

# 1 Stable choice coding during changes of mind

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## 13 Abstract

14 How do we choose the best action in a constantly-changing environment? Many natural  
15 decisions unfold in dynamic environments where newer observations carry better information  
16 about the present state of the world. Recent work has shown that rats can learn to optimally  
17 discount old evidence, updating their provisional decision when the environmental state changes.  
18 Provisional decisions are thought to be represented in the Frontal Orienting Fields (FOF), but  
19 this has only been tested in static environments where the provisional and final decisions are  
20 not easily dissociated. Here, we characterize the representation of accumulated evidence in rat  
21 FOF during decision-making in a dynamic environment. We find that FOF encodes evidence  
22 throughout decision formation with a temporal gain modulation that rises until the period when  
23 the animal may need to act. Using a behavioral model to predict the timing of changes of mind  
24 revealed that FOF neurons respond rapidly to these events, representing the new provisional  
25 decisions in their firing rates. Our results suggest that the FOF represents provisional decisions  
26 even in dynamic, uncertain environments, allowing for rapid motor execution when it is time to  
27 act.

## 28 Introduction

29 When making decisions, animals must weigh and combine the available evidence in favor  
30 of each alternative. With each new observation, evidence about the underlying state of the  
31 environment gradually accumulates until the animal is ready to act. This accumulation model  
32 successfully describes a wide array of decisions<sup>1,2,3,4</sup>. Neural correlates of this accumulation  
33 process are also present across many brain regions in animals performing perceptual categoriza-  
34 tion tasks<sup>1,4</sup>. Delivering perceptual evidence in streams of discrete pulses, like randomly-timed  
35 auditory clicks, provides additional power to estimate the evolution of each subject's latent ac-  
36 cumulated evidence variable on individual trials<sup>5</sup>. Accurate prediction of the decision variable  
37 during each trial increases the resolution for estimating neural encoding across brain regions<sup>6,7,8</sup>.  
38 Hanks et al.<sup>6</sup> characterized the neural representation of accumulating evidence in rats performing  
39 accumulation of trains of auditory click evidence. In the task, two streams of randomly-timed  
40 auditory clicks were emitted from either side of a fixation location and rats were trained to  
41 orient toward the side that played a greater number of clicks. Experimenters recorded from  
42 the frontal orienting fields (FOF), a frontal cortical structure implicated in short term memory  
43 and preparation of orienting movements<sup>9,10,11</sup>. They proposed that FOF neurons used a single  
44 code throughout accumulation, which represented the accumulated evidence value categorically,  
45 providing a readout of the animal's provisional decision<sup>6</sup>. However, perturbations of this signal  
46 in FOF only impaired the animal's choice when they overlapped with the final time points of  
47 accumulation. These results, along with a two-node model of the FOF<sup>12</sup>, indicate that the FOF  
48 is critical for maintaining the decision variable after it has been transformed into a categorical  
49 representation of the animal's choice.

50 While these experiments were conducted using stationary environments, many natural de-  
51 cisions unfold in dynamic environments. In stationary settings, all evidence samples in a trial  
52 reflect the same underlying environmental state. This means the best strategy is to equally  
53 weigh all samples of evidence throughout stimulus presentation<sup>13</sup>. In this regime it is difficult  
54 to dissociate the provisional from the final decision. In dynamic environments, the state of  
55 the world can change while the animal is deliberating. This means the animal should learn to  
56 discount old evidence via leaky integration, weighing more recently presented samples of infor-  
57 mation more heavily than older samples<sup>14,15,16,17,18</sup>. Unlike stationary environments, adopting  
58 the optimal strategy in a dynamic environment leads to frequent fluctuations in the animal's  
59 provisional decision, or changes of mind.

60 Recent work has shown that rats and humans can learn to adopt the optimal discounting rate  
61 in a dynamic environment<sup>14,16</sup>. However, it is unknown whether neural correlates of accumulated  
62 evidence observed in animals performing non-leaky integration in stationary environments persist  
63 in animals performing leaky integration in dynamic environments. Here, we recorded from FOF  
64 in rats during a dynamic accumulation of evidence task. We tested whether the stable code  
65 observed in the stationary environment persisted in the dynamic environment by applying and  
66 extending a method developed to characterize neural tuning to accumulated evidence<sup>6</sup>. Evidence  
67 tuning in FOF was described by a single sigmoidal tuning curve multiplied by a time varying  
68 gain modulation, which increased with time early in the trial and stabilized at the time of the  
69 earliest possible go cue. We reasoned that if FOF neurons track the accumulated evidence  
70 throughout the entire accumulation period, firing rates should respond rapidly to changes in  
71 the provisional decision. Using the behavioral model to predict the timing of changes of mind,  
72 we find that FOF neurons respond within 100ms, reflecting the new provisional decision in  
73 their activity. By recomputing the evidence tuning curves aligning to these change of mind  
74 events, we confirmed that FOF neurons encode evidence with a single tuning curve before and  
75 after changes of mind. These results suggest that FOF maintains a stable representation of  
76 accumulated evidence despite dynamic uncertainty in the environment. The time varying gain  
77 modulation may help ensure that the animal is ready when it is time to act. Our study opens

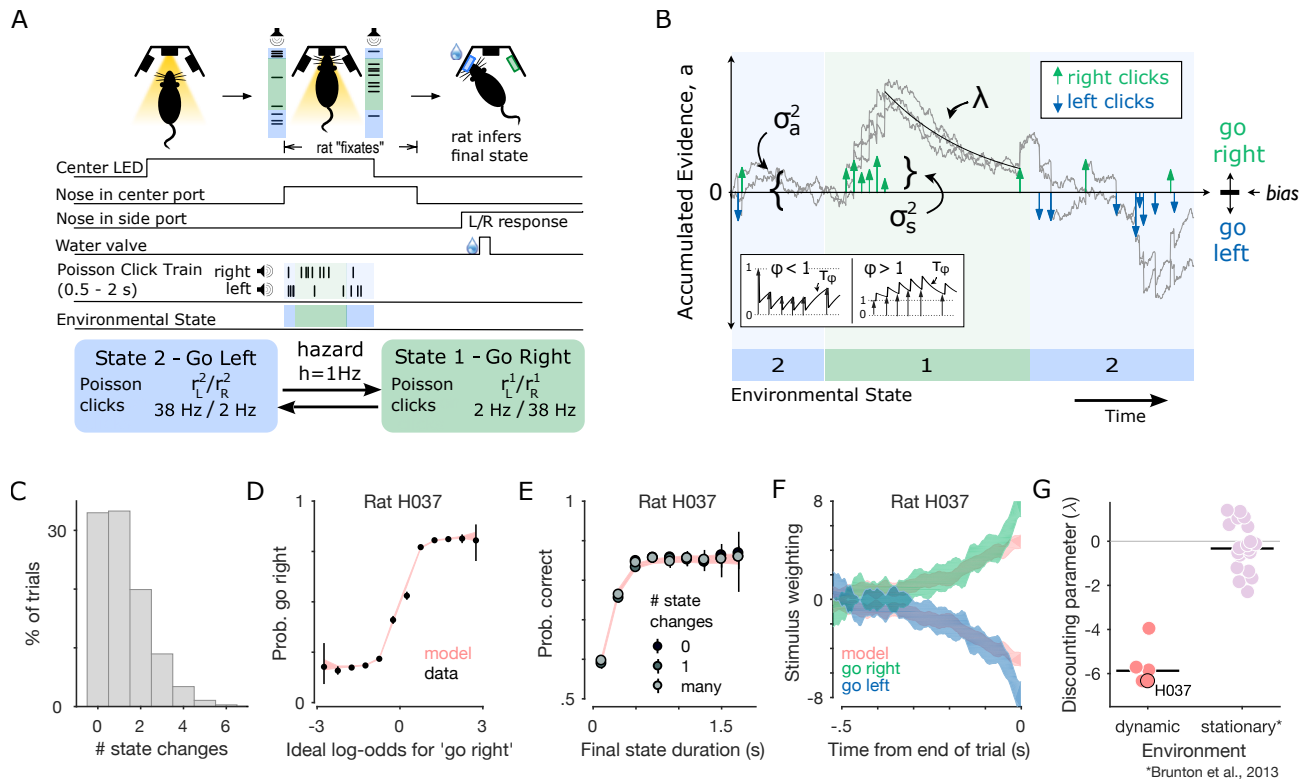
78 up the opportunity for future work on the neural circuit level understanding of how animals  
79 integrate and decide in a volatile environment.

