Supplementary Information for manuscript entitled: A framework to improve predictions of warming effects on consumer-resource interactions

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Supplementary Information 1: Framework for general form functional type response

Here we demonstrate how the framework can be applied to the Rosenzweig-MacArthur model(Rosenzweig & MacArthur 1963) with the general form of the functional response (i.e. not confined to the Holling type II). The model equations are:

$$\frac{dR}{dt} = r\left(1 - \frac{R}{K}\right)R - \frac{aR^q}{1+ahR^q}C \qquad \text{(S1.1)}$$

$$\frac{dC}{dt} = \left(e\frac{aR^q}{1+ahR^q} - m\right)C \qquad \text{(S1.2)}$$

R is the resource species biomass density and C the consumer species biomass density $[\frac{g}{m^3}]$. Resource growth is assumed to be logistic, with an intrinsic growth rate r $[\frac{1}{d}]$ and carrying capacity K $[\frac{g}{m^3}]$. Resource biomass density is limited by the consumer through the functional response with consumer attack rate a $[\frac{m^{3q}}{g^q d}]$ and handling time, h $[d]$. Consumer growth is proportional to the assimilated consumed biomass, with e the dimensionless assimilation efficiency, while losses occur due to metabolic costs, m $[\frac{1}{d}]$. q – a dimensionless parameter – determines the shape of the functional response; for the Holling type II response, $q = 1$, for Holling type III, $q = 2$. We should note that the biomass densities and carrying capacity need not be with respect to volume, $[\frac{g}{m^3}]$; the equations apply equally to surface, $[\frac{g}{m^2}]$. This would require the attack rate to have units, $[\frac{m^{2q}}{g^q d}]$.

The coexistence equilibrium for the general form is:

$$R_S = \left(\frac{m}{a(e-mh)}\right)^{1/q} \tag{S1.3}$$

$$C_S = \frac{r}{aK}\left(\frac{m}{a(e-mh)}\right)^{1-q}\left(K - \left(\frac{m}{a(e-mh)}\right)^{\frac{1}{q}}\right)\left(\frac{e}{e-mh}\right) \tag{S1.4}$$

$$\mathcal{B} = \frac{C_S}{R_S} = \frac{r}{aK}\left(\left(\frac{m}{a(e-mh)}\right)^{\frac{1}{q}}\right)^{1-q}\left(K - \left(\frac{m}{a(e-mh)}\right)^{\frac{1}{q}}\right)\left(\frac{e}{e-mh}\right) \tag{S1.5}$$

The aggregate parameters are the maximal energetic efficiency, ρ, and interaction strength, κ. ρ is determined as the maximal possible energetic gains of the consumer, which is reflected by the ratio of maximal potential feeding rate to metabolic losses (Yodzis & Innes 1992). Maximal feeding rate means a saturated functional response (i.e. unlimited resources and no consumer self-limitation). The functional response, $f(R) = \frac{aR^q}{1+ahR^q}$ saturates at $\max(f(R)) = \frac{1}{h}$. Thus, the term for ρ will be:

$$\rho = \underbrace{e}_{\substack{consumer \\ assimilation \\ efficiecny}} * \underbrace{\frac{1}{h}}_{\substack{maximal \\ consumption\,rate}} * \underbrace{\frac{1}{m}}_{\substack{consumer \\ metabolic \\ loss}} = \frac{e}{mh} \tag{S1.6}$$

The interaction strength, κ, is defined as the ratio of the resource equilibrium in the absence of consumers (i.e. carrying capacity) to the resource equilibrium with consumers (i.e. coexistence resource equilibrium)(Berlow *et al.* 1999; Gilbert *et al.* 2014).

$$\kappa = \frac{K}{\left(\frac{m}{a(e-mh)}\right)^{1/q}} = K\left(ah\left(\frac{e}{mh} - 1\right)\right)^{1/q} \tag{S1.7}$$

Biologically meaningful values require that R_S and C_S are positive, which gives us two conditions on the existence of the coexistence steady state:

$$e - mh > 0 \Leftrightarrow \rho > 1 \qquad \text{(S1.8)}$$

$$K\left(a(e - mh)\right)^{\frac{1}{q}} - m^{\frac{1}{q}}) > 0 \Leftrightarrow \kappa > 1 \qquad \text{(S1.9)}$$

As explained in the main text, condition (S1.9) is necessary and sufficient for coexistence. Therefore, the community feasibility boundary is determined by (S1.9).

The Hopf bifurcation condition can be expressed in terms of ρ and κ for the general form of the functional response. The Hopf bifurcation occurs when the trace of the characteristic matrix, calculated at the equilibrium values of resource and consumer, becomes zero. The condition is:

$$m^{\frac{1}{q}}(K\left(\frac{a(e-mh)}{m}\right)^{\frac{1}{q}} - 1))(1 - \frac{q(e-mh)}{e}) - m^{\frac{1}{q}} = 0 \qquad \text{(S1.10)}$$

By introducing ρ and κ this becomes:

$$(\kappa - 1)\left(1 - \frac{q(\rho-1)}{\rho}\right) - 1 = 0 \Leftrightarrow \kappa = \frac{\rho(2-q)+q}{\rho(1-q)+q} \qquad \text{(S1.11)}$$

Thus, we have shown how the aggregate parameters can be applied to determine the community feasibility threshold (S1.9) and the Hopf bifurcation (S1.11) for the general form of the functional response.

