25.315 | 26.012 |
| 4 | WT | 40 | 37.049 | 38.005 | 39.049 |
| 4 | WT | 80 | 46.374 | 47.601 | 48.746 |
| 4 | WT | 250 | 45.807 | 46.889 | 48.064 |
| 4 | mut | 5 | 6.517 | 6.877 | 7.264 |
| 4 | mut | 10 | 12.027 | 12.547 | 13.114 |
| 4 | mut | 20 | 15.649 | 16.369 | 17.065 |
| 4 | mut | 40 | 19.839 | 20.665 | 21.484 |
| 4 | mut | 80 | 19.157 | 19.943 | 20.770 |
| 4 | mut | 250 | 20.907 | 21.769 | 22.651 |

TABLE S30. $\mathrm{CV}_{X}^{2}$ HIV

| k | SNP | tet | low.HPD | median | upp.HPD |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 4 | WT | 5 | 0.080 | 0.084 | 0.087 |
| 4 | WT | 10 | 0.055 | 0.057 | 0.058 |
| 4 | WT | 20 | 0.038 | 0.040 | 0.041 |
| 4 | WT | 40 | 0.026 | 0.026 | 0.027 |
| 4 | WT | 80 | 0.021 | 0.021 | 0.022 |
| 4 | WT | 250 | 0.021 | 0.021 | 0.022 |
| 4 | mut | 5 | 0.137 | 0.145 | 0.153 |
| 4 | mut | 10 | 0.076 | 0.080 | 0.083 |
| 4 | mut | 20 | 0.059 | 0.061 | 0.064 |
| 4 | mut | 40 | 0.047 | 0.048 | 0.050 |
| 4 | mut | 80 | 0.048 | 0.050 | 0.052 |
| 4 | mut | 250 | 0.044 | 0.046 | 0.048 |

## S8. MICROSCOPIC GENE EXPRESSION MODEL

We referred to the models of section S2 as the phenomelogical models. In fact, our main concern there was to exploit a minimal description of the statistics of the transcription events and the stationary mRNA distribution-which is the (observed) phenomenology, indeed. Due to their simplicity, these models allowed us to attain the important goal of separating the technical noise (due to background fluorescence and measurement process) from the biological noise encoded into $X_{i}$.

Nevertheless, there are specific microscopic biological mechanisms, more difficult to observe, that may give rise to the observed phenomena. In our tetracycline-inducible genes, Tet repressor (TetR) homodimers bind to the operator $\mathrm{TetO}_{2}$ downstream of the transcription start site (TSS). When such a binding event occurs, the transcription is inhibited as the elongation is impeded. Adding Tet in turn alters the conformation of TetR and hinders the binding events, having the net effect of inducing the gene expression. Crucially, the transcription rate is proportional to the abundance of PolII (law of mass action), which can be thought of as waiting in a compartment upstream of the TSS [25]. Therefore, when the gene is actively transcribing, its rate can vary in time according to the amount of PolII ready to initiate transcription. After transcription, PolII can either be re-injected into the compartment and set ready for a new initiation event (PolII recycling), or disposed into the nuclear environment. Also, the compartment recruits PolII from the nuclear environment. This can be described by means of the following chemical reaction scheme:

$$
\begin{gather*}
\varnothing \xrightarrow{\gamma} \text { PolII, }  \tag{42}\\
\mathrm{DNA}_{\text {on }}+\mathrm{PolII} \xrightarrow{l \beta} \mathrm{mRNA}+\mathrm{DNA}_{\mathrm{on}}+\mathrm{PolII},  \tag{43}\\
\mathrm{DNA}_{\text {on }}+\text { PolII }^{(1-l)} \beta  \tag{44}\\
\mathrm{DNA}_{\text {on }} \xrightarrow{\lambda_{\text {off }}} \mathrm{DNRA}_{\text {off }},  \tag{45}\\
\text { DNA }_{\text {off }} \xrightarrow{\lambda_{\text {on }}} \mathrm{DNA}_{\text {on }},  \tag{46}\\
\mathrm{mRNA}_{\text {on }},  \tag{47}\\
\text { PolII } \xrightarrow{\delta} \varnothing, \tag{48}
\end{gather*}
$$