## 80 Results

81 **The dynamic evidence accumulation task.** We trained rats ( $n=5$ ) to perform a previously  
82 developed dynamic evidence accumulation task<sup>16</sup>. This task requires the rat to report which  
83 of two hidden states the environment is in at the time of a go cue. At the beginning of each  
84 trial, the center port in an array of three nose ports is illuminated by an LED. This invites the  
85 rat to poke its nose into the center port, initiating presentation of an auditory stimulus. The  
86 stimulus is composed of two trains of auditory pulses (clicks) delivered in stereo from speakers  
87 positioned on either side of the center port. The left and right click trains are generated from  
88 different Poisson processes with rate parameters,  $r_R^i$  and  $r_L^i$ , that depend on the state  $i$ . When  
89 the environment is in state 1, the ‘Go Right’ state, the generative click rate is higher for the right  
90 speaker than the left ( $r_R^1 = 38Hz$  and  $r_L^1 = 2Hz$ ). In state 2, the ‘Go Left’ state, the click rates  
91 are reversed ( $r_R^2 = 2Hz$  and  $r_L^2 = 38Hz$ ). Trials begin in either state with equal probability  
92 and switch stochastically between states with a fixed hazard rate  $h = 1Hz$ . After a randomized  
93 duration, drawn from a uniform distribution between 500 and 2000 ms, the stimulus ends and  
94 the center LED turns off. This ‘go’ cue signals the rat to withdraw from the center port and  
95 poke its nose into one of two reward delivery ports on either side. The animal receives a drop  
96 of water (18  $\mu$ L) if it chooses the side port corresponding to the final value of the hidden state.  
97 Incorrect choices were signaled with a white noise stimulus (Fig. 1A). In our dataset, roughly  
98 33% of trials had no state changes, 33% had one, and 34% had more than one (Fig. 1C).

99 **Behavioral model captures leaky integration strategy.** We fit a previously-developed  
100 behavioral model<sup>5,16</sup> to rats’ choices using an average of 108,126 trials per rat (63,494 to 185,091  
101 trials each from 118 to 308 sessions). The model parameterizes the process by which the evidence  
102 available in each auditory click is integrated to produce a decision. At the start of each trial,  
103 time  $t = 0$ , this distribution has zero mean and initial variance  $\sigma_i^2$ . Each right and left click,  
104  $\delta_{R,i}$  and  $\delta_{L,i}$ , increments or decrements the accumulation value  $a$ , subject to sensory adaptation  
105 governed by parameters  $\phi$  and  $\tau_\phi$ . Each click also introduces additional noise with variance  
106  $\sigma_s^2$ . Memory noise with variance  $\sigma_a^2$  is introduced at each time step. Evidence is discounted  
107 with rate  $\lambda$  which parameterizes the rate at which, in the absence of further input,  $a$  decays  
108 with time ( $\lambda < 0$ ) or increases with time ( $\lambda > 0$ ). When  $\lambda < 0$ , older pieces of evidence  
109 are discounted relative to newer evidence. While decision makers in stationary environments  
110 perform best when discounting is minimal ( $\lambda = 0$ ), ideal observers in our task adopt a high-level  
111 of discounting of old evidence ( $\lambda < 0$ ), reducing the impact of older clicks that may have been  
112 presented before a change in the hidden state<sup>16,15,14,17</sup>. As previously described<sup>16</sup>, the optimal  
113 discounting rate in a dynamic environment depends on the quality of evidence, including the  
114 observer’s per-click noise ( $\sigma_s^2$ ). At the end of the trial, time  $t = t_N$ , the rat chooses to go right  
115 if the accumulation value  $a$  is greater than the decision boundary  $B$ , except on a fraction of  
116 “lapse” trials where, with some probability  $l$ , the rat chooses randomly (Fig. 1C) (see methods  
117 for mathematical details). For each rat, we found the best-fitting parameter set  $\theta$  to describe  
118 this process using maximum likelihood estimation. These parameter sets were used these to  
119 estimate the probability distribution over accumulation values on each trial  $P(a|t, \delta_R, \delta_L, \theta)$ .

120 We present several assessments of task performance and model validation. Psychometric  
121 curves show a rat’s choices as a function of the ideal observer log-odds favoring a rightward  
122 choice, as well as the correspondence with predictions from the behavioral model fit to an  
123 example rat (Fig. 1D) and all rats used in this study (Fig. S2). Final state chronometric curves  
124 show that performance was dependent on the final state duration, the elapsed time between the  
125 final state change and the ‘go’ cue (Fig. 1E and Fig. S3). Radillo et al.<sup>17</sup> demonstrated the



**Figure 1: Rats accumulate and discount evidence in a dynamic accumulation task**  
 (A) Schematic showing task events and timing. The center port is illuminated by an LED. The rat pokes its nose into the port to initiate playback of randomly timed auditory clicks from speakers on either side. Clicks on each side are generated with different underlying Poisson rate parameters that depend on a hidden environmental state. The stimulus duration is drawn from a uniform distribution between 500 and 2000ms. During that time the hidden state switches stochastically at a fixed hazard rate,  $h = 1\text{Hz}$ . At the end of the stimulus presentation, the center LED turns off and reward is baited in the side port corresponding to the final state.  
 (B) Schematic of the evolution of the accumulation model on an example trial. Three example accumulation traces are shown for different instantiations of the noise applied at each time point ( $\sigma_a$ ) and the noise applied to each click ( $\sigma_s$ ). Neighboring clicks can either depress or facilitate each other according to the adaptation parameters ( $\phi$  and  $\tau_\phi$ ). The evidence discounting rate ( $\lambda$ ) determines how quickly the decision variable  $a$  decays back to zero. At the end of the trial, a choice is made by comparing the decision variable to the bias parameter.  
 (C) Frequency of state changes per trial across all rats' datasets.  
 (D) Example psychometric plot showing the probability that the rat chooses 'go right' as a function of the ideal observer log-odds supporting a 'go right' choice. Rat data (black points) is overlaid on predictions of the accumulation model with parameters fit to this rat (red traces). Errorbars for rat data represent 95% binomial confidence intervals around the mean.  
 (E) Example final state chronometric plot showing accuracy (mean with 95% binomial confidence intervals) as a function of the duration of a trial's final state and the number of switches in a given trial.  
 (F) Psychophysical reverse correlation kernel for an example rat. Green and blue patches indicate strength (mean  $\pm$  s.d.) of evidence favoring rightward choice as a function of time until the trial ends for rightward and leftward choices, respectively. The red patches are corresponding predictions from the accumulation model.  
 (G) Discounting parameters for each rat in this study (red points) compared to each rat in a previously published stationary environment (lilac points; Brunton et al.<sup>5</sup>). Group medians are plotted as black horizontal lines.

