

1 **The Neural Basis of Predictive Pursuit**

2
3 **Short title**

4 The neural basis of predictive pursuit

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6 **One-sentence summary**

7 In a dynamic pursuit environment, monkeys actively predict future prey positions and
8 dACC neurons encode these future positions.

9
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28

ABSTRACT

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It remains unclear how and to what extent non-human animals make demanding on-the-fly predictions during pursuit. We studied this problem in a novel laboratory pursuit task that incentivizes prediction of future prey positions. We trained three macaques to perform joystick-controlled pursuit of prey that followed intelligent escape algorithms. Subjects reliably aimed towards the prey's likely future positions, indicating that they generate internal predictions and use those predictions to guide behavior. We then developed a generative model that explains real-time pursuit trajectories and showed that our subjects use prey position, velocity, and acceleration to make predictions. We identified neurons in the dorsal anterior cingulate cortex (dACC) whose responses track these three variables. These neurons multiplexed prediction-related variables with a distinct and explicit representation of the prey's future position. Our results provide a clear demonstration that the brain can explicitly represent future predictions and highlight the critical role of anterior cingulate cortex for future-oriented cognition.

41 **INTRODUCTION**

42 Many foragers pursue fleeing prey. The ability to effectively pursue prey is thus a critical
43 element in our behavioral repertoires ^{1,2}. To pursue effectively, a forager needs to perform a
44 series of computations: it must maintain a representation of its current position relative to that of
45 the prey, then compute a best path to capture the prey, then execute that path. Because the ability
46 to perform such computations can determine foraging success, pursuit has likely been an
47 important driver of our cognition and its underlying brain systems ³⁻⁶.

48 One way to improve pursuit effectiveness is to predict the future position of the prey and
49 head towards the predicted position ⁷. Estimating future positions can be done using the prey's
50 basic Newtonian variables (most importantly, its current position, velocity, and acceleration) and
51 can be improved using additional (potentially even recursive) variables, such as predictions
52 about the likely evasive strategy of the prey in response to the predator's own future path. By
53 using such information, the forager may be able to formulate a representation of the predicted
54 future position of the prey. The ability of non-human animals to actively predict positions of prey
55 during pursuit is poorly understood. Nonetheless, predictive pursuit is an important part of the
56 repertoire of many species.

57 Prediction is important for many cognitive and behavioral processes, not just foraging.
58 These include motor control, economic decision-making, and abstract long-term planning ⁸⁻¹⁴.
59 There is some evidence that foraging animals can predict the long-term future - that is, they may
60 be able to travel mentally in time and see themselves in the future ^{15,16}. However, observations
61 about animal prediction tend to be limited to a small number of highly adapted species in unique
62 contexts. And, while future planning of movements is relatively well-studied, the ability to
63 predict future positions of prey during dynamic behavior with rapidly changing goals – which

64 feed into but are distinct from motor plans – is not. In the context of pursuit, a critical question is
65 whether future-predicting foragers maintain a specific representation of potential future prey
66 positions and whether those representations (assuming they exist) make use of specialized
67 processes.

68 Although the neural bases of predictive pursuit remain unclear, we can draw some
69 inferences about its likely neuroanatomy. In particular, the dorsal anterior cingulate cortex
70 (dACC) has been implicated in prediction, prospection, and related processes^{17–20}. For example,
71 neuroimaging studies indicate that human dACC is a key region for economic prediction²¹, for
72 prospective reasoning¹¹ and for more open-ended prospective processes^{21,22}. The dACC is well-
73 positioned for this role: it receives broad inputs from limbic and cognitive systems, integrates
74 these, and generates high-level control signals that regulate behavior in an abstract and high-level
75 way^{19,22–24}.

76 Here, we examined the future predicting abilities of rhesus macaques using a novel
77 *virtual pursuit task*. Subjects used a joystick to move an avatar in an open two-dimensional field
78 displayed on a computer screen. Subjects, controlling the avatar, pursued a fleeing prey item that
79 used an artificially intelligent (AI) algorithm to avoid predation. By examining the properties of
80 a generative model fit to our data, we found that our subjects moved towards extrapolated future
81 positions of prey rather than just pointing towards the preys' present positions. Our subjects'
82 made their predictions based on three Newtonian variables associated with the current state, but
83 not other factors that could further improve predictions (such as the effect of the subject's
84 movements on the future position of the prey). We also found that neurons in dACC were
85 selective for those three Newtonian variables (and not others), indicating that responses in this
86 region provide sufficient information to generate the types of predictions our subjects made.

87 Finally, we found that dACC neurons used a spatial code to explicitly represent the predicted
88 future position of the prey, and that this future representation is multiplexed with the
89 representation of current Newtonian variables.

90 **RESULTS**

91 **Behavioral results**

92 Three macaques (*Macaca mulatta*, subjects K, H, and C) used a joystick to control the
93 position of an avatar (a yellow or purple circle) moving continuously and smoothly in a
94 rectangular field on a computer screen (**Figure 1** and **Methods**). On each trial, subjects had up to
95 20 seconds to capture a prey item (a fleeing colored square) to obtain a juice reward. Prey
96 avoided the avatar with a deterministic strategy that combined repulsion from the subject's
97 current position with repulsion from the walls of the field. The prey item was drawn randomly
98 from a set of five, identified by color, that differed in maximum velocity and associated reward
99 size.

100 All subjects showed stable behavior within twelve 2-hour training sessions that followed
101 a longer training period on joystick use (**Figure S1** and **Figure S2**). All data presented here were
102 collected after the training sessions (number of trials, K: 3229; H: 3890; C: 2512). Subjects
103 successfully captured the prey in over 95% of trials and, on successful trials, did so in an average
104 of 5.04 seconds (K: 4.26 sec, H: 5.32 sec, C: 5.54 sec) and median of 3.62 seconds (K: 3.36sec,
105 H: 3.73 sec, C: 3.93 sec). Subjects' performance varied lawfully with prey type, indicating
106 sensitivity to manipulation of reward and/or difficulty (**Figure S1**).

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108 **Behavioral evidence of future state prediction**

109 For analysis purposes, we split all data into one-second segments (**Figure S3**). Within
110 each segment, we calculated the error (sum of squares) between the model (see below) and the
111 behavior at each frame (i.e., each 16.67 ms). For each segment, we computed the minimum point

112 on a 201x201 matrix of intensities for each parameter pair (force by time, **Figure 2**, see below).