where $\mathrm{DNA}_{\text {on }}$ and $\mathrm{DNA}_{\text {off }}$ are unlocked and locked DNA configurations, respectively. The presence of the 3 '-5' crosstalk loop is thought to facilitate the recycling of PolII after each transcription event; therefore we can study the effect of the recycling on the simulated expression data by tuning $l$ in the reaction scheme. Obviously, the pA mutation lowers the
recycling probability $l$ with respect to the WT, but $l$ is not supposed to be zero in mutant genes, as the recycling can occur by means of other mechanisms (e.g., diffusion). By the law of mass action

$$
\begin{align*}
\lambda_{\mathrm{off}} & =n K_{\lambda}  \tag{49}\\
\lambda_{\mathrm{on}} & =K_{\lambda} \tag{50}
\end{align*}
$$

where $K_{\lambda}$ is a chemical affinity and $n$ is the concentration of TetR. Hence, we can imitate variations in the Tet dose by fine-tuning $n$, with large values of Tet (high induction levels) corresponding to small values of $n$. Unlike the simpler phenomenological models, we do not have an analytical likelihood for this model, thus parameter inference is more challenging, to be addressed with likelihood-free methods such as the Approximate Bayesian Computation (ABC) [26, 27]. We simulate the model using the Doob-Gillespie algorithm; sample trajectories of mRNA abundances are plotted in Fig. 5 D (Main text).

When all the chemical species are highly abundant and the gene is always in "on" state (this can be achieved in the limit as $k_{\text {off }} \rightarrow 0$ ), it is straightforward to derive the following rate equations,

$$
\begin{align*}
\frac{\mathrm{d}}{\mathrm{~d} t}[\mathrm{PolII}] & =\alpha-[\mathrm{PolII}](\delta-\beta(1-l))  \tag{51}\\
\frac{\mathrm{d}}{\mathrm{~d} t}[\mathrm{mRNA}] & =[\mathrm{PolII}] \beta-[\mathrm{mRNA}] d \tag{52}
\end{align*}
$$

where $[X]$ is the abundance of the species $X$. The stationary mRNA abundance is then

$$
\begin{equation*}
[\mathrm{mRNA}]=\frac{\beta}{d} \frac{\alpha}{\delta+\beta(1-l)}, \tag{53}
\end{equation*}
$$

which corresponds to the vertical lines of Fig. 4 B (Main text). While the parameters $\gamma, \beta, d, \delta, K_{\lambda}$ are chosen to simulate mRNA abundances and noises in ranges consistent with those of the real data, fine-tuning the recycling probability and the induction parameters $l$ and $n$ yields patterns similar to those observed in the experimental setting (i.e., those of Figs. S10, S12, and 2 (Main text)). More specifically, a simple scatter plot of the sample averages versus the $\mathrm{CV}^{2}$ of [mRNA] shows a drift of the noise curve from the Poisson case $\mathrm{CV}_{X}^{2}=1 / \mu_{X}$ as the recycling rate $l$ increases. Fitting a negative binomial (NB) Bayesian model

$$
\begin{align*}
\operatorname{mRNA} & \sim \operatorname{NB}\left(\mu_{X}, k_{\text {on }}\right),  \tag{54}\\
\mu_{X} & \sim \operatorname{Gamma}(0.001,0.001)  \tag{55}\\
k_{\text {on }} & \sim \operatorname{Gamma}(0.001,0.001) \tag{56}
\end{align*}
$$

to 500 simulated stationary mRNA abundances, allowed us to estimate the average burst size $\alpha / k_{\text {off }}=$ $\mu_{X} / k_{\text {on }}$ and the burst frequency $k_{\text {on }}$ shown in Figs. S14 and 5 C (Main text).


FIG. S14. Negative-binomial model fit to 500 mRNA abundances simulated from the microscopic model. The pattern of the inferred average burst sizes and burst frequencies mirrors those obtained from the real data. For each value of the recycling probability $l(l=0.5,0.85,1)$, simulations are performed with $\lambda_{\text {off }}=0.5,1,1.5,2,2.5,3,2.5,3.5,4,4.5 ;$ remaining parameters are $\left(\gamma, \beta, d, \delta, \lambda_{\text {on }}\right)=(10,10,0.01,1,0.01)$. The solid lines in the noise plot (lower-left plot) are fitted $\mathrm{CV}_{X}^{2}=A / \mu_{X}+B$ curves.