126 rate of increase and saturation level of the chronometric curve for an ideal observer depends

127 only on the hazard rate and SNR of the click rates. Psychophysical reverse correlation kernels  
 128 quantify the influence of clicks at each timepoint throughout the stimulus period, providing an  
 129 assay of the rats' evidence discounting that is independent of the behavioral model. Reverse  
 130 correlations for all rats in this study show heavier weighting of clicks presented at the end of the  
 131 trial compared to the beginning (Fig. 1F and Fig. S4).

132 The behavioral model parameter fits for each rat confirm that all rats used a leaky inte-  
 133 gration strategy ( $\lambda < 0$ ). Best fit discounting parameters were significantly different from a  
 134 previously reported<sup>5</sup> dataset of rats integrating in a stationary environment ( $p < .01$ ; two-tailed  
 135 Wilcoxon rank-sum test,  $n=5$  in dynamic and  $n=19$  stationary environments) (Fig. 1G). Con-  
 136 sistent with previous work, rats adopted discounting rates that favor more recent evidence due  
 137 to the environmental volatility<sup>16</sup>.

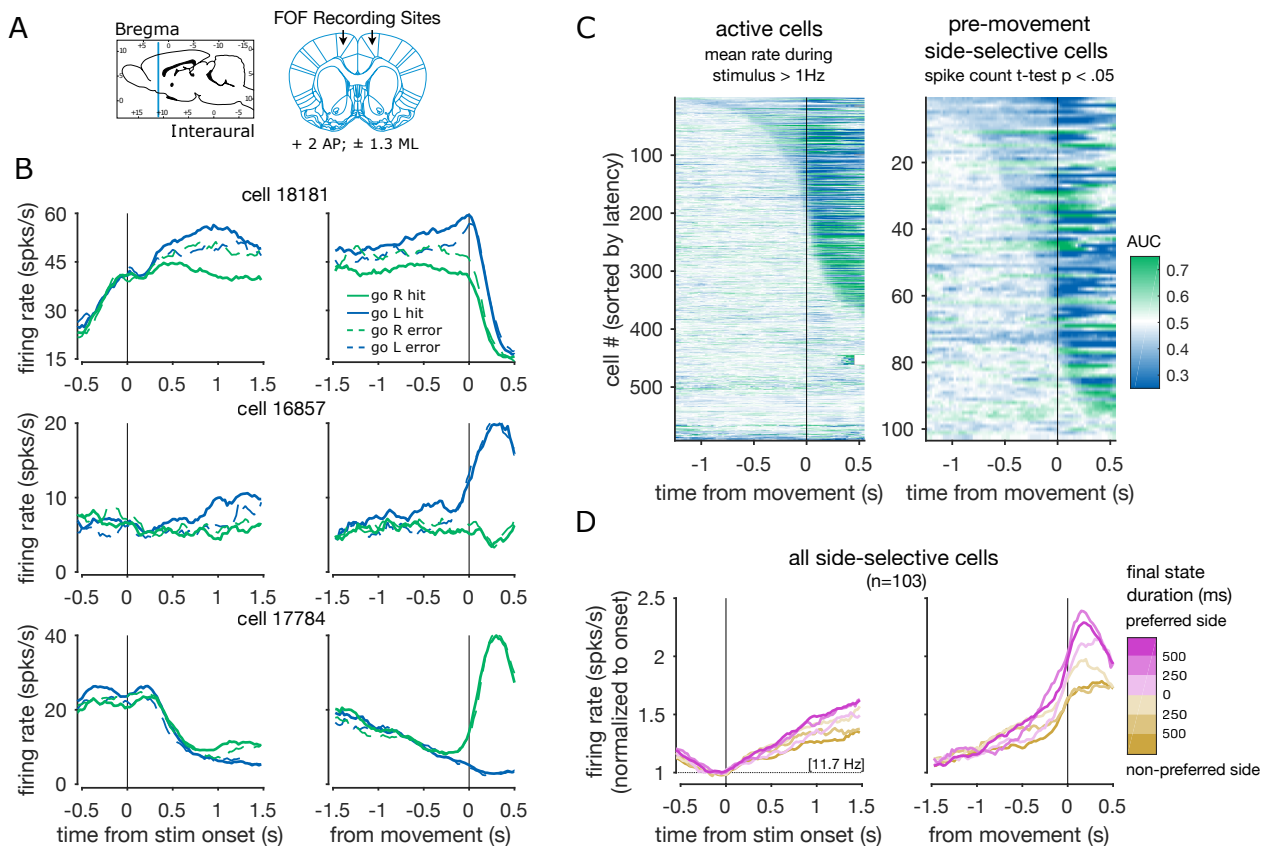
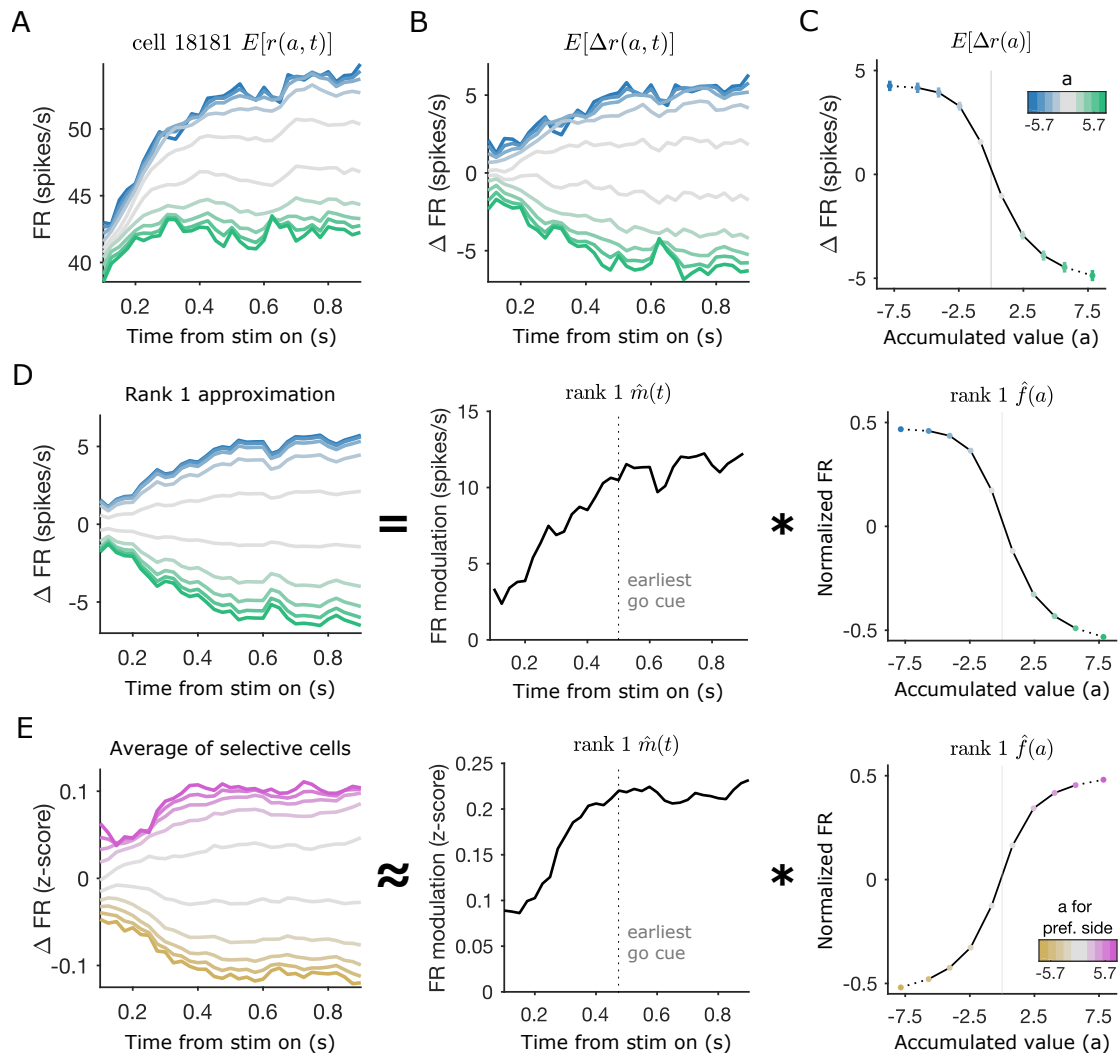