113 We then averaged over all segments and all trials, separately for the three subjects.

114 We developed a generative model of behavior (see **Methods**). We used the variable τ
115 (τ) to refer to the prediction parameter for each subject. The variable τ comes from the model
116 and refers to a fit scalar variable, which is multiplied by future position (see the equations in
117 **Methods** section “**Behavioral Model**”). In practice, it can be interpreted as the distance into the
118 future that the subject prospects to guide his behavior (**Figure 2A**). The variable τ can have
119 positive, negative, or zero values. A positive value for τ indicates that the subject points towards
120 the expected future position of the prey - that is, the strategy reflects prediction. A zero τ
121 indicates that the subject points the joystick directly at the current position of the prey. A
122 negative value for τ indicates that the subject points the joystick towards where the prey was in
123 the recent past. Note that all of these strategies (within limits) are capable of eventually catching
124 all prey, since the subject’s avatar is, by design, faster than the prey. The scalar parameter κ
125 (κ) reflects the amount of force applied toward the direction of the predicted position. Thus,
126 a negative value indicates that force is exerted away from (180 degrees opposite) the prey’s
127 position, whereas a positive value indicates that force is exerted towards it.

128 We also added an inertia term to the model. Specifically, we computed an inertially
129 biased path for each 16.67 ms frame. The biased path is a vector sum of the computed best
130 predicted direction and the previous direction ($P_{\text{subject}}(t) - P_{\text{subject}}(t-1)$). In our implementation,
131 these two terms have equal weighting. Note that in practice, their relative weighting may
132 nonetheless vary because the force term (κ , which is fit in the model), affects the weight of the
133 new direction relative to the past direction. This approach for implementing inertia is designed to
134 align intuitively with how inertia works (see **Methods**, **Figure S2**, and **S4**).

135 We called our first model the *physics variable based prediction model* (PVBP). It
136 assumes that subjects' prediction derives from the the prey's current position, velocity (i.e. both
137 speed and direction), and acceleration (which includes both direction and magnitude of
138 acceleration), as well as further derivatives, see **Figure S5**). For all three subjects, the best fitting
139 τ is positive, indicating that they point the joystick towards the prey's future position. For ease of
140 interpretation, we translated τ into time units by calculating the distance between the current
141 position and estimated position, then divided that quantity by the average velocity of the prey
142 across the session. The results of this calculation indicate that subjects K, H and C pointed the
143 joystick towards the position that the prey would occupy in an average of 800 ms, 767 ms, and
144 733 ms in the future, respectively. In the context of the task, these numbers are substantial: they
145 reflect 18.78%, 14.42%, and 13.23% of the average trial duration for K, H, and C, respectively.

146 To determine whether the positive prediction parameter τ is significantly greater than
147 zero, we performed a bootstrap (randomization test making use of resampling with replacement)
148 of heatmap slices from each segment (individual heatmap from 500 segments). This resampling
149 was performed 500 times and resulting heatmaps were added. Then the τ and κ that best explain
150 each segment (that is, the one resulting in the lowest cost) was selected in each resampling. We
151 confirmed that the estimated value τ and κ are both greater than zero more than 99% of the time
152 (i.e., $p < 0.01$).

153 The distance into the future that our subjects predicted did not detectably depend on the
154 the speed of the prey (linear regression between reward/speed and mean τ , K: $\beta = 3.0316$,
155 $p = 0.1110$; H: $\beta = 4.5798$, $p = 0.1791$; C: $\beta = 7.1007$, $p = 0.0957$; the term β refers to the regression
156 coefficient for speed against neural activity). We next asked whether taking more complex paths
157 (ones with more turns vs. more straight paths) affected prediction span. Prey path complexity (as

158 measured by path curvature estimated by average angle method) affected prediction.
159 Specifically, subjects predicted less far into the future when the prey path had more curves (K:
160 $\beta=-0.0687$; H: $\beta=-0.0567$; C: $\beta=-0.0898$, $p<0.0001$ for each). Thus, subjects had the ability to
161 dynamically adjust their own prediction in light of changing circumstances.

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163 **Alternative models do not predict trajectories as well as physics-based prediction**

164 We next compared the physics-based model to two other models implementing different
165 prediction algorithms (**Figure 2B**). First, the veridical prediction (VP) model assumes that the
166 subjects will make perfect predictions that incorporate all game dynamics, including preys'
167 repulsion from the walls and the subject's avatar. This means that a subject that makes a
168 veridical prediction takes into account the effect his own movements will have on the prey's
169 strategy. Second, the cost contour map prediction (CCMP) model is the same as VP but excludes
170 repulsion from the avatar, meaning that the subject's prediction model for the prey would not
171 take into account their own motion. We compared the performance of each model by computing
172 the sum of squares error between the prediction trajectory and the observed trajectories over all
173 time bins.

174 Using the Akaike Information Criterion (AIC), we found that the PVBP fit better than the
175 other two models in our well-trained subjects (K: 7.529×10^6 , for subject K, second best was VP:
176 7.542×10^6 ; H, PVBP: 8.923×10^6 ; for subject H, second best was CMPP: 8.950×10^6 , **Figure 2D**).
177 We fit each segment with distinct τ and κ parameters, and we fit these same two parameters for
178 each of our three models. As a consequence, the comparison of models can be done directly
179 without concern of potential bias toward any specific model. In other words, by fitting each of
180 the three models subject to identical constraints, we ensured a fair comparison across models.

181 For the less well-trained subject, C, the CCMP model explained trajectories most accurately
182 (7.955×10^6).

183 We speculated that one factor that may influence strategy is the speed of the prey. Indeed,
184 we found that all three subjects used PVBP more frequently when the speed of the prey was
185 faster (**Figure S6**). Note that this observed link between speed and the fit of the PVBP occurs
186 even in our third (less fully trained) subject ($p < 0.001$, logistic regression, **Figure S6**). In any
187 case, our model's classification of strategies appears to be robust: the same results were obtained
188 using a different method. Specifically, we fit all individual segments to the best model and
189 computed the model that fit the most overall number of segments (**Figure 2D** and **Methods**).

190 Doing the fitting this way may seem excessively flexible. That is, using two times the
191 number of segments might allow us to fit only noise. (Consider, for example, the case of fitting 9
192 data points with 9th-order polynomial curve). On the other hand, the extra freedom may allow us
193 to better fit signal - or, of course, it may fit both noise and signal. The key question, then, is
194 whether using a large number of parameters makes the fit better despite the possibility of fitting
195 noise. To answer this question, we directly compared the two approaches (**Figure S3**).
196 Specifically, we compared a model assigning two parameters globally versus one applying two
197 parameters for each one-second segment (i.e., Monkey K: 28,164; Monkey H: 35,308; Monkey
198 C: 20,720 parameters, **Figure S3**). We then used AIC to compare models. We found that the
199 second-by-second fitting resulted in lower AIC values, implying a better fit, than the 2-parameter
200 counterpart. Specifically, in this figure, for all individual subjects, the change in AIC (AIC for
201 global parameter model minus AIC for second-by-second model) was positive - implying the
202 model fit by second-by-second model explains the data better than the counterpart - for the best
203 physics-based model explaining the subject's behavior.

204 Overall, the model comparison results showed that subjects predict the upcoming position
205 of the prey using Newtonian physics but ignore the walls and their own influence on the prey.
206 That is, subjects use a simplified approximation of the structure of the game to make future
207 predictions; presumably this simplified one is sufficient to generate good predictions with lower
208 mental effort costs. Indeed, the correlation between speed of prey and subjects' reliance on
209 physics based prediction (a result confirmed with two different analytical approaches) suggests
210 that prediction might have a computational cost.