## S9. MATERIALS

## A. Cell lines and cell culture

The wildtype HBB and HIV-1-env cell lines have been utilized in previous studies [24, 28, 29]. The nomenclature was changed for the present manuscript, with the cell lines denoted HBB WT, HBB mut, HIV WT and HIV mut, which had been denoted $\beta \mathrm{pA}+, \beta \mathrm{pA}-$, HIV-1 $\mathrm{pA}+$ and HIV-1 pA- in [24], respectively. Cells were maintained in DMEM medium supplemented with $10 \%$ fetal bovine serum and $100 \mu \mathrm{~g} \mathrm{~mL}^{-1}$ penicillinstreptomycin (DMEM-10). Induction of cell lines was carried out for 16 hours before downstream experiments.
Deletion cell line construction. The design and construction of the deletion cell line used the protocol detailed in [30], with the following changes. A dual sgRNA strategy was employed with a 5 ' guide binding between the AmpR promoter and CMV enhancer and a 3 ' guide binding just after the 3 ' FRT site. The use of plasmid $\mathrm{pSpCas} 9(\mathrm{BB})-2 \mathrm{~A}-\mathrm{GFP}$ (PX458) and dual targeting necessitated the transfection with two plasmids, each containing a respective guide. Transfection was carried out using calcium phosphate, followed by washing the cells with warm PBS after 16-24 hours and replacing the me-
dia. Cells were allowed to recover for 48-72 hours before single-cells were isolated in 96 -well plates via FACS (BD ARIAFusion), with the brightest $10 \%$ GFP positive cells being sorted. Testing for deletion was initially verified via genomic DNA extraction and PCR, followed by smFISH assay using flow cytometry.

## B. Single-molecule RNA fluorescence in situ hybridization

Probe sets. Probe sets for HBB and HIV-1-env RNA were designed with the tool at www.biosearchtech.com/stellarisdesigner (see Table S31 for sequences). The probe sets were synthesized by LGC Biosearch Technologies as custom Stellaris ${ }^{\circledR}$ probe sets. AKT1 probes were readymade and ordered from LGC Biosearch.
smFISH. smFISH staining followed the probe manufacturer's protocol. Briefly, cells were grown on poly-L-lysine treated glass coverslips overnight, fixed in $3.7 \%$ formaldehyde for 10 minutes and permeabilized in ethanol for $>1$ hour. After overnight staining at $37^{\circ} \mathrm{C}$ in dextran sulphate and formamide buffered with SSC, cells were washed, followed by mounting onto a slide using Vectashield with DAPI as the mounting medium. Imageing was carried out

TABLE S31. Sequences and details of smFISH probe sets. Product Name: Stellaris ${ }^{\circledR}$ FISH Probes, Custom Assay with Quasar ${ }^{\circledR} 670$ Dye.