Figure 2: **FOF neurons encode the rat's upcoming choice** (A) Coordinates used for FOF recordings (+2 AP; ± 1.3 ML). (B) Average firing rates for three example FOF cells aligned to stimulus onset (left) and movement (right). Activity is conditioned on right (green) vs. left (blue) side choice, as well as hits (solid lines) vs. errors (dashed lines). (C) Side-selectivity at each time point relative to movement for all active cells (left; firing rate > 1 Hz) and for the subset of these cells that meet the spike count pre-movement side-selectivity criterion (right; 2-tailed t-test  $p < .05$ ). AUC is computed on spike rates for right versus left choices. Plots are sorted by latency to 200ms (8 consecutive time bins) of significant AUC values relative to a distribution created by permuting choice labels across trials (2-tailed permutation test, 250 permutations,  $p < .05$ ). (D) Average activity of all pre-movement side-selective cells conditioned on final state duration and cells' side preferences. Grand-average firing rate at stimulus onset (11.7Hz) is written in brackets.

138 **FOF responses during dynamic accumulation.** We recorded from the frontal orienting  
 139 fields (FOF) of rats performing the dynamic evidence accumulation task. In 4 rats, we im-



**Figure 3: FOF neurons encode the accumulated evidence throughout the trial (A)** Firing rate map as a function of accumulated evidence and time for an example neuron. Colors indicate accumulated evidence value with the same colors as in B and C. (B) Residual rate map in which the mean temporal trajectory is subtracted. (C) Average tuning over time. Points indicate mean ( $\pm$  s.e.m.) across time of the change in firing rate relative to temporal average as a function of accumulated evidence value  $a$ . (D) Rank 1 approximation of the residual rate map  $E[\Delta r(a, t)]$  from B. The approximation (left) is equal to the outer product of a modulation over time  $\hat{m}(t)$  (middle) and a tuning curve  $\hat{r}(a)$  (right). (E) Average residual z-scored firing rate map (left). This plot is produced by averaging over the residual z-scored firing rate map of all pre-movement side-selective cells. This map is approximated by the outer product of a modulation curve (middle) and a tuning curve (right).

140 planted unilateral (n=2 left FOF, 2 right FOF) microwire arrays at coordinates (+2 AP;  $\pm$  1.3  
 141 ML) (Fig. 2A). In a 5th rat, we implanted a bilateral tetrode drive over the same coordinates.  
 142 Recordings from 69 sessions yielded 738 units. See Supplementary Table 1 for a breakdown of  
 143 data by rat (Method, location). Cells were considered active and included for further analysis if  
 144 they had a mean firing rate of at least 1 Hz during the trial (n = 592 active cells).

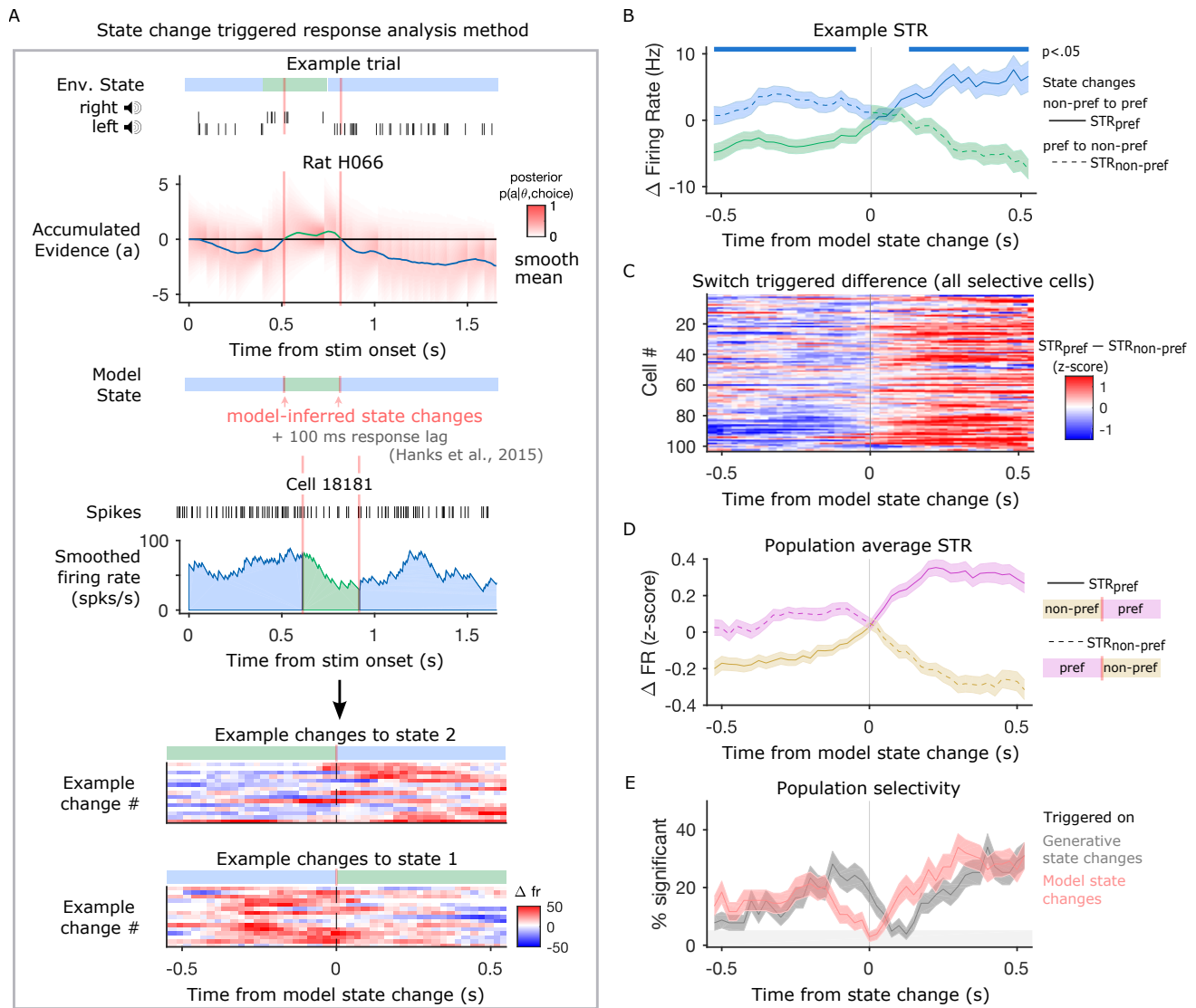
145 Individual cells show stereotyped temporal dynamics aligned to both the onset of the trial  
 146 (entering the center nose port), and the movement following the end of the stimulus (nose out of  
 147 center port) (Fig. 2B). Many individual cells diverged throughout the trial based on the animal's  
 148 upcoming choice. We tested the timecourse of selectivity for single neurons to right versus left  
 149 choices by computing the area under the receiver operating characteristic curve (AUC) and

150 comparing it to a permutation distribution computed by shuffling choice labels across trials.  
151 For purposes of visualization, cells are sorted by latency to 200ms (8 consecutive time bins) of  
152 significant AUC values (2-tailed permutation test, 250 permutations,  $p < .05$ ). We present these  
153 plots for all active neurons and for a subset of pre-movement side-selective neurons (Fig. 2C).  
154 This subset made up 17% of the active population ( $n=103$  selective) and was defined as cells  
155 with spike counts that differed significantly as a function of upcoming animal choice during the  
156 period between the start of the stimulus and the movement away from the fixation port (2-tailed  
157 t-test,  $p < .05$ ). For each neuron, the side associated with the higher spike count is referred  
158 to as the cell's preferred side. Following Hanks et al.<sup>6</sup>, we focus subsequent analyses on these  
159 pre-movement side-selective cells. Pre-movement side selectivity was slightly less common in this  
160 dataset than in previous studies of FOF in stationary environments<sup>6</sup>. This may be a consequence  
161 of frequent changes of mind, which create a dissociation between provisional and final choice  
162 throughout trials in the dynamic task. Across pre-movement side-selective neurons, we computed  
163 the average activity conditioned on final state duration and cell preference (Fig. 2D). We observe  
164 divergences at different latencies depending on the final state duration.