211

212 **Prediction-related information encoded in dorsal anterior cingulate cortex**

213 Based on its role as a nexus for motivational, cognitive, and motor information^{18,23}, and
214 its demonstrated role in human prospection^{11,25}, we hypothesized that dACC would be critical
215 for predictive pursuit (**Figure 3A**). We fit a statistically unbiased Linear-Nonlinear Generalized
216 Linear model (GLM²⁶⁻²⁸) to responses of 150 well-isolated dACC neurons (K: n=31; H: n=119).
217 For this analysis, we focused on the entire trial period rather than pre-selecting epochs.

218 Position, velocity, and acceleration of the prey were all encoded by significant
219 proportions of neurons (**Figure 3**; position: 62.00%, n=93/150; speed: 35.33%, n=53/150;
220 36.67%; direction: n=55/150, acceleration: 24.67%, n=37/150, $p < 0.01$ in all cases, two-way
221 binomial test). The model fit shown in magenta is the shape of reconstructed filter (examples,
222 **Figure 3C, D**). According to the GLM, jerk, the derivative of acceleration, is not encoded (see
223 **Figure S5**). Jerk also did not measurably affect the subject's neural responses (it only modulated
224 2.00% of cells, n=3/150, $p=0.1288$). Together, these results indicate that dACC ensembles carry
225 the major raw ingredients that our subjects use to predict prey positions.

226 We wondered whether ostensible coding for prey variables could be the byproduct of
227 coding for self-position, since self-position and prey position do tend to be somewhat correlated.
228 We therefore repeated our GLM analyses but included self-position, self-direction, and self-
229 speed as explanatory factors and considered variance explained by prey parameters only after
230 accounting for these variables. Doing this, the proportion of neurons selective for the prey's
231 position information remained significant (position: 65.45%; $p < 0.01$, two-way binomial test),
232 as did neurons selective for prey speed (18.56%; $p < 0.01$), and direction (10.78%, $p = 0.021$).

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234 **Neurons in dACC encode future position**

235 We next asked whether dACC neurons encode the future position of the prey. For each
236 neuron, we refit the GLM using an additional parameter, the position of the prey at time t in the
237 future. We selected the time t ($t = 833\text{ms}$) that was most similar to the value of τ resulting from
238 our generative model, that is, the one indicating the most likely time span of prediction (733,
239 766, and 800 for the three subjects, respectively) subject to the additional constraint of being a
240 multiple of 166.67 (i.e. 10 frames). Note that although this value was chosen in advance, it aligns
241 with the empirically derived measure of peak future position coding (**Figure 4D**, see below).

242 Our analysis approach deals with the problem of correlation between the set of current
243 Newtonian variables (including current position) and future position by assigning all explanatory
244 power to the set of current variables first, and only counting as significant any additional
245 variance explained by future position (see **Methods**). Despite this conservative criterion, we
246 found that responses of 24.67% of dACC neurons are selective for the prey's future position at
247 time t ($n = 37/150$).

248 Visual inspection of the neurons' filters shows that their selectivity is complex (examples
249 are shown in **Figure 4C**). That is, they are positionally tuned, but, unlike place cells, have non-
250 point-like shapes. They contain multiple peaks. They do not appear to be smooth gradients.
251 Instead, they appeared to be heterogeneously spatially tuned. In this manner, they resemble
252 recently identified non-grid-like space-selective cells in entorhinal cortex²⁶. Notably,
253 conventional methods for detecting place/grid-like cells will greatly underestimate the proportion
254 of such tuning.

255 We next asked how *strongly* dACC neurons encode the future position of the prey. We
256 calculated the proportion of log likelihood increase (LLi) between the current position model and
257 the current plus future model (**Figure 4A**). Our neurons showed a wide range in marginal
258 variance explained. On average, adding the future position term improved variance explained by
259 6.89% (the mean of this proportion is significantly different from zero, $p < 0.001$, Wilcoxon sign-
260 rank test, **Figure 4A inset**).

261 We then asked whether these newly discovered *future position cells* constitute a separate
262 class of neurons from the cells that tracked the current position of the prey. To do this, we
263 computed the explanatory variance accounted for by future position (variance explained by the
264 combined model minus variance explained by current position) and current position, as defined
265 by log likelihood improvement (LLi) in fitting. We found a positive correlation between these
266 variables (**Figure 4B**), indicating current and future position were multiplexed in the same
267 population of cells ($r = 0.7394$, $p < 0.001$, cf.²⁹).

268 To quantify the difference between current and future position coding, we fit separate
269 models: one incorporated current position plus current Newtonian variables; the other was the
270 same but used future position (assuming $t=833$ ms) instead of current position. For the 36

271 neurons with significant tuning for both current and future position, we calculated the similarity
272 between the filters, using a technique known as *spatial efficiency*, SPAEF³⁰ (**Figure 4C**). A zero
273 SPAEF indicates orthogonal filters; positive SPAEF indicates similar filters; negative SPAEF
274 indicates anticorrelated filters. Although the mean of the spatial efficiency for our neurons was
275 positive, it was not significantly so, and spanned a large range of values from negative to positive
276 (mean of population spatial efficiency=0.0440, Wilcoxon sign-rank test, p=0.3790).

277 Finally, we assessed future encoding by examining the accuracy of model fitting to each
278 of several possible future times, ranging from 0 to 1333 ms in the future. We ran a type of sliding
279 window analysis that involved sampling one frame (16.67 ms) every ten frames (166.67 ms) and
280 ignoring the intervening nine frames. We found that the value of 833 ms fit the largest number of
281 neurons. (Values around it fit many neurons too). Specifically, the plurality, 24.67% of neurons,
282 were tuned for prey position at 833 ms (**Figure 4D**). The roughly equivalent value of the neural
283 and the behaviorally fit prospective distance (733 and 800 ms for those two subjects) suggests
284 that these neurons encode the future position of prey on the same approximate timescale as the
285 subject actively predicts.

286 We considered the possibility that this peak at 833 ms was due to some unanticipated
287 correlation between positions in the future and at the present. If this were so, then the average
288 distance of the self and/or prey would show a local minimum at a point in the future
289 corresponding to the peak. However, we did not see this. On the contrary, we found that the
290 distance increases monotonically for both subjects (**Figure 4E**).