1) Oligo Name: HIV
tcactaaacgagctcgtcga ggtcaaaacagcgtggatgg
taaacgctagagtccggagg gagctcggtaccaagcttaa
agaattccaccacactggac cagcagttgttgcagaatta ctatgtcgacacccaattct tctgtcgagtaacgcctatt agtctaggatctactggagg tggtacaagcagttttaggc ggcaatgaaagcaacacttt cctaaggcttttgtcatgaa gagtctgactgttctgatga gctgctttgatagagaagct cttcttcttctattccttcg aggatccgttcactaatcga cagatcgtcccagataagtg tagctgaagaggcacaggct agagtaagtctctcaagcgg ttccacaatcctcgttacaa aatatttgagggcttcccac ccaatactgtaggagattcc gcactattctttagttcctg ctgtggcattgagcaagtta cttctataaccctatctgtc agctctataagctgcttgta tattcttctaggtatgtggc agcaaaatcctttccaagcc tactttttgaccacttgcca ttacagcaggccatccaatc ctcagctcgtctcattcttt cctctagactcgagatactg gctgatcagcgggtttaaac ctggcaactagaaggcacag accttccagggtcaaggaag taggaaaggacagtgggagt
2) Oligo Name: HBB
tcactaaacgagctcgtcga aaacagcgtggatggcgtct cggtgtcttctatggaggtc tttaaacgctagagtccgga gtcagaagcaaatgtaagct ggttgctagtgaacacagtt tgcaccatggtgtctgtttg gcagtaacggcagacttctc caacttcatccacgttcacc aaagaacctctgggtccaag gagtggacagatccccaaag cttagggttgcccataacag gagcactttcttgccatgag caggccatcactaaaggcac cttgaggttgtccaggtgag cactcagtgtggcaaaggtg acgtgcagcttgtcacagtg agcctgaagttctcaggatc aaagtgatgggccagcacac ctggtggggtgaattctttg caccactttctgataggcag cgcttagtgatacttgtggg tggacagcaagaaagcgagc cttagggaacaaaggaacct tagacccagtttggtagttg tcatgttttctacagctaga tccagcagacatgggtgatc tcctcatgttttctacagtc ctagacagcagacatgggtg gtgatcctcatgttttctac tacagtcgtccagcagacat atgggtgatcctcatgtttt ttctacagctagacagcaga agacatgggtgatcctcatg ctcatgttttctacagtcgt tagacagcagacatgggtga ttatctagatccggtggatc ttgtggtttgtccaaactca gcatttttttcactgcattc tgcagcttataatggttaca gcaattgttgttgttaactt
on a brightfield microscope.
Flow cytometry. DNA staining was carried out with FxCycle ${ }^{\text {TM }}$ Violet Stain (ThermoFisher, F10347) at a concentration of $1 \mu \mathrm{~g} \mathrm{~mL}^{-1}$. Fixed cells were analysed on a BD Fortessa. Processing and data analysis of raw flow cytometry data was carried out using the flowcore R package [1] (v1.48, $R$ version 3.3).
SmFISH spot counting. Quantification of RNA was carried out using FISH-quant [31]. Images were imported with the following settings: XY 64.8 nm ; Z 200 nm ; Refractive index 1.515; NA 1.40; Em 592; Ex 546; Microscope widefield. Cell outlines were
drawn manually. A single image was then processed and the settings used to batch-process the remaining set. The threshold and quality score parameters of FISH Quant were set to quantify as many spots as possible while reducing spurious detection through batch-specific selection of these parameters.

## C. RNA isolation and preparation, and degradation rate estimation

Total RNA was extracted from the respective cell lines following the RNeasy Mini Kit (Qiagen, 47104) protocol, using QIAshredder (Qiagen, 79654). RNA for RNA-seq analysis was treated with TURBO DNA-free ${ }^{\mathrm{TM}}$ kit (ThermoFisher, AM1907).

To estimate the mRNA degradation rate, RNA was reverse transcribed using random primers (Promega, C118A) and M-MLV reverse transcriptase (Promega, M170A) followed by qPCR using SensiMix ${ }^{\text {TM }}$ SYBR ${ }^{\circledR}$ No-Rox (Bioline, QT650-02) on a Qiagen Rotor-Gene Q. Gene-specific primers were used (HIV Forward TCTCCTACGGCAGGAAGAAG; HIV Reverse GGTAGCTGAAGAGGCACAGG). Analysis was carried out by calculating the CT values using the qpcR R package [32] (v1.4-1) and from this $2^{-\Delta \Delta C t}$ were calculated using the mut time 0 concentration as the reference sample. A degradation time series was carried out by standard induction method at 250 $\mathrm{ng} \mathrm{mL}{ }^{-1}$ tetracycline for 16 hours, followed by removal of media and washing with warm DMEM-10. Cells were then placed in fresh medium and samples were taken at different time points following on from this.

## D. Nanostring

Cells were seeded, induced and processed as indicated previously (subsection S9A), with the following alteration: after trypsinisation cells were resuspended in 1 mL of PBS and kept on ice. Counting of cells was carried out via Countess (ThermoFisher) cell counter with $100 \mu \mathrm{~L}$ (50 : 50) PBS to trypan blue. Samples were spun down at 500 g for 5 minutes and were then resuspended in RLT buffer from RNeasy Mini Kit (Qiagen, 47104) with beta-mercaptoethanol to obtain a concentration of 6500 cells per $\mu \mathrm{L}$. Samples were then vortexed for 1 minute and placed at $-80^{\circ} \mathrm{C}$. Cell lysis was verified under a microscope. Samples were shipped on dry ice to an external provider for processing. Custom probe sets, including probes targeting HIV-1$e n v$ along with GAPDH and AKT1 as house-keeping genes, were designed and shipped by NanoString Technologies.