165 **Stable accumulator tuning in dynamic environment.** The choice-selectivity metrics pre-  
166 sented above reveal coding of the final choice in average neural activity. However, during the  
167 trial, the hidden state can change multiple times ( $1.22 \pm 1.20$  state changes per trial). This cre-  
168 ates dissociations during the trial between the animal's provisional choice and the final choice.  
169 To better describe tuning to the provisional decision throughout the trial, we applied and ex-  
170 tended a method developed to quantify the tuning of single neurons to the accumulated value  
171 throughout the trial. Using this method, Hanks et al.<sup>6</sup> found that FOF neurons had a stable  
172 encoding of evidence throughout accumulation in a stationary environment. Here, we sought to  
173 test whether the FOF continues to encode the evidence throughout trials when the environment  
174 is dynamic. Further, we asked whether this encoding can still be captured by a single tuning  
175 curve in the dynamic environment.

176 We used the approach described by Hanks et al.<sup>6</sup>, to produce an evidence tuning map of  
177 each neuron. First, we computed the joint distribution  $P(r, a, t)$  of each cell's firing rate  $r$ , the  
178 instantaneous accumulation value  $a$ , and time in the trial  $t$ . The distribution over  $a$ , from the  
179 behavioral model described above, was further constrained using the animal's choice  $y$  on each  
180 trial, giving the posterior distribution  $P(a|t, \delta_R, \delta_L, \theta, y)$ . We improve on the method used by  
181 Hanks et al.<sup>6</sup> by using an analytical computation of the posterior distribution of accumulated  
182 evidence, allowing for more accurate estimation of  $P(r, a, t)$  (see methods). Marginalizing the  
183 joint distribution with respect to firing rate provides an estimate of the expected firing rate as  
184 a function of accumulated evidence and time, for each cell  $E[r(a, t)]$ . We present this rate map  
185 for an example cell which is strongly tuned to the accumulator throughout the trial, firing more  
186 when accumulated evidence favors left choices (Fig. 3A). Because our neurons have stereotyped  
187 temporal dynamics aligned to stimulus onset, we subtract out the average temporal dynamics  
188 to get the expected firing rate modulation  $E[\Delta r(a, t)] = E[r(a, t)] - E[r(t)]$  (Fig. 3B). Following  
189 Hanks et al.<sup>6</sup>, a summary tuning curve was computed by averaging over time to get  $E[\Delta r(a)]$   
190 (Fig. 3C).

191 We extend the method by computing the rank 1 approximation of the residual firing rate  
192 map  $E[\Delta r(a, t)]$  using the singular value decomposition (Fig. 3D). For the example cell, this  
193 approximation captures 99.6% of the variance in the estimated residual firing rate map. The  
194 mean variance explained by this approximation for all pre-movement side-selective cells was  
195  $89.7\% \pm 9.8\%$ . The approximation is equal to the outer product of the first left singular vector  
196  $u_1$  and the first right singular vector  $v_1$ , scaled by the first singular value  $s_1$ . These terms can be  
197 rearranged and interpreted as the outer product of a firing rate modulation,  $\hat{m}(t) = u_1 s_1 \text{range}(v_1)$   
198 and a tuning curve  $\hat{f}(a) = v_1 / \text{range}(v_1)$ . Scaling by  $\text{range}(v_1)$  gives  $\hat{f}(a)$  unit scale and  $\hat{m}(t)$



**Figure 4: FOF neurons track changes in the provisional decision** (A) Schematic explaining method used to compute state change triggered responses (STR). A given trial has a hidden environmental state (blue and green bar) used to generate click trains from each speaker. We compute the posterior distribution of accumulated evidence  $P(a|\theta, \delta_R, \delta_L, y)$  (labeled  $P(a|\theta, \text{choice})$ ). We find time points where the smoothed posterior mean crosses the decision boundary and label these model-inferred state changes. We then select the residual smoothed firing rates from the 550ms before and after each state change and average together the residual responses for changes into state 1 and changes into state 2. (B) STR (mean  $\pm$  s.e.m.) for the example cell used in panel A. Significance bars indicate time points when  $d'$  for discriminating model state is different from chance (2-tailed permutation test, 250 permutations,  $p < .05$ ). The trace showing changes into the cell's preferred state (state 2 for this cell) is labeled STR<sub>pref</sub> (solid line) and the trace for changes into the cell's non-preferred state is labeled STR<sub>non-pref</sub> (dashed line). (C) Heat map showing difference between responses for changes into the preferred and non-preferred state (STR<sub>pref</sub> - STR<sub>non-pref</sub>) for each of the pre-movement side-selective cells. (D) Average z-scored STR (mean  $\pm$  s.e.m.) across all pre-movement side-selective cells for state changes into cells' preferred states and non-preferred states. (E) Percentage of included cells (mean  $\pm$  s.e.m.) with significant encoding across time relative to model predicted state changes (red trace) and generative state changes (gray trace).



199 units of spikes/s. Our complete tuning curve approximation becomes:

$$r(a, t) \approx E[r(t)] + \hat{m}(t) * \hat{f}(a). \quad (1)$$

200 We computed a population average residual rate map across all pre-movement side-selective  
 201 cells by computing the residual firing rate map  $E[\Delta r(a, t)]$  for each cell using z-scored firing rates.  
 202 The accumulated value axis was inverted for left choice preferring cells and then the residual  
 203 firing rate maps were averaged together (Fig. 3E). We computed the rank 1 approximation of  
 204 this population residual rate map. This approximation explained 99.7% of the variance in the  
 205 population residual rate map (Fig. 3E middle, right). The population firing rate modulation  
 206 curve  $\hat{m}(t)$  rises for the first 500 milliseconds and then plateaus at its maximum value. Therefore,  
 207 the population tuning can be described as a single tuning curve whose modulation increases  
 208 during the period of the trial before a ‘go’ cue is possible. The modulation stabilizes at its  
 209 maximum value during the period in which the trial may end and the animal may need to  
 210 report its decision. Despite the dynamic environment, and changing provisional choice, we find  
 211 FOF neurons continue to stably encode the evidence with a single tuning curve throughout  
 212 evidence accumulation.