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292 **State information is not confounded with gaze information**

293 Activity in dACC is selective for saccadic direction, and may therefore also correlate
294 with gaze direction (although this has not, to our knowledge, been shown³¹). Consequently, it is
295 possible that our spatial kernels may reflect not task state but gaze information. Specifically,
296 what appears to be tuning for future position may instead be attributable to the fact that monkeys
297 looked towards the predicted future prey position. We tested this possibility by calculating the
298 Euclidean distance between eye position and prey position in a range from -80 to +80 frames
299 (**Figure 5A**). The distance between eye and prey position was the closest at -5 frames (77.09
300 pixels), indicating that eye position *lagged* prey position. Thus, if gaze direction were a major
301 confound, it would show up as increased selectivity for past positions, not prediction of future
302 positions. Likewise, the chance that prey velocity encoding is a by-product of eye velocity
303 encoding was belied by the stark differences between gaze speed and prey speed ($p < 0.001$,
304 Wilcoxon sign rank test, but also clear from visual inspection of **Figure 5B**). Finally, we
305 repeated our GLM analyses (see above) but included eye position (only for the one subject from
306 which we collected gaze data). We found that that the number of tuned neurons for the prey did
307 not substantially change; that is, that adding in gaze position as a regressor did not qualitatively
308 change our results (**Figure 5C**).

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310 **Encoding of reward and reward proximity in dACC**

311 Research based on conventional choice tasks indicates that dACC neurons track values of
312 potential rewards³². We next asked how dACC encodes anticipated rewards in our more
313 complex task. We found that, averaging over all other variables, the value of the pursued reward
314 modulates activity of 8.67% of neurons (using a simple linear regression of firing rate against
315 value; this proportion is greater than chance, $p=0.038$, one-way binomial test). Note that this

316 analysis ignores the potential encoding of prey speed, which is perfectly correlated with static
317 reward in our task design. We hypothesized that reward/speed would be encoded in a modulatory
318 manner³³, that is, that the pursued reward/speed would alter the shape of the tuning for other
319 task variables, rather than be multiplexed (**Figure 6A**). To test this hypothesis, we split our
320 dataset by reward size and, as a control, split it randomly. We found that for several variables
321 (prey position, prey direction, and prey speed), value splits produced greater differences than
322 random ones (purple bar, $p = 0.0221$ for prey speed, and $p < 0.001$ for other prey variables,
323 **Figure 6B**). This result indicates that the reward information encoded in dACC interacts
324 mathematically with encoding of other variables. In other words, selectivity is mixed.

325 A good deal of research suggests that dACC neurons also signal the approach in time of
326 impending rewards³⁴⁻³⁶, even in continuous tasks^{37,38}. We thus asked whether it does so here.
327 We repeated our GLM, including relative (self-to-prey) distance as an explanatory variable. We
328 found that 38.67% of neurons ($n=58/150$) were tuned for self-prey distance. Interestingly, this
329 relationship is heterogeneous - of these 58 neurons, 31.03% ($n=18/58$) showed a positive slope
330 and 18.97% ($n=11/58$) showed a negative slope. This bias is not itself significant ($p=0.2649$ for
331 rise and fall bias, $n = 18/29$; $p = 1.000$ for monotonic bias, $n = 29/59$, binomial test in all cases).
332 This result indicates that while dACC neurons do track the approach to reward, they do not show
333 an overall rise or fall in activity as they do so.

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DISCUSSION

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Pursuit is an important element of the behavioral repertoire of many foragers^{2,6}. The

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algorithmic bases of pursuit have recently attracted the interest of scholars in ecology,

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engineering, psychology and other disciplines^{4,7,39-44}. Nonetheless, we know very little about

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how pursuit decisions occur in real time, and we know even less about their neuronal

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underpinnings. Here, we examined how macaques pursue virtual prey in a continuous, time-

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varying task. We developed a generative model based on a large dataset. The result from this

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model suggests that our subjects follow a predictive strategy. That is, instead of pointing towards

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the position of the prey, they extrapolate the future positions of prey and use this prediction to

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adjust their heading. This strategy is more efficient (yields more reward per unit time) but may

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be more computationally demanding than a simpler one that would involve pointing at and

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tracking the current position of the prey. These results demonstrate that pursuing animals can

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adopt complex future-predicting strategies that improve performance.

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We found that dACC neurons track the elemental physical variables our subjects use to

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predict the future and explicitly encode the prediction. Specifically, we found that firing rate

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responses of neurons in dACC encode three Newtonian variables (position, velocity, and

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acceleration) that our subjects used to track the prey and predict future prey positions. The same

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neurons carry an additional representation of the future position of the prey that is multiplexed

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with the Newtonian variables rather than maintained in a separate pool of specialized neurons.

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Both representations make use of a two-dimensional response field, akin to place fields in

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hippocampus, but not localized to a single position. Specifically, spatial representation in dACC

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is qualitatively similar to place representations of non-grid cells in entorhinal cortex²⁶. It is

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notable that dACC uses partially distinct spatial tuning functions to track the present and future

357 positions of the prey, thus in principle allowing unambiguous decoding for a given population
358 response.

359 Our work is directly inspired by important studies identifying mechanisms underlying
360 pursuit in other animals^{39,40,45}. Our work goes beyond these studies by developing a generative
361 model, that is, a model that seeks to understand how the data are generated⁴⁶. One benefit of the
362 generative model is that it lets us probe how the decision is made at every time step and make
363 guesses about the underlying mental process leading to decision. The generative model in turn is
364 vital for extending our understanding of mechanism to the neuronal level.

365 This model allows us to generate results that provide novel insight into the role of dACC
366 in cognition. First, our results emphasize the core role of dACC in prediction, a role that is
367 central to other theories, albeit not ones that directly involve pursuit^{11,17,20,21,47,48}. One recent
368 study is particularly relevant to these results²⁰. The authors examined hemodynamic activity in
369 human dACC during a complex decision-making task in which subjects had to track previous
370 rewards and use a reinforcement learning-like mechanism to formulate a future prediction and
371 make the best choice. They found that dACC tracks multiple variables, but was particularly
372 selective for long-term estimates of expected prediction errors. These results highlight the key
373 role of dACC in prediction in general and suggest its role is conserved across species (see also
374¹⁷). Second, our findings highlight the importance of dACC to navigation. While studies of
375 navigation typically focus on the medial temporal lobe, a growing body of work has begun to
376 explore the role of cingulate cortex, which receives direct projections from medial temporal
377 regions^{25,49}.

378 There are several important limitations to the present work. First, and most obviously, our
379 subjects were not performing a truly naturalistic task; they were performing a laboratory task that

380 required specialized training. Future studies will be needed to ascertain whether these results
381 relate to natural pursuit contexts that are ostensibly similar, such as pursuit of insects in the
382 peripersonal space^{50,51}. Second, and relatedly, the task space we used was greatly constrained -
383 both agents were restricted to a small rectangular space and had strict speed limits. Subjects had
384 full information about the position of the prey at all times. To understand prediction more fully,
385 it will be critical to extend to contexts in which some information is hidden.

386 Traditional laboratory tasks that study topics of interest to cognitive neuroscience -
387 decision-making and executive control - have discrete steps and force the brain to adjust to that
388 structure^{52,53}. One reason we developed the prey pursuit task is that it embeds those cognitive
389 processes in a continuous time-varying task. Doing so allows us to study one of the brain's
390 greatest strengths - its ability to adjust and change its mind on the fly as new evidence comes in
391⁵³⁻⁵⁷, and to incorporate that into future plans.