## E. RNA-seq

Library preparation. RNA-seq libraries were prepared using 500 ng of total input RNA and the NEBNext ${ }^{\circledR}$ UltraTM II Directional RNA Library Prep Kit for Illumina (E7760L), along with the NEBNext ${ }^{\circledR}$ rRNA Depletion Kit (E6310L) and NEBNext ${ }^{\circledR}$ Multiplex Oligos for Illumina Set 1 and 2 (E7335, E7500). Ribo-depletion was carried out to capture transgene RNA regardless of the absence or presence of a poly (A) tail. The manufacturer's manual was followed, with the final PCR amplification using 9 cycles. Libraries were assessed via Bioanalyser, diluted and mixed before being sequenced on an Illumina ${ }^{\circledR}$ NextSeq 500 , generating paired-end reads with read length 42.

RNA-seq analysis. Data quality control was performed with FastQC v0.11.5. Read and adapter trimming was carried out using TrimGalore! v0.4.3 with cutadapt v0.4.3 using default settings [33].

Indices for STAR to map to were constructed from the human genome (GRCh38.p12, Gencode primary annotation) and the respective (HBB WT/mut and HIV-1-env WT/mut) transgenic sequence. The GTF was modified to include these genes as a separate chromosome (chrHBB or chrHIV). To mask the existing HBB sequence, bedtools' (v2.25.0) maskfasta command was used [34]. RNA-seq reads were mapped to the genome using STAR software v2.5.3a with parameter --outSAMattributes XS [35]. Counts per gene were calculated using LiBiNorm [36] acting in an HTSeq-count [37] compatible mode with the following parameters: --format=bam --minaqual=10 --stranded=reverse
--mode=intersection-strict. Coverage statistics were generated using deepTools' (v3.1.3) bamCoverage [38]. Fold changes for the HBB and HIV genes were calculated using DESeq2 v1.22.1 [39] from Bioconductor release 3.8 and $R$ v3.5.1.
[1] F. Hahne, N. LeMeur, R. R. Brinkman, B. Ellis, P. Haaland, D. Sarkar, J. Spidlen, E. Strain, and R. Gentleman, BMC Bioinformatics 10, 106 (2009).
[2] H. M. Shapiro, Practical Flow Cytometry (John Wiley \& Sons, Inc., Hoboken, NJ, USA, 2003) pp. 1733.
[3] K. Lo, R. Brinkman, and R. Gottardo, Cytometry A (2008).
[4] K. Lo, F. Hahne, R. Brinkman, and R. Gottardo, BMC Bioinformatics (2009), R package version 3.5.0.
[5] B. Munsky, G. Neuert, and A. van Oudenaarden, Science 336, 183 (2012).
[6] J. Kim and J. C. Marioni, Genome Biology 14, R7 (2013).
[7] S. Tiberi, M. Walsh, M. Cavallaro, D. Hebenstreit, and B. Finkenstädt, Bioinformatics 34, i647 (2018).
[8] M. B. Elowitz, A. J. Levine, E. D. Siggia, and P. S. Swain, Science 297, 1183 (2002).
[9] J. R. S. Newman, S. Ghaemmaghami, J. Ihmels, D. K. Breslow, M. Noble, J. L. DeRisi, and J. S. Weissman, Nature 441, 840 (2006).
[10] A. Bar-Even, J. Paulsson, N. Maheshri, M. Carmi, E. O'Shea, Y. Pilpel, and N. Barkai, Nature Genetics 38, 636 (2006).
[11] R. D. Dar, B. S. Razooky, L. S. Weinberger, C. D. Cox, and M. L. Simpson, PLoS ONE 10, 1 (2015).
[12] M. Soltani, C. A. Vargas-Garcia, D. Antunes, and A. Singh, PLoS Computational Biology 12, e1004972 (2016).
[13] M. Dobrzynski and F. J. Bruggeman, Proceedings of the National Academy of Sciences 106, 2583 (2009).
[14] E. Meinelt, M. Reunanen, M. Edinger, M. Jaimes, A. Stall, D. Sasaki, and J. Trotter, Standardizing