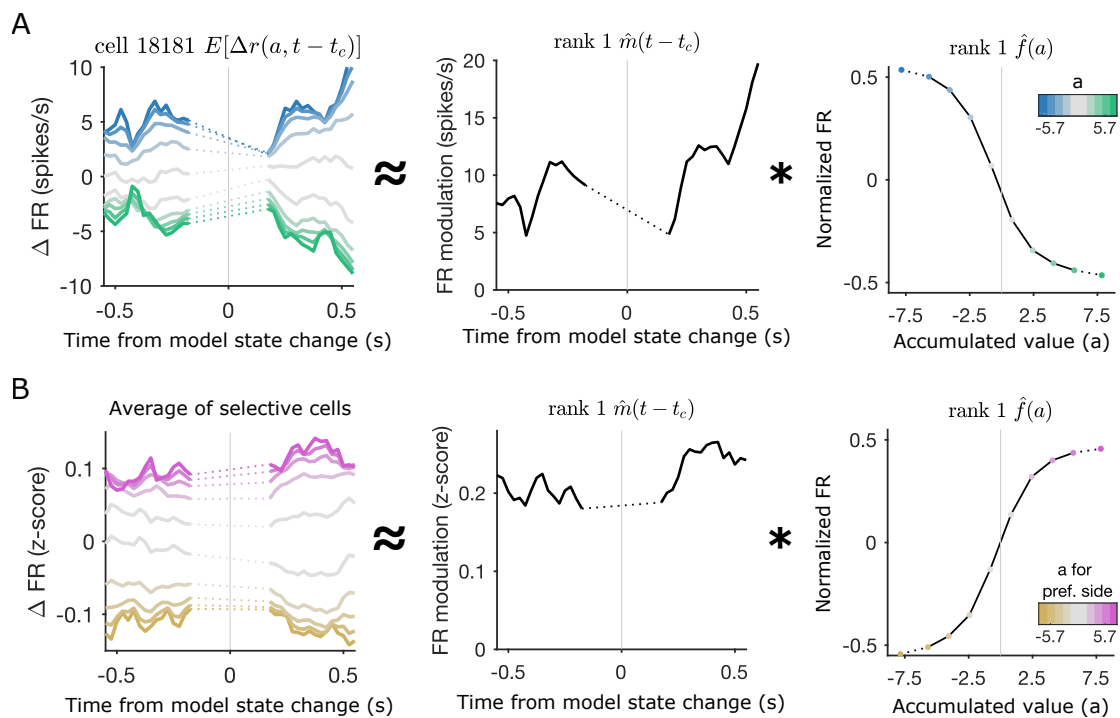


Figure 5: **State change triggered tuning curves show post state change modulation increase** (A) Example cell tuning map triggered on model-predicted state changes with rank 1 approximation derived temporal modulation and evidence tuning. Data is excluded from the 300ms around the state change where the accumulated value distribution is too narrow to estimate tuning (dotted lines). (B) Average of all pre-movement side-selective cells’ tuning maps computed with z-scored firing rates and triggered on model-predicted state changes along with rank 1 approximation derived temporal modulation and evidence tuning for the average map.

213 **Neurons track changes in provisional decision.** If cells are stably tuned to the accumulated  
 214 evidence throughout deliberation, we should be able to see rapid responses in their firing rates to  
 215 changes in the animal’s provisional decision. To look at responses to these changes of mind, we  
 216 computed each cell’s average deviation from its mean temporal trajectory aligned to time points  
 217 when the behavioral model predicted a change in the animal’s estimate of the environmental

218 state (Fig. 4A). Following Hanks et al.<sup>6</sup>, we introduced a 100ms response lag between model-  
219 predictions and FOF responses. For this analysis, changes of mind were selected at time points  
220 when a 100ms running average of the posterior mean crossed the decision boundary ( $a = 0$ ).  
221 To avoid introducing noise into this analysis, changes of mind in the first and last 200ms of  
222 the trial were excluded, as were state changes that immediately reversed to the previous state  
223 (see methods). For each cell, this method produced two state-change triggered response curves  
224 describing responses to changes into states 1 ( $STR_1$ ) and 2 ( $STR_2$ ). STRs are also referred  
225 to as  $STR_{pref}$  and  $STR_{non-pref}$  according to cells' previously determined side-preference. STRs  
226 are shown for an example neuron (Fig. 4B). Discriminability before and after model-predicted  
227 changes of mind was measured using  $d'$  and tested for significance by permuting the state-change  
228 labels across trials (2-tailed permutation test, 250 permutations,  $p < .05$ ).

229 To visualize the state change triggered response across the neural population, each cell's  
230 response is summarized by computing the difference between the z-scored STR for state changes  
231 into the preferred state and into the non-preferred state ( $STR_{pref} - STR_{non-pref}$ ). We present  
232 these data as a heat map for all pre-movement side-selective cells (Fig. 4C). The z-scored STRs  
233 were averaged across these cells to give the average state-change triggered response across the  
234 population (Fig. 4D). We apply the permutation procedure described above to each cell and  
235 compute the fraction of the included cells that significantly encode state at each timepoint  
236 relative to the state change (Fig. 4E). If cells are encoding the animal's provisional decision, we  
237 should expect them to take intermediate firing rates during both the state change into and out  
238 of the preferred state. This would mean that cells are not significantly discriminating between  
239 states at the time of the state change. Consistent with this, we find that the population reaches  
240 its minimum fraction of cells differentiating between states at the time of the model-predicted  
241 state change. We recomputed the timecourse of discriminability across cells triggered on changes  
242 in the veridical environmental state, rather than the model-predicted changes. When we do this,  
243 we find the dip in the fraction of cells significantly discriminating the state is delayed relative  
244 to the dip produced by triggering on model-predicted changes. This is consistent with the  
245 FOF tracking changes in the sign of accumulated evidence rather than simply responding to  
246 the instantaneous stimulus. At the level of individual cells and across the population, we see  
247 rapid responses to changes of mind providing further evidence that neurons track the animal's  
248 provisional decision throughout the accumulation process.

249 **Stable evidence tuning before and after changes of mind.** To further characterize cell  
250 tuning to accumulated evidence during changes of mind, we recomputed the tuning maps aligning  
251 time to model-predicted state changes instead of the start of the trial. The computation and  
252 rank 1 decomposition of the tuning curves proceeded in the same manner as before except time  
253 in each trial was aligned to changes of mind:

$$r(a, t - t_c) \approx E[r(t)] + \hat{m}(t - t_c) * \hat{f}(a) \quad (2)$$

254 where  $t_c$  is the timing of changes of mind. Consistent with the state-change triggered responses  
255 and previous tuning curve analysis, we see that tuning in example neurons and the population  
256 is well described by a single evidence tuning curve multiplied by a temporal modulation before  
257 and after state changes (Fig. 5A,B). The rank 1 approximation for the example cell presented in  
258 Figure 5A explains 98.7% of the variance in the tuning map and the average variance explained  
259 for all selective cells is  $84.9\% \pm 9.7\%$ . The population average across z-scored tuning maps for  
260 all pre-movement side-selective cells is also well-described by the rank 1 approximation, which  
261 captures 95% of the variance (Fig. 5B). This demonstrates that neurons encode the accumulated  
262 evidence with a single tuning curve even at the times when the hidden state and provisional  
263 decision fluctuate.

## Discussion

We recorded neural activity from the frontal orienting fields (FOF) of rats performing a dynamic decision-making task designed to induce frequent changes of mind. In our study, rats integrated sequential pieces of information, discounting older evidence, to track changes in a volatile hidden state. FOF responses have been characterized previously during a similar task in a stationary environment where rats learn to equally weigh all evidence and changes of mind are rare<sup>6</sup>. This previous work revealed categorical encoding of population activity to the accumulated evidence, characterized by a single tuning curve throughout the trial. This suggested that FOF encoded the provisional decision during decision-making. However, in a stationary environment, the provisional decision rarely differs from the final choice meaning that preparatory activity could begin without needing to be reversed. In a dynamic environment, where changes of mind are frequent, it might be advantageous to suppress choice coding until the final decision is reached. It was not clear whether FOF would play a similar role in representing evidence during decision-making in a constantly-changing environment and while the provisional decisions were still highly flexible.