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555

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559

560 **Competing interests**

561 The authors have declared that no competing interests exist.

562

563 **Author Contributions**

564 SBMY and BYH conceptualized and designed the experiment. SBMY collected the data.
565 SBMY, and STP developed the behavioral model, SBMY, JCT, and BYH developed the
566 physiological model and analyzed the data. SBMY and BYH wrote the manuscript.

567 **Supplementary Material**

568 **Material and Methods**

569 **Subjects.** All animal procedures were approved by the University Committee on Animal
570 Resources at the University of Rochester and/or the University of Minnesota and were designed
571 and conducted in compliance with the Public Health Service's Guide for the Care and Use of
572 Animals. Three male rhesus macaques (*Macaca mulatta*) served as subjects for the behavior; two
573 of them also served as subjects for the physiology. Subjects had never previously been exposed
574 to decision-making tasks in which they could use a joystick to pursue a moving prey. Previous
575 training history for these subjects included two types of foraging tasks^{37,57}, intertemporal
576 choice tasks⁵⁹, several types of gambling tasks⁶⁰⁻⁶², attentional tasks (similar to those in ref⁶³),
577 and two types of reward-based decision tasks^{64,65}.

578 **Experimental Apparatus.** The joystick was a modified version of commercially
579 available joysticks with a built-in potentiometer (Logitech Extreme Pro 3D). The control bar was
580 removed and replaced with a control stick (a 15 cm plastic dowel) topped with a 3 cm diameter
581 plastic sphere designed to be easy for macaques to manipulate. The joystick position was read
582 out by a custom coded program in Matlab running on the stimulus-control computer. The
583 joystick was controlled by an algorithm that detected the positional change of the joystick and
584 limited the maximum pixel movement to within 23 pixels in 16.67 ms.

585 **Task Design.** At the beginning of each trial, two shapes appeared on a gray computer
586 monitor placed directly in front of the subject. The yellow (subject K) or purple (subjects H and
587 C) circle (15-pixel diameter) represented the subject. Subject position was determined by the
588 joystick and was limited by the screen boundaries. A square shape (30 pixel length) represented
589 the prey. The movement of the prey was determined by a simple AI (see below). Each trial ended

590 with either the successful capture of the prey or after 20 seconds, whichever came first.
591 Successful capture was defined as any spatial overlap between the avatar circle and the prey
592 square. Capture resulted in immediate juice reward; juice amount corresponded to prey color:
593 orange (0.3 mL), blue (0.4 mL), green (0.5 mL), violet (0.6 mL), and cyan (0.7 mL).

594 The path of the prey was generated interactively using A-star pathfinding methods,
595 which are commonly used in video gaming⁶⁶. For every frame (16.67 ms), we computed the cost
596 of 15 possible future positions the prey could move to in the next time-step. These 15 positions
597 were spaced equally on the circumference of a circle centered on the prey's current position, with
598 radius equal to the maximum distance the prey could travel within one time-step. The cost in turn
599 was based on two factors: the position in the field and the position of the subject's avatar. The
600 field that the prey moved in had a built-in bias for cost, which made the prey more likely to move
601 towards the center (**Figure 1B**). The cost due to distance from the subject's avatar was
602 transformed using a sigmoidal function: the cost became zero beyond a certain distance so that
603 the prey did not move, and it became greater as distance from the subject's avatar decreased.
604 Eventually, the costs from these 15 positions were calculated and the position with the lowest
605 cost was selected for the next movement. If the next movement was beyond the screen range
606 (1920x1080 resolution), then the position with the second lowest cost was selected, and so on.
607 The maximum speed of the subject was 23 pixels per frame (and each frame was 16.67 ms). The
608 maximum and minimum speeds of the prey varied across subjects and were set by the
609 experimenter to obtain a large number of trials (**Figure 1**). Specifically, speeds were selected so
610 that subjects could capture prey on <85% of trials; these values were modified using a staircase
611 method. If subjects missed the prey three times consecutively, then the speed of the prey was
612 reduced. Once the subject intercepted the prey in a trial where the staircase method was used,

613 then the selection of prey speed was randomized again. To ensure sufficient time of pursuit, the
614 minimum distance between the initial position of each subject avatar and prey was 400 pixels.

615 **Training Level Estimation.** Three subjects were trained for the same amount of time (8
616 weeks). As training progressed, each subject was exposed to a progressively more difficult
617 (faster) suite of prey, up to a fixed maximum. Subject K and subject H reached a similar range
618 for maximum speed of prey during the training period (K:15 pixels per frame; H: 14 pixel per
619 frame). However, subject C only attained a maximum speed of 8 pixels per frame (**Figure S6**). It
620 is for this reason we refer to him as the less well-trained subject.

621 **Behavioral Model.** To fit each subject's movement, each trial was divided into 1 second-
622 long segments. Each segment included 61 data points (because we used 16.67 ms resolution).
623 We modeled these trajectories using a single prediction and a single force parameter for the
624 entire trial, as a simplifying assumption. Nonetheless, it is reasonable to assume that throughout
625 a long, 20-second period, there would be active adjustment of prediction and force. Actual
626 comparison by AIC supported our intuition, and we used segment as the unit of analysis
627 throughout (values of 'AIC of segment/AIC of trial' was 0.9328, 0.9214, 0.9227, for subjects K,
628 H, and C (or whatever) respectively).

629 Overall, the position of the subject was generated according to the following:

630
$$P_{subject}(t + 1) = \kappa f(P_{subject}(t)) + m$$

631 where $P_{subject}(t)$ is position of the subject at time t , m is the inertia of subject as calculated from
632 the joystick, and κ is the force parameter. The vector $\kappa f(P_{subject}(t))$ was then summed with the
633 inertia m that was defined as following:

634
$$m = P_{subject}(t) - P_{subject}(t - 1)$$

635 $P_{prey}(t)$ indicates the position of the prey at time t . The function with respect to subject position at
636 time t was defined as:

$$f(P_{subject}(t)) = P_{prey}(t + 1) - P_{subject}(t)$$

637 Then the position of the prey at time $t+1$ was:

$$638 \quad P_{prey}(t + 1) = P_{prey}(t) + \tau \sum_{n=0}^k \frac{d^n P_{prey}(t)}{dt^n}$$

639 Where the n indicates the order of derivation with respect to the time. Thus, $n=1$ indicates
640 velocity, and $n = 2$ indicates acceleration.

641 The Physics Variable-Based Prediction (PVBP) model incorporates one previous time
642 step to predict the prey's next position. This approach is similar to a Kalman filter⁶⁷. The other
643 two models we tested do not utilize any past information. The model assuming prediction using
644 the cost contour map (CCMP model) considers only the lowest cost location at the next time
645 step. The model assuming veridical prediction (VP) automatically finds the exact position of the
646 prey at the next time step. Once the prey's position on the next time step is predicted, the model
647 computes how far this predicted position is from the agent's current position. A prediction value
648 of 1 indicates that the future position will be as far as from the agent's current position as the
649 prey's current position. The best-fitting parameter pairs were determined by performing a grid
650 search across the ranges of both parameters.