Below we demonstrate that our framework can be applied to the biomass ratio and stability for the general form of the functional response. For the biomass ratio, we derive the elasticities, but for stability no metric exists that covers the general form of the functional response. Therefore, we will illustrate how the qualitative aspects of the framework's approach can still be applied to investigate stability properties in the $\rho - \kappa$ plane.

Biomass ratio:

The biomass ratio was determined to be $\mathcal{B} = \frac{r}{aK}\left(\left(\frac{m}{a(e-mh)}\right)^{\frac{1}{q}}\right)^{1-q}(K - \left(\frac{m}{a(e-mh)}\right)^{\frac{1}{q}})(\frac{e}{e-mh})$.

The elasticities with respect to each parameter, x, are defined as $\partial_x \mathcal{B} = \frac{\partial \ln(\mathcal{B})}{\partial \ln(x)}$. Then, the elasticities for the general form functional response will be:

Table S1: Elasticities of the biomass ratio ($\partial_x \mathcal{B}$) with respect to the six original model parameters. All sensitivities are expressed in terms of ρ and κ.

Parameter	Consumer-resource biomass ratio, $\mathcal{B}$
x	$\partial_x \mathcal{B} = \dfrac{\partial \ln(\mathcal{B})}{\partial \ln(x)}$
r	1
K	$\dfrac{1}{\kappa - 1}$
a	$\dfrac{1}{q}\dfrac{1}{\kappa - 1}$

h	$-\dfrac{1}{q}\dfrac{1}{(\rho-1)(\kappa-1)}$
e	$\dfrac{1}{q}\dfrac{\rho}{(\rho-1)(\kappa-1)}+1$
m	$-\dfrac{1}{q}\dfrac{\rho}{(\rho-1)(\kappa-1)}-1$

As we argued, all elasticities are expressible in terms of ρ and κ. Interestingly, the important results of our sensitivity analysis hold. In particular, far from the consumer extinction, when κ is large, $\partial_K\mathcal{B}$, $\partial_a\mathcal{B}$, $\partial_h\mathcal{B}$ all tend towards 0, $\partial_e\mathcal{B}$ and $\partial_m\mathcal{B}$ towards 1 and -1, respectively. Therefore, $\partial_e\mathcal{B} = -\partial_m\mathcal{B} \approx \partial_r\mathcal{B} = 1$ will be the group of largest elasticities and e, m and r will be have the highest relative impact on biomass distributions away from the community boundaries. Similarly, close to the community feasibility boundary, $\kappa = 1$, all elasticities $\partial_e\mathcal{B}, \partial_m\mathcal{B}, \partial_K\mathcal{B}, \partial_a\mathcal{B}, \partial_h$ diverge apart from $\partial_r\mathcal{B} = 1$ which remains constant.

Stability

We have already shown that the condition for the Hopf bifurcation can be expressed solely in terms of ρ and κ (S1. 11, Fig. S1). The stability metric used in the main study (Johnson & Amarasekare 2015), effectively measures the distance from the Hopf bifurcation in the $\rho - \kappa$ plane. Therefore, any distance metric with respect to the Hopf bifurcation and its sensitivities will be expressible in terms of of ρ and κ, as will its sensitivities.

We see that with increasing q, the Hopf bifurcation condition becomes steeper. This means that the area below the curve which corresponds to stable equilibrium dynamics grows and hence that a type III functional response leads to more stable dynamics, as has been previously