We found that FOF responses to accumulation in a dynamic environment were similar to FOF responses during accumulation in a stationary environment. First, a subpopulation of about 17% of active neurons showed significant side-selectivity during the pre-movement stimulus period. This was a smaller fraction than previously reported, but was an expected result of a task with more frequent stimulus-induced changes of mind. Using a method developed by Hanks et al.<sup>6</sup>, we measured the encoding of the decision variable in single neurons and across the population. We improved this method by using a rank 1 approximation to explain the evidence-encoding component of neural firing rates as the product of a temporal modulation and an evidence tuning curve. The rank 1 approximation supported the description of FOF neurons with a single evidence tuning curve that was modulated over the trial. Across the population, we found that the temporal modulation increased until the timing of the earliest possible ‘go’ cue and then plateaus at a maximum modulation strength during the rest of the trial.

The dynamic nature of the task allowed measurements that are not possible in stationary tasks, where evidence is drawn from a single distribution during each decision, and changes of mind are rare. We used our behavioral model to predict the timing of changes in the provisional decision during evidence accumulation and measured the response to these changes of mind events in neural activity. If the neurons use a single evidence tuning curve throughout accumulation, we expect the neural firing rates to encode the provisional decision before and after changes of mind. Computing state change triggered responses for each neuron, showed that FOF cells rapidly responded to changes of mind, reflecting the new provisional decision in their firing rates. Critically, neurons encoded provisional decisions both before and after changes of mind, which implies that provisional decisions are encoded even when they differ from the final choice. Neuronal responses were better aligned to changes of mind predicted by the behavioral model than to changes in the true environmental state, suggesting that these responses were not simply reflecting a change in sensory experience. Combining this approach with the method for computing accumulated evidence tuning maps, we found, as described above, that the product of a single evidence tuning curve and temporal modulation was still sufficient to explain the evidence response across changes of mind (rank 1 approximation). We observed that, after the moment when ‘go’ cues could arrive, the temporal modulation of evidence tuning was, on average, stable. Together, our results demonstrate that FOF neurons encode the animal’s provisional decision and respond rapidly, updating this representation following changes of mind.

Changes of mind are not unique to dynamic environments and can also occur during evidence accumulation in stationary environments. These events can occur during stimulus presentation due to noise in the decision making process and can be predicted from neural activity<sup>19</sup>. Changes of mind may also occur after the subject begins to execute their choice due to post-processing

314 delays<sup>20</sup> or constraints placed on action<sup>21</sup>. Our work differs from these studies, in that we use an  
315 environment designed to induce changes of mind and ask how neurons respond to these model-  
316 predicted events. To our knowledge, only one other study<sup>22</sup> has examined neural responses to  
317 behaviorally predicted changes of mind during evidence accumulation in a dynamic environment,  
318 and ours is the only such study in an animal model.

319 Previous inactivation studies suggest that while FOF is critical for performing actions and  
320 reporting decisions, it is not necessary for the integration of evidence<sup>6,23</sup>. This is consistent  
321 with the FOF representing the evidence after categorization into a provisional choice<sup>12</sup>. Work in  
322 mouse anterior lateral motor (ALM), a comparable cortical region, shows that categorical signals  
323 in this region recover quickly following photoinhibition, suggesting categorical input from other  
324 brain regions<sup>24</sup>. In a recent study, Finkelstein et al.<sup>25</sup> found that ALM choice signals were  
325 robust to distractors delivered during a delay period after the typical evidence presentation  
326 period, suggesting local circuitry maintained the choice signal. Our study considered a similar  
327 brain structure operating in a regime where, rather than ignoring distractors, it needed to flexibly  
328 update provisional decisions in response to new information. These studies, along with recent  
329 modeling work<sup>12,26</sup>, suggest a common role for the FOF and the ALM in maintaining choice  
330 signals that are either robust to or responsive to new information according to task demands.

331 The dynamic decision-making task offers a complementary approach to typical studies of  
332 evidence accumulation in static environments. Here, we showed that in constantly-changing  
333 environments FOF neurons encode provisional choices and respond rapidly to changes of mind  
334 predicted from our behavioral model. Our quantitative methods and behavioral paradigm will  
335 be useful tools for investigation of the brain circuitry supporting evidence accumulation and the  
336 decision-making process more generally.

## 337 **Methods**

338 **Subjects** Animal use procedures were approved by the Princeton University Institutional Ani-  
339 mal Care and Use Committee and carried out in accordance with NIH standards. All subjects  
340 were adult male Long Evans rats (Vendor: Taconic and Harlan, USA). Rats were pair-housed  
341 prior to implantation with recording electrodes and single-housed subsequently. Rats were placed  
342 on a water restriction schedule to motivate them to perform the task for water rewards.

343 **Behavioral training** We trained rats on the dynamic clicks task<sup>16</sup> (Figure 1). Rats went  
344 through several stages of an automated training protocol. In the final stage of training, each  
345 trial began with the illumination of a center nose port by an LED light inside the port. This  
346 LED indicated that the rat could initiate a trial by placed its nose into the center port. Rats  
347 were required to keep their nose in the center port (nose fixation) until the light turned off as  
348 a ‘go’ signal. During center fixation, auditory cues were played indicating the current hidden  
349 state. The duration of the stimulus period was drawn from a uniform distribution between 500  
350 and 2000ms. After the ‘go’ signal, rats were rewarded for entering the side port corresponding to  
351 the final value of the hidden state. The hidden state did not change after the ‘go’ cue. Correct  
352 choices were rewarded with 18 microliters of water. Incorrect choices were signaled by a white  
353 noise stimulus (spectral noise of 1 kHz for a 0.7 seconds duration). The rats were put on a  
354 controlled water schedule where they receive at least 3% of their weight every day. Rats trained  
355 each day in training session of around 120 minutes. Training sessions were included for analysis if  
356 the overall accuracy rate exceeded 70%, the center-fixation violation rate was below 25%, and the  
357 rat performed more than 50 trials. In order to prevent the rats from developing biases towards  
358 particular side ports an anti-biasing algorithm detected biases and probabilistically generated  
359 trials with the correct answer on the non-favored side.

360 **Psychometric and chronometric curves** Task performance was assessed using psychometric

361 curves, chronometric curves and psychophysical reverse correlations. For all task performance  
362 plots, rat data was overlaid on predictions from the accumulation model described below. These  
363 predictions were made by using the probability of a right or correct choice on each trial given  
364 by the accumulation model in place of the actual choice observed.

365 Psychometric plots show the probability that the rat chose to go right as a function of the  
366 ideal observer log-odds supporting a ‘go right’ choice. Final state chronometric plots show the  
367 probability of a correct choice as a function of the final state duration, the elapsed time between  
368 the final hidden state change (or the beginning of the stimulus, if there are no state changes)  
369 and the end of the stimulus. Data is plotted separately for trials with 0, 1, or more than 1 state  
370 changes.

371 **Psychophysical reverse correlation** The computation of the reverse correlation curves was  
372 similar to methods previously reported<sup>5,6,23</sup>. An additional step was included, as in Piet et al.<sup>16</sup>,  
373 to deal with the changing hidden state. First, the click trains on each trial were smoothed with  
374 a causal Gaussian filter  $k(t)$ . The left click train was subtracted from the right, creating one  
375 smooth click rate for each trial. The filter had a standard deviation of 5 msec.

$$r_i(t) = \delta_{R,t} * k(t) - \delta_{L,t} * k(t) \quad (3)$$

376 Then, the smoothed click rate on each trial was normalized by the expected click rate for that  
377 time step, given the current state of the environment. This gives us the deviation (the excess  
378 click rate) from the expected click rate for each trial.

$$e_i(t) = r_i(t) - \langle r(t) | S_i(t) \rangle \quad (4)$$

379 Finally, we compute the choice triggered average of the excess click rate by averaging over trials  
380 based on the rat’s choice.

$$\text{excess-rate}(t|\text{choice}) = \langle e(t) | \text{choice} \rangle \quad (5)$$

381 The excess rate curves were then normalized to integrate to one. This was done to remove  
382 distorting effects of a lapse rate, as well to make the curves more interpretable by putting the  
383 units into effective weight of each click on choice.