651 During this search, we tested the range of the prediction parameter between -400 to 400
652 subjects H and C, and -200 to 200 for subject K. (Units for this range correspond to the distance
653 the prey moved in the previous timestep). Subjects H and C had a larger range, because over 5%
654 of their trajectories resulted either in -200 or 200 in prediction parameter value. Representative
655 parameters for explaining each segment were selected based on the value of the sum of squared
656 error between the actual segment and the segment generated by the model.

657 **Significance Testing.** To determine whether the positive prediction parameter was
658 significantly greater than zero, we performed a bootstrap of heatmap slices from each segment.
659 This resampling was performed 500 times, and selected heatmaps were added. Then, the
660 parameter set resulting in the lowest cost was selected in each resampling.

661 **Model Evaluation.** To evaluate model performance and compare among models, we
662 computed the Akaike Information Criteria (AIC) using the likelihood of each model (**Figure 2**,
663 and **Figures S4 and S5**). We first calculated the mean and variance of all the sum-of-squared
664 errors across trajectories. Then we estimated the likelihood assuming a normal distribution
665 centered on the mean of the sum-of-squared errors with a variance equivalent to the variance of
666 the sum-of-squared errors across all trajectories. To validate whether subjects used a single
667 prediction and force across the all the trials or adaptively changed their prediction method, we
668 compared the AIC value between cases where the parameter pair varied across all trajectories,
669 using only the single best parameter pair.

670 **Electrophysiological recording.** One subject (H) was implanted with multiple floating
671 microelectrode arrays (FMAs, Microprobes for Life Sciences, Gaithersburg, MD) in the dorsal
672 anterior cingulate cortex (dACC). This is the region that we define as Area 24¹⁸ and that
673 corresponds to dACC in most other primate studies, including those from our lab^{37,60,68}. Each
674 FMA had 32 electrodes (impedance 0.5 MOhm, 70% Pt, 30% Ir) of various lengths to reach
675 multiple layers within dACC. Neurons from subject K were recorded with laminar V-probes
676 (Plexon, Inc, Dallas, TX) that had 24 contact points with 150 μ m inter-contact distance.
677 Continuous, wideband neural signals were amplified, digitized at 40 kHz and stored using the
678 Grapevine Data Acquisition System (Ripple, Inc., Salt Lake City, UT). Spike sorting was done

679 manually offline (Plexon Offline Sorter). Spike sorting was performed blind to any experimental
680 conditions to avoid bias.

681 **Details of LN model.** To test the selectivity of neurons for various experimental
682 variables, we constructed Generalized Linear Models with navigational variables (GLM^{26,28}).
683 The GLM models estimated the spike rate (r_i) of one neuron during time bin t as an exponential
684 function of the weighted sum of the relevant value of each variable at time t , which the weights
685 are determined by set of coefficients (w_i). The estimated firing rates from the GLM models can
686 be expressed as:

687
$$r = \exp\left(\sum_i X_i^T w_i\right) / dt$$

688 Where r denotes a vector of firing rates for one neuron over T time points across the session, and
689 i indexes the variables of interest, e.g. position of avatar on screen. The vector of firing rates over
690 T time points provides the benefit for modeling the neural activity without specific time-locking
691 to behavioral event. X_i is a matrix in which each column represents a set of “state variables” of
692 the animal (e.g. one of twelve speeds, determined by post-hoc binning) obtained from binning
693 the continuous variable so that all the columns for a particular row are 0, except for one column.
694 Unlike conventional tuning curve analysis, GLM analysis does not assume the parametric shape
695 of the tuning curve *a priori*. Instead, the weights, which define the shape of tuning for each
696 neuron, were optimized by maximizing the Poisson log-likelihood of the observed spike train
697 given the model-expected spike number, with additional regularization for the smoothness of
698 parameters in a continuous variable, and a lasso regularization for parameters in a discrete
699 variable. Position parameters were smoothed across rows and columns separately. The
700 regularization hyperparameter was chosen by maximizing the cross-validation log-likelihood
701 based on several randomly selected neurons. The unconstrained optimization with gradient and

702 Hessian was performed (MATLAB `fminunc` function). Model performance of each neuron was
703 quantified by the log-likelihood of held out data under the model. This cross-validation
704 procedure was repeated 10 times (10-fold cross-validation), and overfitting was penalized.
705 Through multiple levels of penalties, we can compare performance of models with varying
706 complexity.

707 **Forward model selection.** Model selection was based on the cross-validated log-
708 likelihood value for each model. We first fit n models with a single variable, where n is the total
709 number of variables. The best single model was determined by the largest increase in spike-
710 normalized log-likelihood from the null model (i.e., the model with a single parameter
711 representing the mean firing rate). Then, additional variables ($n-1$ in total) were added to the best
712 single variable model. The best two-variable model was preferred over the single variable model
713 only if it significantly improved the cross-validation log-likelihood (Wilcoxon Signed Rank Test,
714 $\alpha = 0.05$). Likewise, the procedure was continued for the three-variable model and beyond if
715 adding more variables significantly improved model performance, and the best, simplest model
716 was selected. The cell was categorized as not tuned to any of the variables considered if the log-
717 likelihood increase was not significantly higher than baseline, which was mean firing rate of
718 fitted neurons across the session.

719 **Future position models.** We examined effect of future position by fitting a GLM having
720 ‘future position’ and ‘current position’ together as the input variable. Then we compared to the
721 GLM model with only current position. Difference between the two models was evidence that
722 additional variance was explained by including future position.

723 **Comparison between current and future position filters.** For this purpose, we
724 constructed two GLMs: one with current position and current Newtonian variables (velocity and

725 acceleration), and another with future position and current Newtonian variables. Then we
726 selected the neurons that showed significant tuning for both models. To compare the similarity
727 between two positional filters, we used the SPAtial EFficiency metric (SPAEF) that prior
728 literature suggests to be more robust than the 2D spatial correlation³⁰. It quantifies the similarity
729 between two maps:

$$730 \quad \xi_{SPAEF} = 1 - \sqrt{(A-1)^2 + (B-1)^2 - (C-1)^2}$$

731 A is the Pearson correlation between two maps, B is the ratio between the coefficients of
732 variation for each map, and C is the activity similarity measured by histogram profiles. Values
733 near -1 indicate anticorrelated maps (one tends to be high when the other is low); 0 indicates
734 uncorrelated maps; 1 indicates perfect matching between the two.

735 **Velocity Dependent Physics Variable-Based Model (PVMP) Prediction Bias.** We
736 examined whether PVBP is preferred when the velocity of prey is high (**Figure S6**). We first
737 obtained the average velocity of the prey at each segment, and then categorized each segment as
738 belonging to either the physics or non-physics variable-based prediction based on which fit result
739 was best. With the prey velocity and segment category, we performed logistic regression with
740 velocity as a predictor and category as the dependent variable (glmfit in MATLAB).