demonstrated (Uszko *et al.* 2017; Daugaard *et al.* 2019). In terms of the effects of warming on stability, two distinct regions emerge: at low maximal energetic efficiency, $\rho < 2$, changes in interaction strength, κ, determine whether warming will stabilise (decreasing κ) or destabilise (increasing κ) dynamics. At higher values of ρ, $\rho > 2$, dynamics are effectively stable.
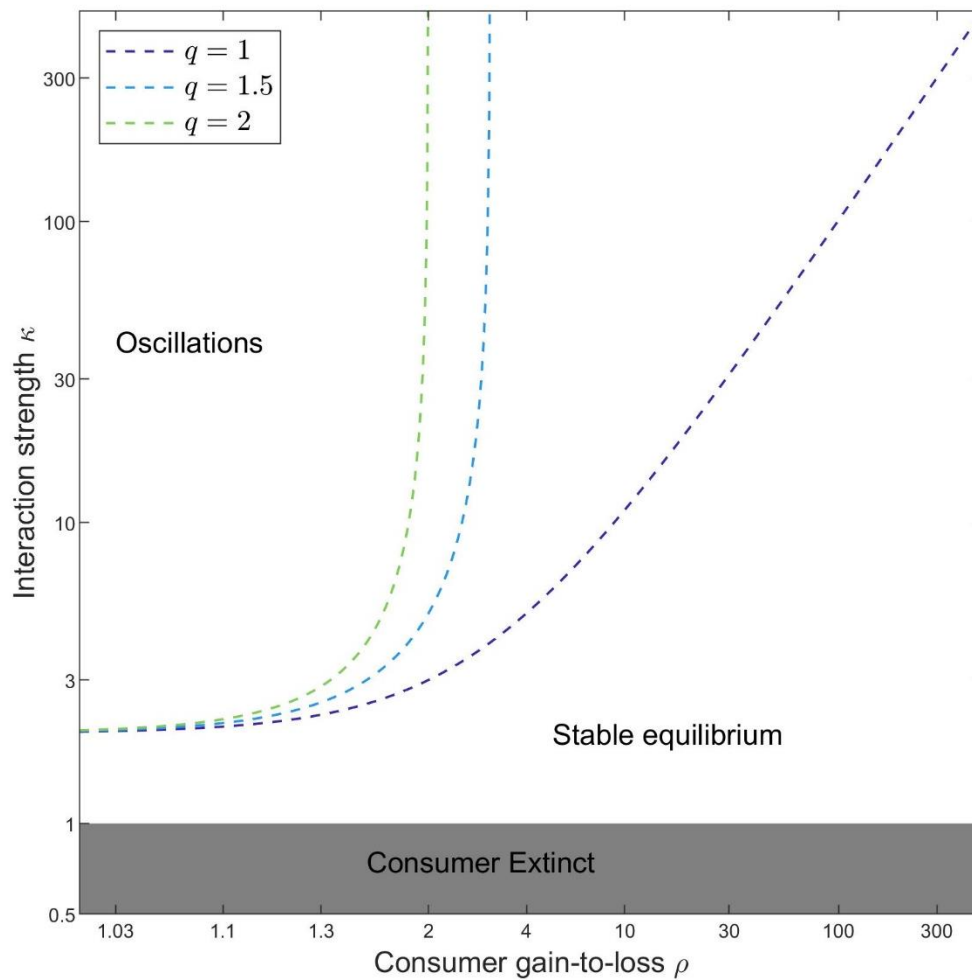

Figure S1. The Hopf bifurcation in the of $\rho - \kappa$ plane for different values of of q, i.e. functional response types. $q = 1$ corresponds to the Holling type II and $q = 2$ to the Holling type III.

Supplementary Information 2: type II functional response alternative formulation

The relationship between consumer per capita feeding rate and resource density, $f(R)$ $[\frac{1}{time}]$, simply known as the functional response can be represented in terms of attack (or search) rate, a $[\frac{volume}{mass*time}]$, and handling time (or inverse maximal intake rate), h $[time]$ (Holling 1959), for a type II response:

$$f(R) = \frac{aR}{1+ahR} \qquad \text{(eq S2. 1)}$$

Alternatively, the functional response can be reformulated as the Michaelis-Menten equation where the shape of the curve is controlled by the maximum consumption rate, J $[\frac{1}{time}]$, and half-saturation density, R_0 $[\frac{mass}{volume}]$ - the resource density corresponding to the point of half-saturation of the functional response, $f(R_0) = \frac{1}{2}\max f(R)$. Thus, the functional response becomes:

$$f(R) = \frac{JR}{R_0+R} \qquad \text{(eq. S2.2)}$$

The two formulations are interchangeable through a simple transformation:

$$J = \frac{1}{h} \qquad \text{(eq. S2.3)}$$

$$R_0 = \frac{1}{ah} \qquad \text{(eq. S2.4)}$$

The coexistence steady state becomes:

$$R_S = \frac{mR_0}{eJ-m} \qquad \text{(eq. S2.5)}$$

$$C_S = erR_0 \frac{K(eJ-m)-mR_0}{K(eJ-m)^2} \qquad \text{(eq. S2.6)}$$

The aggregate parameters:

$$\rho = \frac{eJ}{m}, \kappa = \frac{K(eJ-m)}{mR_0} = \frac{K}{R_S} \qquad \text{(eq. S2.7)}$$

Parameter	C:R biomass ratio, $\mathcal{B}$	Stability metric, $\mathcal{S}$
x	$\partial_x \mathcal{B} = \dfrac{\partial \ln(\mathcal{B})}{\partial \ln(x)}$	$\partial_x \mathcal{S} = \dfrac{\partial \mathcal{S}}{\partial \ln(x)}$
r	1	0
K	$\dfrac{1}{\kappa - 1}$	$-\dfrac{\kappa}{\rho - 1}$
R_0	$-\dfrac{1}{\kappa - 1}$	$\dfrac{\kappa}{\rho - 1}$
a	$\dfrac{1}{\kappa - 1}$	$-\dfrac{\kappa}{\rho - 1}$
h	$-\dfrac{1}{(\rho - 1)(\kappa - 1)}$	$\dfrac{\kappa(1-\rho) + 2\rho}{(\rho - 1)^2}$

J	$\dfrac{\rho}{(\rho-1)(\kappa-1)}+1$	$-\dfrac{2\rho}{(\rho-1)^2}$
e	$\dfrac{\rho}{(\rho-1)(\kappa-1)}+1$	$-\dfrac{2\rho}{(\rho-1)^2}$
m	$-\dfrac{\rho}{(\rho-1)(\kappa-1)}-1$	$\dfrac{2\rho}{(\rho-1)^2}$

Table S2. Sensitivities of biomass ratio and stability to all parameters, including maximum consumption rate, J, and half-saturation density, R_0.

