

## Appendix A

For across-group analyses, we want to calculate the ISC between two groups of subjects, with two different sets of shared responses,  $g_1$  and  $g_2$ , and different levels of variance,  $\epsilon_i$  and  $\epsilon_j$ . To simplify the notation, we define  $g_1^T g_1 = a_1$ ,  $g_2^T g_2 = a_2$ ,  $g_1^T g_2 = a_B$ ,  $\mathbb{E}[\epsilon_i^T \epsilon_i] = b_1$  and  $\mathbb{E}[\epsilon_j^T \epsilon_j] = b_2$ . We would like to estimate the correlation between  $g_1$  and  $g_2$ , regardless of the difference in signal-to-noise levels between the two groups. That is, our goal is to compute:

$$ISC_b = \frac{a_B}{\sqrt{a_1}\sqrt{a_2}}$$

If we know the ISCs for each group, from equation 1:

$$shISC_1 = \frac{N^2 a_1}{N^2 a_1 + 4N b_1}$$

and

$$shISC_2 = \frac{N^2 a_2}{N^2 a_2 + 4N b_2},$$

where  $N$  is the number of subjects in each group, and the ISC between the two groups is:

$$shISC_B = \frac{N^2 a_B}{\sqrt{N^2 a_1 + 4N b_1} \sqrt{N^2 a_2 + 4N b_2}}.$$

Dividing  $shISC_B$  by the geometric mean of  $shISC_1$  and  $shISC_2$  allows us to calculate the between-group ISC without the  $b$ 's:

$$\begin{aligned} ISC_b &= \frac{shISC_B}{\sqrt{shISC_1} \sqrt{shISC_2}} \\ &= \frac{a_B}{\sqrt{a_1} \sqrt{a_2}}. \end{aligned}$$

## Appendix B

We start by assuming that the response timecourse for every subject consists of a response that is shared across subjects,  $g$ , and a subject-specific response,  $\epsilon$ . The response timecourse for subject  $i$  can therefore be represented as  $s_i = g + \epsilon_i$ . For simplicity, all subject timecourses,  $s$ , have zero mean, we define  $a$  as the norm of  $g$  ( $g^T g = a$ ), and we define

$\mathbb{E}[\epsilon_i^T \epsilon_i] = b$ . If we have  $N$  subjects in total, the split half ISC (*shISC*) is:

$$\begin{aligned}
shISC &= \mathbb{E} \left[ \rho \left( \sum_{i=1}^{N/2} s_i, \sum_{j=N/2+1}^N s_j \right) \right] \\
&= \mathbb{E} \left[ \rho \left( \frac{N}{2}g + \sum_{i=1}^{N/2} \epsilon_i, \frac{N}{2}g + \sum_{j=N/2+1}^N \epsilon_j \right) \right] \\
&= \frac{\frac{N^2}{4} g^T g}{\sqrt{\frac{N^2}{4} g^T g + \mathbb{E}[\sum_{i=1}^{N/2} \epsilon_i^T \epsilon_i]} \sqrt{\frac{N^2}{4} g^T g + \mathbb{E}[\sum_{j=N/2+1}^N \epsilon_j^T \epsilon_j]}} \\
&= \frac{N^2 a}{N^2 a + 2N b} \\
&= \frac{N f}{N f + 2} \tag{Equation 1}
\end{aligned}$$

where  $f = a/b$ , or the signal to noise ratio (the ratio of norms between  $g$  and  $\epsilon_i$ ).

If we rearrange the variables in Equation 1, we find that:

$$f = \frac{2 * shISC}{-N * shISC - N}. \tag{Equation 2}$$

We can thus calculate *pwISC* and *looISC* from *shISC* as they relate to  $f$  as follows. For *pwISC* between subjects  $i$  and  $j$ :

$$\begin{aligned}
pwISC &= \mathbb{E}[\rho(s_i, s_j)] \\
&= \mathbb{E}[\rho(g + \epsilon_i, g + \epsilon_j)] \\
&= \mathbb{E} \left[ \frac{(g + \epsilon_i)^T (g + \epsilon_j)}{\sqrt{(g + \epsilon_i)^T (g + \epsilon_i)} \sqrt{(g + \epsilon_j)^T (g + \epsilon_j)}} \right] \\
&= \frac{g^T g}{\sqrt{g^T g + \mathbb{E}[\epsilon_i^T \epsilon_i]} \sqrt{g^T g + \mathbb{E}[\epsilon_j^T \epsilon_j]}} \\
&= \frac{a}{a + b} \\
&= \frac{f}{f + 1},
\end{aligned}$$

and for *looISC*, between one left-out subject, and all others:

$$\begin{aligned}
looISC &= \mathbb{E} \left[ \rho \left( s_1, \sum_{j=2}^N s_j \right) \right] \\
&= \mathbb{E} \left[ \rho \left( g + \epsilon_1, (N-1)g + \sum_{j=2}^N \epsilon_j \right) \right] \\
&= \frac{(N-1)a}{\sqrt{a+b}\sqrt{(N-1)^2a + (N-1)b}} \\
&= \frac{(N-1)f}{\sqrt{f+1}\sqrt{(N-1)^2f + (N-1)}} \\
&= \frac{\sqrt{N-1}f}{\sqrt{f+1}\sqrt{(N-1)f+1}}.
\end{aligned}$$

Thus, for example, if there are 40 subjects, and *shISC* is 0.5, the estimated *pwISC* = 0.05 and the estimated *looISC* = 0.18.