741 **Data availability.** The datasets generated during the current study are available on the
742 Hayden lab website, <http://www.haydenlab.com/>, or from the authors on reasonable request. The
743 code generated to perform the analyses for the current study is available from the corresponding
744 author.

Figure Captions

745
746

747 **Figure 1.** Experimental paradigm and behavioral results. **(A)** Cartoon of *virtual pursuit task*.
748 Subject uses a joystick to control an avatar (circle) and pursue prey (square) on a computer
749 screen. **(B)** Raincloud plot showing each subject's capture times in an example session (limit was
750 20 seconds). The box plot indicates 2nd and 3rd quartile of the data; midline indicates the
751 median of the data (K: 3.36 sec, H: 3.73 sec, C: 3.93 sec). The dots under the probability density
752 functions indicate individual data points. **(C)** Avatar and prey trajectories on example trials.
753 Grey: path of avatar; red/blue: path of prey. Color gradient indicates the time progression
754 through the trial.

755

756 **Figure 2.** Model description and fitting results. **(A)** Cartoon of model for generating future
757 position based on prediction. Solid black arrow indicates movement from previous time frame to
758 the current one. Subjects are assumed to aim at a point that leads (red solid arrow) or lags (cyan
759 solid arrow) the prey. The resulting movement (red/blue dashed arrow) vectors are constrained to
760 a maximum speed and inertia (black dashed arrow). **(B)** Fitting results: Akaike Information
761 Criterion (AIC, left) across all the trajectories and percentage of trials best explained by each
762 model (right). For calculating the AIC, we summed the log-likelihood across the whole data set
763 from each subject individually and used the quantity (2 x number of the segments) as the number
764 of free parameters. This quantity was: subject K, 28,164; subject H, 35,308; subject C, 20,720
765 parameters. Predictive models provide better fits than zero prediction ones. **(C)** Example
766 trajectories and corresponding fit trajectories generated by predictive and non-predictive models.
767 **(D)** Heatmap plots of model performance explaining subject's pursuit segment across parameter
768 space from a single subject (Subject K) for physics based model (left), cost model (center), and
769 veridical (right). The small gray circle at the peak indicates the best parameter combination
770 explaining that subject's behavior, that is, the one that generates the smallest distance between
771 the actual segment and model-predicted segment.

772

773 **Figure 3.** Basic neural results. **(A)** Cartoon showing location of recorded brain areas in dACC;
774 sagittal and coronal views. **(B)** Filters (tuning surfaces) of two example neurons showing
775 selectivity for current position of the prey. **(C)** Example neuron showing tuning for speed (black
776 line) and the corresponding model fit (magenta line). **(D)** Example neuron showing tuning for
777 prey direction (black line) and the corresponding model fit (blue line). **(E)** Preponderance of
778 tuning for the Newtonian physics variables tested. Tuning for future position is counted only if
779 the neuron is selectively tuned for future position above and beyond current position.

780

781 **Figure 4.** Properties of future position selectivity. **(A)** Proportion of variance explained by
782 including future position in each neuron (only neurons that are selective for current position are
783 shown). Neurons are sorted according to amount of additional variance explained by future
784 position. **(B)** Log likelihood increase (LLi, a measure of explanatory power) for current and
785 future position are correlated on a cell-by-cell basis. Red solid line indicates the linear regression
786 line. **(C)** Example filters from neurons that are significantly tuned for both current and future
787 prey position. Spatial efficiency (SPAEF), a measure of the similarity of two-dimensional
788 filters²⁹, is shown on the y-axis of the central plot. A more positive SPAEF indicates that the

789 matrices are more similar to each other; low values indicate orthogonality. Only significant
790 neurons are shown; cells are sorted by spatial efficiency. **(D)** Sliding window analysis for future
791 position encoding strength. Plot shows proportion of neurons significantly selective for future
792 position at several possible future delays. This curve peaks at around 700-800 ms, which
793 corresponds to the average prediction distance for all three subjects. **(E)** The distance between
794 current prey position and future prey position at time t rises roughly linearly with time. This
795 finding indicates that the peaks found in panel D are not likely to be an artifact of some
796 unforeseen periodicity in the relative paths of the subject and prey.

797
798 **Figure 5.** Analyses that control for potential gaze confounds. **(A)** The Euclidean distance
799 between the eye position at $t=0$ and prey position (orange solid line) / self position (blue solid
800 line). Error bar = SEM, and is the width of the lines shown. **(B)** Speed distribution of prey
801 movement and smooth eye pursuit. **(C)** Proportion of neurons tuned for three key variables using
802 the standard GLM described above and another version that assigns variance to eye position first.
803 All three variables are still significant in the population when including gaze position.

804
805 **Figure 6.** Modulatory effect of reward size on tuning for prey variables. **(A)** Responses of an
806 example neuron selective for the angle between self and prey; changes in the reward size of prey
807 (divided into three bins) appear to change the gain and not the offset of the neurons; that is,
808 reward interacts multiplicatively with angle. **(B)** This pattern is also observed in the population.
809 The proportion of neurons significantly tuned for prey variables (prey position, prey direction,
810 and prey speed) when splitting data randomly (grey bar) or according to value of pursued prey
811 (purple bar). The difference of value split was significant ($p = 0.0221$ for prey speed, and $p <$
812 0.001 for other prey variables).

813

Supplementary Figure Captions

814

815 **Figure S1. Subjects' behavior varies according to prey speed/reward.** (A) Mean prey
816 velocity in each segment plotted separately for each subject. Pursuit result differs according to
817 color (equivalent to maximum speed) of prey. The maximum speed of prey increases from
818 orange (slowest with smallest reward) to cyan (fastest with largest reward). As maximum speed
819 increases, the mean capturing time (B) and percent of failed trials increases (C). However,
820 reward rate also increases, since the amount of reward is larger for faster prey (D). Errorbars are
821 the standard error of the mean, obtained by bootstrapping (1000 bootstraps).

822

823 **Figure S2. Different prediction strategies and influence of different amounts of inserted**
824 **force.** (A) The strategic difference between physics variable based prediction (PVBP, red lines)
825 and veridical prediction (VP, blue lines). This generates different predictive points. (B) Effect of
826 inserted force, shown between small (pink) and large (purple) forces. Vector-summation with
827 inertia yields different outcomes for different force conditions.

828

829 **Figure S3. Dynamic changes of parameter sets at each segment explain each subject's**
830 **segments better than identical single parameter set across all the trajectories.** AIC
831 comparison between the case of the single parameter set across all the sessions (case 1) or
832 adaptively changing parameter set at each segment (case 2). Delta AIC indicates the difference
833 between the cases (case 1 - case 2), and a positive value indicates adaptively changing the
834 strategy explains subject's segment better, even if there is a penalty for having more parameters.
835 Each column shows an individual subject's result.