Thus, we can see that:

$$\partial_K \mathcal{B} = \partial_a \mathcal{B} = -\partial_{R_0} \mathcal{B} \text{ and } \partial_K \mathcal{S} = \partial_a \mathcal{S} = -\partial_{R_0} \mathcal{S}$$

$$\partial_e \mathcal{B} = \partial_J \mathcal{B} = -\partial_e \mathcal{B} \text{ and } \partial_e \mathcal{S} = \partial_J \mathcal{S} = -\partial_e \mathcal{S}$$

Therefore, our results in the main text directly apply to the Michaelis-Menten formulation of the functional response. We also see that when using J and R_0 the sensitivities can be reduced to two groupings, which simplifies analyses further.

Supplementary Information 3: Stability metric adjustment

In our study, we qualified stability in terms of the steady state being a stable equilibrium or a limit cycle (oscillations around unstable equilibrium). To this end we used a metric which quantifies the tendency for oscillations in consumer-resource dynamics (Johnson & Amarasekare 2015). The metric was defined as $\phi = ahK$; the smaller the value the more 'stable' the dynamics. In particular, if $\phi < 1$ the dynamics are stable, if $\phi > 1$ they are unstable. Effectively, this metric was deduced as, what its authors referred to, a 'conservative' criterion for stability (for details see derivation of eq. (5) in (Johnson & Amarasekare 2015).

Expressed in terms of the the aggregates ρ and κ, $\phi = \frac{\kappa}{\rho-1}$. We adjusted $\phi = 1$ to create a metric, S, which vanished ($S = 0$) at the Hopf bifurcation, was positive for the stable equilibrium ($S > 0$) and negative for oscillations around the unstable equilibrium ($S < 0$). The Hopf bifurcation in terms of ϕ is given by $\phi_H = \frac{\rho+1}{\rho-1}$. Therefore, for S to vanish at the Hopf bifurcation, we took $S = \phi_H - \phi = \frac{\rho+1}{\rho-1} - \frac{\kappa}{\rho-1} = \frac{-(\kappa-\rho-1)}{\rho-1}$. Thus, $S = 0$ at the Hopf bifurcation, $S > 0$ when the equilibrium is stable and $S < 0$ when the oscillations around the unstable equilibrium occur.

Supplementary Information 4: Temperature parameterisations from the literature

In our study, we used the Rosenzweig-MacArthur model (Rosenzweig & MacArthur 1963) to derive the framework combining a sensitivity analysis and the aggregate parameters, maximal energetic efficiency, ρ, and interaction strength, κ. We argued that the framework can be applied to any temperature parameterisation which can be fitted to the Rosenzweig-MacArthur model. We implemented parameterisations from six studies from the literature with empirically-derived thermal dependencies of certain parameters and assumptions about the thermal dependence of others. We should note that not all studies measured or reported a thermal performance curve for the resource growth rate, r. Therefore, the consumer-resource biomass ratio (eqs. 5 and S1.5) was evaluated and presented only for two studies (Fig. 2). However, biomass ratio sensitivities can be expressed in terms of $\rho = \frac{e}{mh}$ and $\kappa = ahK\left(\frac{e}{mh} - 1\right)$ and hence no r thermal dependence is necessary to plot the parameterisation trajectories in the $\rho - \kappa$ plane. As such, all six studies utilised in our application of the framework (Fig. 4 and 7) reported parameterisations enabling us to derive the thermal dependencies of ρ and κ. Finally, we note that these paramtersiations all assumed a type II functional response. Below we briefly describe each study before providing details of how we implemented their parameterisations.

Brief description of studies:

Uszko *et al.* (2017)investigated the effects of warming on the stability of consumer-resource communities using the Rosenzweig-MacArthur model. Their model was parameterised for the crustacean *Daphnia hyaline* feeding on the green alga *Monoraphidium minutum*.

Fussmann *et al.* (2014) combined a meta-analysis of global invertebrate data and literature sources to determine the thermal dependencies of biological rates implemented in a bioenergetic model (Yodzis & Innes 1992). The functional response was represented in terms of maximum consumption rate, J, and resource half-saturation density, R_0 (see Supplementary Information S2).

Vucic-Pestic *et al.* (2011): an empirical study of how the functional response (attack rate, handling time) and consumer metabolism scale with temperature in a predator-prey community with three different size-classes of predators and two types of prey (mobile and resident). Other parameter values were taken from the literature. No model was employed, though the energetic efficiency term was based on Vasseur and McCann (2005).

Binzer *et al.* (2016) performed a modelling study of the interactive effects of eutrophication and warming on food webs parameterised from the literature. The population dynamics between successive trophic levels were described using a bio-energetic model (Yodzis & Innes 1992).