Application Setup Across Multiple Flow Cytometers Using BD FACSDiva ${ }^{T M}$ Version 6 Software (2012).
[15] C. Zechner, J. Ruess, P. Krenn, S. Pelet, M. Peter, J. Lygeros, and H. Koeppl, Proceedings of the National Academy of Sciences 109, 8340 (2012).
[16] J. Paulsson, Nature 427, 415 (2004).
[17] A. Hilfinger and J. Paulsson, Proceedings of the National Academy of Sciences 108, 12167 (2011).
[18] D. Adler, "vioplot: Violin plot," (2005), https://cran.r-project.org/package=vioplot.
[19] D. J. Spiegelhalter, K. R. K. R. Abrams, and J. P. Myles, Bayesian approaches to clinical trials and health-care evaluation (John Wiley \& Sons, 2004) p. 391.
[20] A. Patil, D. Huard, and C. Fonnesbeck, Journal of Statistical Software 35, 1 (2010).
[21] H. Haario, E. Saksman, and J. Tamminen, Bernoulli 7, 223 (2001).
[22] S. L. Scott, A. W. Blocker, F. V. Bonassi, H. A. Chipman, E. I. George, and R. E. McCulloch, International Journal of Management Science and Engineering Management 11, 78 (2016).
[23] O. Tange, ;login: The USENIX Magazine 36, 42 (2011).
[24] C. K. Mapendano, S. Lykke-Andersen, J. Kjems, E. Bertrand, and T. H. Jensen, Molecular Cell 40, 410 (2010).
[25] W.-K. K. Cho, N. Jayanth, B. P. English, T. Inoue, J. O. Andrews, W. Conway, J. B. Grimm, J.-H. H. Spille, L. D. Lavis, T. Lionnet, and I. I. Cisse, eLife 5 (2016), 10.7554/eLife. 13617.
[26] M. Sunnåker, A. G. Busetto, E. Numminen, J. Corander, M. Foll, and C. Dessimoz, PLoS Computational Biology 9, e1002803 (2013).
[27] J. Lintusaari, M. U. Gutmann, R. Dutta, S. Kaski, and J. Corander, Systematic Biology 66, syw077 (2016).
[28] C. K. Damgaard, S. Kahns, S. Lykke-Andersen, A. L. Nielsen, T. H. Jensen, and J. Kjems, Molecular Cell 29, 271 (2008).
[29] A. B. Eberle, S. Lykke-Andersen, O. Mühlemann, and T. H. Jensen, Nature Structural and Molecular Biology 16, 49 (2009).
[30] F. A. Ran, P. D. Hsu, J. Wright, V. Agarwala, D. A. Scott, and F. Zhang, Nature Protocols 8, 2281 (2013).
[31] F. Mueller, A. Senecal, K. Tantale, H. MarieNelly, N. Ly, O. Collin, E. Basyuk, E. Bertrand, X. Darzacq, and C. Zimmer, Nature Methods 10, 277 (2013).
[32] A. Spiess, "qpcR: Modelling and Analysis of Real-

Time PCR Data," (2013), https://CRAN.Rproject.org/package=qpcR.
[33] M. Martin, EMBnet.journal 17, 10 (2011).
[34] A. R. Quinlan and I. M. Hall, Bioinformatics 26, 841 (2010).
[35] A. Dobin, C. A. Davis, F. Schlesinger, J. Drenkow, C. Zaleski, S. Jha, P. Batut, M. Chaisson, and T. R. Gingeras, Bioinformatics 29, 15 (2013).
[36] N. P. Dyer, V. Shahrezaei, and D. Hebenstreit, PeerJ 7, e6222 (2019).
[37] S. Anders, P. T. Pyl, and W. Huber, Bioinformatics 31, 166 (2015).
[38] F. Ramírez, F. Dündar, S. Diehl, B. A. Grüning, and T. Manke, Nucleic Acids Research 42, W187 (2014).
[39] M. I. Love, W. Huber, and S. Anders, Genome Biology 15, 550 (2014).


[^0]:    * To whom correspondence should be addressed. E-mail: d.hebenstreit@warwick.ac.uk m.cavallaro@warwick.ac.uk