836

837 **Figure S4. Including an inertia term improves model performance.** (A) Model segment
838 comparison between models with and without inertia. (B) Histogram results suggest that
839 incorporating an inertia component to the model leads to a better fit of the data (mean of sum-of-
840 squared error difference below zero at x-axis). 95% of data fall to the right of the black, dashed
841 line. Bootstrapping of difference in performance between the model with and without inertia was
842 performed in randomly sampled trajectories (number of resamples: 1000; randomly selected
843 trajectories: 2000).

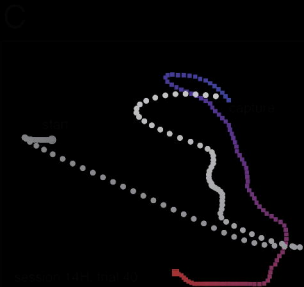
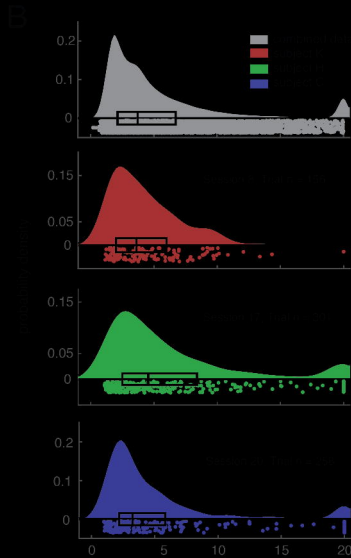
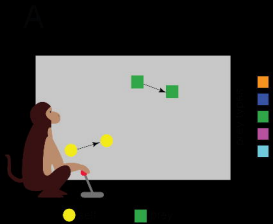
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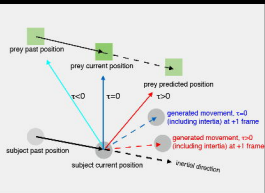
845 **Figure S5. Additional terms after acceleration don't improve model performance.** (A) Each
846 heatmap indicates the addition of more physical derivatives of position. The black circle
847 indicates the best parameter set for the model. (B) Summary bar graph. Physics include within-
848 physics prediction model comparison (from velocity to pop, the 6th derivative).

849

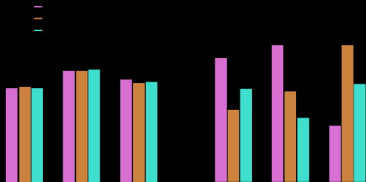
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851 **Figure S6. Prey velocity dependent strategy selection.** All the subjects consistently show
852 biases using PVBP when the prey velocity is faster. Logistic regression was performed between
853 prey velocity and a categorical dependent variable (0: non-PVBP, 1: PVBP). The p-values of all
854 logistic coefficients were significant ($p < 0.001$).





screen X



physics-based model



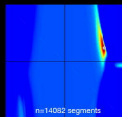
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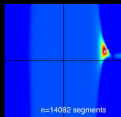
veridical model



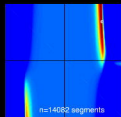
Y



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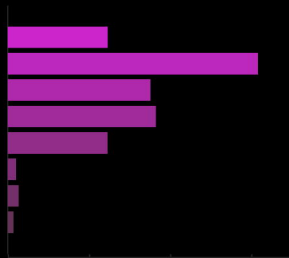
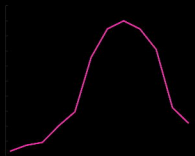
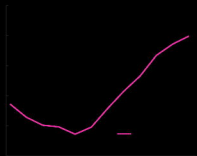
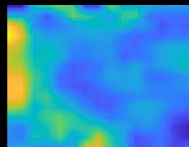
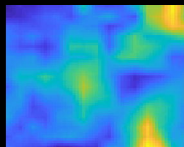
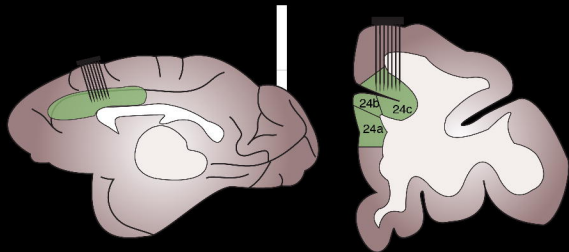


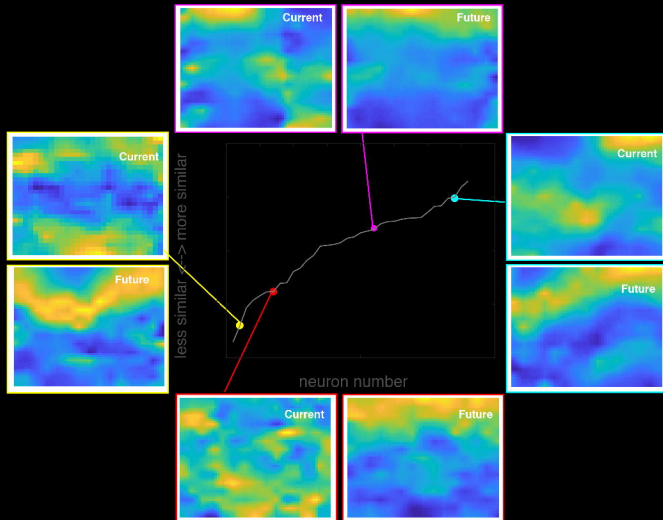
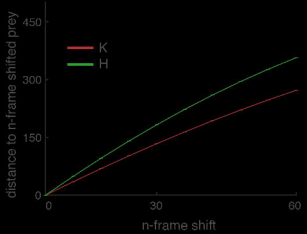
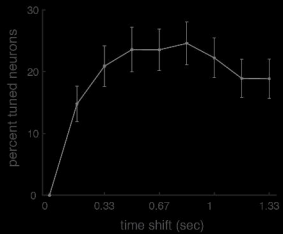
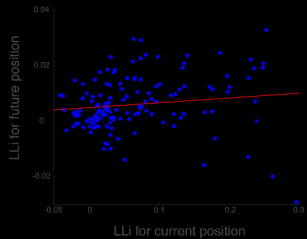
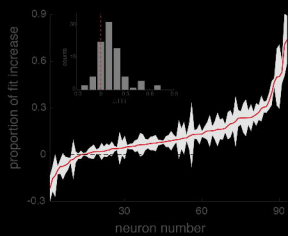
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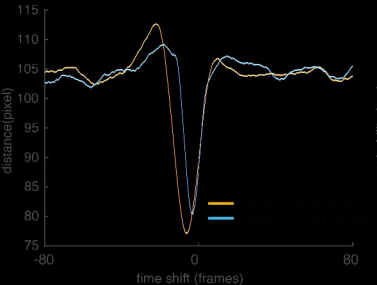
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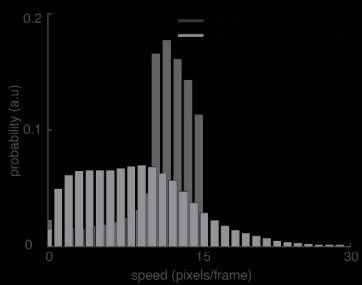




A



B



C