Sentis *et al.* (2012): experimental data from a ladybeetle (*C. Maculata lengi*) feeding on a green peach aphid (*Myzus Persicae*) used to estimate how the functional response scaled with temperature. No model was employed, though the energetic efficiency term was based on Vasseur and McCann (2005).

Archer *et al.* (2019): a comparison of the thermal dependence of the functional responses derived from field and lab measurements. A sedentary (larvae of *L. riparia*) and a mobile (larvae of *P. cingulatus*) predator fed on a common prey (blackfly larvae from the *Simuliidae*

family). A per capita energy feeding rate was derived, which was then transformed to the dimensionless energetic efficiency – as in the previous studies.

Parameter values in studies:

In the following equations, temperature T is in Kelvin, where 273.15K corresponds to 0°C. In all Arrhenius-based parameterisations, $k = 8.62 \times 10^{-5}$ $[eVK^{-1}]$ is the Boltzmann constant. r is the resource population growth rate (time^{-1}), a is the attack or search rate (surface or volume per unit of time and resource unit), h is the handling time (time), m is the metabolic rate (time^{-1}) or m_E the standard metabolic rate (energy*time^{-1}), J is the maximum feeding rate (time^{-1}), R_0 the resource-half saturation density (biomass or resource unit per surface or volume), K is the resource carrying capacity (biomass or resource unit per surface or volume) and e is the consumer assimilation efficiency (unitless).

<u>Uszko et al. (2017):</u>

$$r(T) = r_0 e^{-\frac{\left(T - T_{opt}\right)^2}{2s^2}} \text{ with } r_0 = 2.2, T_{opt} = 298.15, s = 12, [d^{-1}]$$

$$a(T) = a_0 e^{-\frac{\left(T - T_{opt}\right)^2}{2s^2}} \text{ with } a_0 = 6, T_{opt} = 296, s = 9.4, [m^3 g^{-1} d^{-1}]$$

$$h(T) = h_0 e^{\frac{\left(T - T_{opt}\right)^2}{2s^2}} \text{ with } h_0 = 0.2, T_{opt} = 294.1, s = 7.2, [d]$$

$$m(T) = m_0 e^{\frac{-E_m}{kT}} \text{ with } m_0 = 4.4 * 10^8, E_m = 0.55, [d^{-1}]$$

K and e are not functions of temperature; we used $e = 0.385$ and $K = 2, [gm^{-3}]$.

Fussmann et al. (2014)

Reference temperature $T_{ref} = 20°C = 293.15K$. Carnivore assimilation efficiency was used:

$e = 0.85$

$$r(T) = r_0 e^{\frac{E_r(T-T_{ref})}{kTT_{ref}}} \text{ with } r_0 = 8.715 * 10^{-7}, E_r = 0.84, [s^{-1}]$$

$$K(T) = K_0 e^{\frac{E_K(T-T_{ref})}{kTT_{ref}}} \text{ with } K_0 = 5.623, E_K = -0.772, [gm^{-2}]$$

$$R_0(T) = R_{0_0} e^{\frac{E_{R_0}(T-T_{ref})}{kTT_{ref}}} \text{ with } R_{0_0} = 3.664, E_{R_0} = -0.114, [gm^{-2}]$$

$$J(T) = J_0 e^{\frac{E_J(T-T_{ref})}{kTT_{ref}}} \text{ with } J_0 = 8.408 * 10^{-6}, E_J = 0.467, [s^{-1}]$$

$$m(T) = m_0 e^{\frac{E_m(T-T_{ref})}{kTT_{ref}}} \text{ with } m_0 = 2.689 * 10^{-6}, E_m = 0.639, [s^{-1}]$$

Vucic-Pestic et al. (2011):

$T_{ref} = 290.65K$ and $e = 0.85$.

Predators were split into three mass classes.Predator masses within the same size-classes varied with temperature, so we used the mean value over all temperatures to obtain the mean mass of each size class. Additionally, predator masses varied between the experiments for the two prey types.

Prey mass (m_R) [mg]	1.91 (mobile)	23.26 (resident)
Small predator mass (m_C) [mg]	66.1617	72.6217
Medium predator mass (m_C) [mg]	121.2167	134.8133
Large predator mass (m_C) [mg]	147.8683	160.6550

Parameter thermal dependencies also scale with body mass. For the dimensions in the equations to be correct, we introduce a reference body mass for consumers and resources, $m_{C_0} = 1mg$ and $m_{R_0} = 1mg$, respectively.

We also note that the study does not explicitly measure or define a resource carrying capacity. However, it does provide an equation of the thermal dependence of the prey population. We assumed this to be the carrying capacity in our calculation of κ.

Finally, in this study, the authors measure the standard metabolic rate with units of energy over time [Jh^{-1}]. For the calculation of maximal energetic efficiency, the maximal feeding rate [$\frac{1}{handling\ time}$] was converted from [$\frac{1}{time}$] to an energetic equivalent, [Jh^{-1}]. The conversion – described in the original study - is from per capita to per capita biomass (multiply with prey biomass m_R) and then to energy per capita (multiply with the weight [kg] - energy [J] conversion factor of 7×10^6). Note, $m_R = [mg] = 10^{-6}[kg]$.

$$\rho = 7 \times 10^6 \times 10^{-6} \times m_R \times \frac{1}{h} \times \frac{e}{m}$$

Similarly, for $\kappa = ahK(\frac{e}{mh} - 1)$, the first term required a correction, since h needed a conversion from per capita $[h\ Ind^{-1}]$ to per capita biomass time $[h\ mg^{-1}]$ to match the dimensions of K. Hence, we divided handling time by the prey biomass:

$$\kappa = a(\frac{1}{m_R}h)K(\rho - 1)$$

The parameter values we used for calculating ρ and κ:

The carrying capacity:

$$K(T) = K_0 \left(\frac{m_R}{m_{R_0}}\right)^S e^{-\frac{E_K}{kT}}\sigma^Z e^{tl_0(tl-1)}$$

where $K_0 = e^{-31.15}$, m_R is the prey body mass, $s = -0.72$, $E_K = -0.71$, $z = 1.03$, $tl_0 = 2.86$, $tl = 1.5$.

$$\sigma = \sigma_0 e^{\frac{-E_\sigma(T_{ref}-T)}{kTT_{ref}}} [gCm^{-2}y^{-1}], \text{ where } \sigma_0 = 600 \text{ and } E_\sigma = 0.35.$$

Attack rate $[m^2 h^{-1}]$ and handling time $[h\ Ind^{-1}]$ varied with prey type:

$$a(T) = a_0 \left(\frac{m_C}{m_{C_0}}\right)^S e^{-\frac{E_a(T_{ref}-T)}{kTT_{ref}}}$$

with:

$a_0 = e^{-1.77}, E_a = 0.37, s = -.48$ for mobile prey

$a_0 = e^{-5.94}, E_a = -0.27, s = 0.91$ for resident prey

$$h(T) = h_0 \left(\frac{m_C}{m_{C_0}}\right)^s e^{-\frac{E_h\left(T_{ref}-T\right)}{kTT_{ref}}}$$

with:

$h_0 = e^{2.99}, E_h = -0.24, s = -0.66$ for mobile prey and

$h_0 = e^{6.85}, E_h = -0.23, s = -1$ for resident prey

Metabolic rate was measured in terms of energy per unit of time, $m_E(T)$.

$$m_E(T) = m_0 \left(\frac{m_C}{m_{C_0}}\right)^s e^{-\frac{E_m\left(T_{ref}-T\right)}{kTT_{ref}}} \text{ with } m_0 = e^{-3.91}, E_m = 0.61, s = 0.62, \text{ with units } [Jh^{-1}].$$

m_C varieswith the predator size-class.

Binzer et al. (2016):

m_R and m_C are the unit biomass $[g]$ of prey and predator, respectively. For the dimensions in the following equations to be correct, we introduce a reference body mass for resource and consumer, $m_{R_0} = 1g$ and $m_{C_0} = 1g$, respectively. We assumed a 100:1 consumer-resource body-mass ratio. $T_{ref} = 20°C = 293.15K$ and $e = 0.85$.

$$r(T) = r_0 \left(\frac{m_R}{m_{R_0}}\right)^s e^{\frac{E_r(T_{ref}-T)}{kTT_{ref}}} \text{ with } r_0 = e^{15.68}, E_r = -0.84, s = -0.25, [s^{-1}]$$

$$K(T) = K_0 e^{\frac{E_K\left(T_{ref}-T\right)}{kTT_{ref}}} \text{ with } E_K = 0.71, [gm^{-2}]$$

In the original study K_0 varied between 1 and 20, so we selected two scenarios, $K_0 = 5$ for 'low enrichment' and $K_0 = 15$ for 'high enrichment' (Fig. 4c, 7c).

$$a(T) = a_0 \left(\frac{m_R}{m_{R_0}}\right)^{s_R} \left(\frac{m_C}{m_{C_0}}\right)^{s_C} e^{\frac{E_a(T_{ref}-T)}{kTT_{ref}}} \text{ with } a_0 = e^{13.1}, E_a = -0.38, s_R = 0.25, s_C = -0.8,$$

$$[m^2 s^{-1}]$$

$$h(T) = h_0 \left(\frac{m_R}{m_{R_0}}\right)^{s_R} \left(\frac{m_C}{m_{C_0}}\right)^{s_C} e^{\frac{E_h(T_{ref}-T)}{kTT_{ref}}} \text{ with } h_0 = e^{9.66}, E_h = 0.26, s_R = -0.45, s_C = 0.47,$$

$$[s]$$

$$m(T) = m_0 \left(\frac{m_C}{m_{C_0}}\right)^{s} e^{\frac{E_m(T_{ref}-T)}{kTT_{ref}}} \text{ with } m_0 = e^{16.54}, E_m = -0.69, s = -0.31, [s^{-1}]$$

<u>Sentis et al (2012):</u>

Certain parameterisations were not fully provided in the original manuscript. We have retrieved them from the authors and used these for the calculation of maximal energetic efficiency, ρ, and interaction strength, κ.

The prey and predator masses are $m_R = 0.5031$ [mg] and $m_C = 8.7 \times 10^{-3}$ [g], respectively. For the dimensions in the parameter equations to be correct, we introduce a reference body mass for the consumer, $m_{C_0} = 1g$.

We note that this study too does not explicitly measure or define a resource carrying capacity. However, it does provide a gradient of prey densities based on which their energetic efficiency

was calculated. We used a low and a high value from the gradient as our carrying capacity values.

Similarly to Vucic-Pestic *et al.* (2011), standard metabolic rate was measured [energy*time^{-1}]. Hence we needed to convert h from $[d\ Ind^{-1}]$ into an energetic equivalent for its units to cancel out with m $[Js^{-1}]$. We applied the conversion described in the original study, to per capita biomass (multiply with m_R) and then to energy per capita (multiply with the weight $[kg]$ - energy $[J]$ conversion factor of 7×10^6). Note, $m_R = [mg] = 10^{-6}[kg]$. Additionally, we scaled m up from seconds to a daily rate.

$$\rho = 7 \times 10^6 \times 10^{-6} \times m_R \times \frac{1}{h} \times \frac{1}{86400m} \times e$$

$\kappa = ahK(\rho - 1)$ did not require any scalings, since the product ahK was unitless.

The parameter values for calculating ρ and κ were:

$a(T) = a_0(T - T_0)(T_1 - T)^{\frac{1}{2}}$ with $a_0 = 0.061, T_0 = 11.05 + 273.15, T_1 = 38.00 + 273.15$, $[m^2 d^{-1}]$

$h(T) = h_0 e^{\frac{E_h}{T - 273.15}}$ with $h_0 = 0.0083, E_h = 44.7569, [d\ ind^{-1}]$

$m_E(T) = m_0 \left(\frac{m_C}{m_{C_0}}\right)^s e^{\frac{-E_m}{kT}}$ with $m_0 = 2.86 * 10^7, E_m = 0.65, s = 0.75, [Js^{-1}]$

K [$ind\ m^{-2}$] corresponnds to prey density values of the original study. We used $K = 5$ and $K = 60$ for the 'low enrichment' and 'high enrichment' scenarios, respectively (Fig. 4d, 7d).

Archer et al. (2019):

$T_{ref} = 283.15K$

For the dimensions in the parameter equations to be correct, we introduce a reference body mass for the consumer, $m_{C_0} = 1mg$.

No carrying capacity values were explicitly defined or provided. We derived the carrying capacity, K, from Fig. 3c of the manuscript which demonstrates the temperature-depedence of the prey density.

For the metabolic rate of *P. cingulatus* we did not include additional the quadratic term used in the fitting of the curve to the original data.

To attain a unitless aggregate parameter $\rho = \frac{e}{mh}$ we converted h from [$d\ Ind^{-1}$] into an energetic equivalent for its units to cancel out with m [Jh^{-1}] – as in the original study. The per capita handling time was converted to per capita energy handling time using the prey body mass ($0.546\ mg$) and the prey energy content ($23.1\ Jmg^{-1}$) - these scaling were provided in the original study. Finally, we converted m from an hourly to a daily rate to match the feeding rate temporal scale.

$$\rho = 0.546 \times 23.1 \times \frac{1}{h} \times \frac{1}{24m} \times e$$

$\kappa = ahK(\rho - 1)$ did not require any scalings, since the product ahK was unitless.

The parameter values used were the following:

For the sedentary (*L. riparia*) predator:

$a(T) = a_0 e^{-\frac{E_a(T_{ref}-T)}{kTT_{ref}}}$ with $a_0 = 0.241$ (Lab 2013), $a_0 = 0.802$ (Lab 2015) or $a_0 = 1.889$ (Field 2015) and $E_a = 0.74$ (for all measurements), with units $[m^2 d^{-1}]$

$h = 4.033$, $[d\ ind^{-1}]$ – no temperature dependence

$m_E(T) = m_0 \left(\frac{m_C}{m_{C_0}}\right)^S e^{\frac{-E_m}{kT}}$ with $m_0 = e^{-4.171} * 10^7, E_m = 0.687, m_C = 10^{0.25}, s = 0.525,$

$[Jh^{-1}]$

For the mobile predator (*P. cingulatus*):

$a(T) = a_0 e^{-\frac{E_a(T_{ref}-T)}{kTT_{ref}}}$ with $a_0 = 1.529$ (Lab 2015) or $a_0 = 5.515$ (Field 2015) and $E_a = 0.229$ (for all measurements), with units $[m^2 d^{-1}]$

$h = 0.644$, $[d\ ind^{-1}]$ – no temperature dependence

$$m_E(T) = m_0 e^{\frac{-E_m}{kT}} \text{ with } m_0 = e^{-1.056} * 10^7, E_m = 1.072, \text{ with no effects of body mass, } [Jh^{-1}]$$

And for all calculations:

$$K(T) = 10^{0.16(T-273.15)}, \text{ with units } [ind\ m^{-2}]$$

$$e(T) = \frac{e_0 e^{\frac{-E_e(T_0-T)}{kTT_0}}}{e_0 e^{\frac{-E_e(T_0-T)}{kTT_0}} + 1}, \text{ with } e_0 = e^{2.266}, E_a = 0.164, T_0 = 293.15K$$

Supplementary Information 5: Biomass ratio and stability sensitivity values in the $\rho - \kappa$ plane

Since all parameter sensitivities can be expressed in terms of ρ and κ, the sensitivity values of each parameter are fully determined in the $\rho - \kappa$ plane. Here we present figures with contours of the parameter values in the aggregate parameter plane. These combined with the 'rankings regions' from the main manuscript, provide a complete picture of the parameter sensitivities in the $\rho - \kappa$ plane.
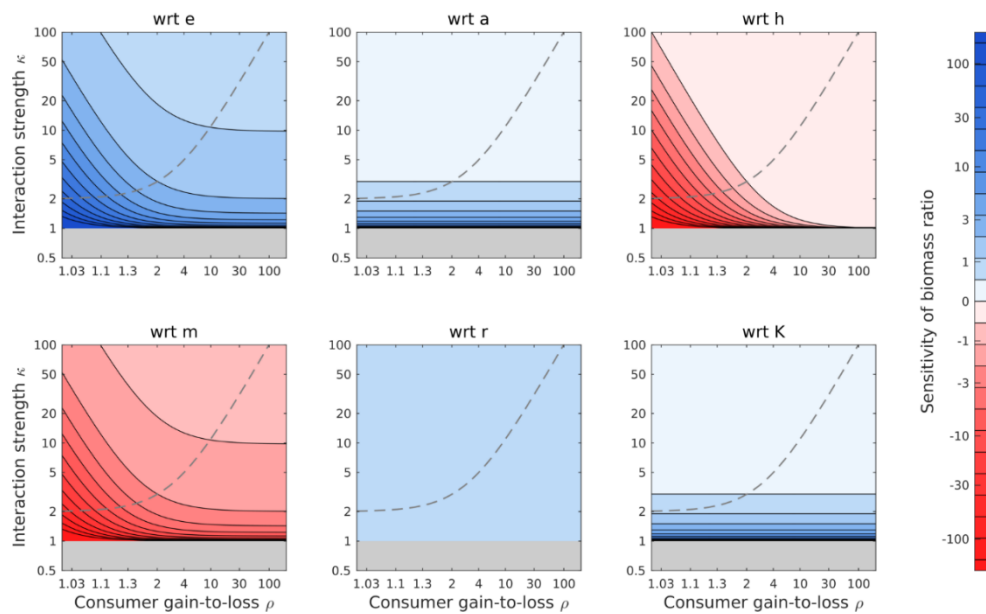

Figure S5.1. Values of biomass ratio sensitivities with respect to ('wrt') individual parameters (different panels) derived from the expressions in Table 1 of the main manuscript.
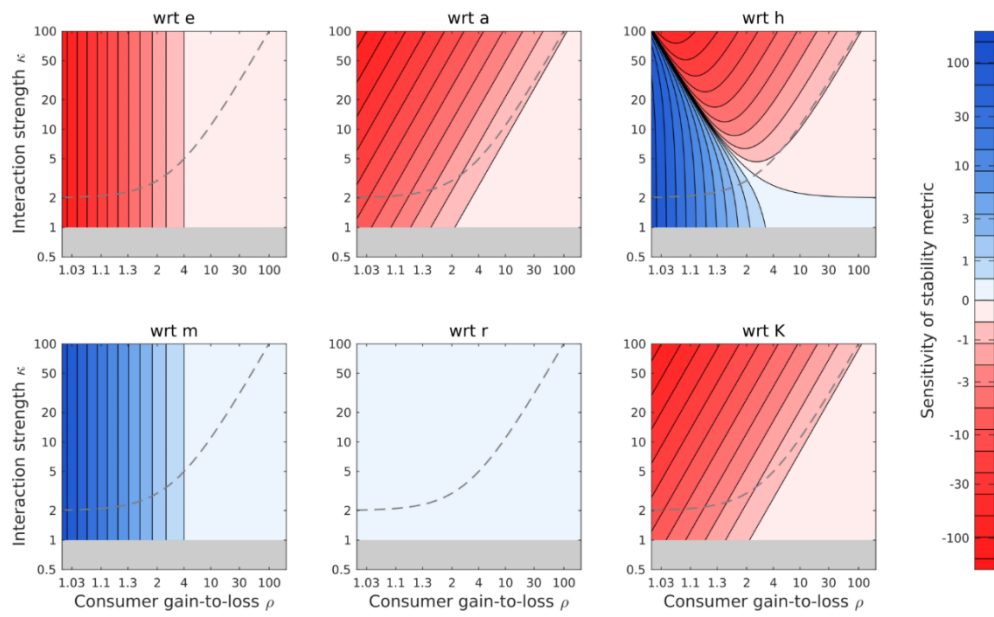
Figure S5.2. Values of stability metric sensitivities with respect to ('wrt') to individual parameters (different panels) derived from the expressions in Table 1 of the main manuscript.

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