384 **Accumulation Model** The accumulation model characterizes the decision-making process as  
385 the evolution over time  $t$  of an accumulation value  $a$  in response to left and right click trains,  $\delta_L$   
386 and  $\delta_R$ , with dynamics governed by a parameter set  $\theta$ . Each rat’s behavioral data is used to find  
387 the parameter set that maximizes the probability under the model of the rat’s choices  $y$ . Evaluating  
388 this model with the best fit parameters produces a probability distribution over values of  $a$   
389 at every timepoint in the trial. We refer to this as the forward model distribution  $P(a|t, \delta_R, \delta_L, \theta)$ .  
390 The forward model was described previously in Piet et al.<sup>16</sup> and will be reviewed in detail below.  
391 To characterize neural encoding of the accumulation value, we further constrained the accumu-  
392 lation value distribution on trials where we had simultaneous neural recordings by incorporating  
393 the rat’s choice,  $y$ , to find the posterior distribution  $P(a|t, \delta_R, \delta_L, \theta, y)$ . To do this, we computed  
394 a distribution that we refer to as the backward model distribution, which we describe in the  
395 next section.

396 At each moment in the trial, the forward model  $P(a|t, \delta_R, \delta_L, \theta)$  predicts a Gaussian distri-  
397 bution of accumulation values with mean  $\mu(t)$  and variance  $\sigma^2(t)$ . At start of the trial, time  
398  $t = 0$ , this distribution has zero mean and initial variance  $\sigma_i^2$ . At all timepoints, the mean and

399 variance are given by:

$$\begin{aligned}\mu(t) &= \mu_0 e^{\lambda t} + \int_0^t (\delta_{R,s} \cdot C(R(s)) - \delta_{L,s} \cdot C(L(s))) ds \\ &= \sum_i^{\#R_t} e^{\lambda(t-R(i))} C(R(i)) - \sum_i^{\#L_t} e^{\lambda(t-L(i))} C(L(i))\end{aligned}\quad (6)$$

$$\begin{aligned}\sigma^2(t) &= \sigma_i^2 e^{2\lambda t} + \frac{\sigma_a^2}{2\lambda} (e^{2\lambda t} - 1) + \int_0^t \sigma_s^2 (\delta_{R,s} \cdot C(R(s)) - \delta_{L,s} \cdot C(L(s))) e^{2\lambda t} ds \\ &= \sigma_i^2 e^{2\lambda t} + \frac{\sigma_a^2}{2\lambda} (e^{2\lambda t} - 1) + \sum_i^{\#R_t} \sigma_s^2 C(R(i)) e^{2\lambda(t-R(i))} + \sum_i^{\#L_t} \sigma_s^2 C(L(i)) e^{2\lambda(t-L(i))}\end{aligned}\quad (7)$$

400 Where  $\#R_t$  is the number of right clicks on this trial up to time  $t$ ,  $R(i)$  is the time of the  $i^{\text{th}}$   
401 right click and  $\delta_{R,i}$  is a delta function at time  $R(i)$ .  $C(R(i))$  tells us the effective adaptation for  
402 that click.

403 The model parameters  $\theta$  can be described in words as an initial noise variance  $\sigma_i^2$ , a per-click  
404 noise variance  $\sigma_s^2$ , a memory noise variance  $\sigma_a^2$ , a discounting rate  $\lambda$ , the strength and time  
405 constant of adaptation  $\phi$  and  $\tau_\phi$ , a bias  $B$  and a lapse rate  $l$ .

406 To determine the probability of a right versus left choice, we first integrate the accumulation  
407 value distribution in the last timepoint of the trial  $P(a|t = t_N, \delta_R, \delta_L, \theta) = \mathcal{N}(\mu(t_N), \sigma^2(t_N))$   
408 from the bias parameter  $B$  to  $+\infty$

$$P(a > B|t = t_N, \delta_R, \delta_L, \theta) = \frac{1}{2} \left( 1 + \operatorname{erf} \left( \frac{-(B - \mu(t_N))}{\sigma(t_N)\sqrt{2}} \right) \right).\quad (8)$$

409 On each trial, the rat makes a random choice with probability determined by lapse rate  $l$ . Then,  
410 the probability of a ‘go right’ choice is given by

$$P(y^i = 1|\theta) = (1 - l)P(a > B|t = t_N^i, \delta_R^i, \delta_L^i, \theta) + l/2\quad (9)$$

$$P(y^i = -1|\theta) = (1 - l)(1 - P(a > B|t = t_N^i, \delta_R^i, \delta_L^i, \theta)) + l/2\quad (10)$$

411 Where

$$y^i = \begin{cases} 1, & \text{if rat chooses right on trial } i \\ -1, & \text{if rat chooses left on trial } i \end{cases}\quad (11)$$

412 Parameters  $\theta$  were fit to each rat individually by maximizing the likelihood function:

$$L = \prod_i^{\#\text{trials}} P(y^i|\theta).\quad (12)$$

413 A half-gaussian prior was included on the initial noise  $\sigma_i$  and accumulation noise paramete-  
414 rs  $\sigma_a$ . The priors were set to match the respective best fit values from Brunton et al.<sup>5</sup>. The  
415 numerical optimization was performed in MATLAB, using the function `fmincon`. To estimate  
416 the uncertainty on the parameter estimates, we used the inverse hessian matrix as a parameter  
417 covariance matrix<sup>27</sup>. To compute the hessian of the model, we performed automatic differen-  
418 tiation in *julia* to exactly compute the local curvature<sup>28</sup>. See the Supplemental Materials for

419 parameter estimates and uncertainty values. Brunton et al.<sup>5</sup> extensively analyzed how well a  
420 similar model with an additional bound parameter recovers generative parameters, finding the  
421 model contains one maximum likelihood point in parameter space (See Section 2.3.3-6 of the  
422 supplement to Brunton et al.<sup>5</sup>). We compared parameter fits in this task to those reported in  
423 Brunton et al.<sup>5</sup>, which developed the stationary version of this task.

424 **Posterior model** The forward model gives us a probability distribution over accumulation  
425 values at each time point in a trial as well as an estimated probability of the rat choosing to  
426 go right or left on that trial. Observing the rat's choice  $y$  at the end of each trial allows us to  
427 constrain the distribution of possible trajectories that the accumulation value could have taken.  
428 The resulting posterior distribution (previously called the backward pass distribution in Brunton  
429 et al.<sup>5</sup>) is useful for analyzing the neural encoding of accumulated evidence.

430 The posterior distribution can be computed by taking the product of the forward model  
431 distribution and a backward distribution. While the forward distribution is constrained to the  
432 start the trial with mean 0 and variance  $\sigma_i^2$ , the backward distribution is constrained to finish  
433 the trial with the probability density uniformly distributed on one side of the decision boundary  
434  $B$  according to the rat's choice. To distinguish between the two distributions and to make the  
435 constraints on the initial and final distributions explicit, we will write the forward distribution

$$f(a) = P(a|t, \delta_R, \delta_L, \theta, a_0 = \mathcal{N}(0, \sigma_i^2)) \quad (13)$$

436 and the backward distribution

$$b(a) = \begin{cases} P(a|t, \delta_R, \delta_L, \theta, a_N = \mathcal{U}(B, \infty)), & \text{if } y_i = 1 \\ P(a|t, \delta_R, \delta_L, \theta, a_N = \mathcal{U}(-\infty, B)), & \text{if } y_i = -1. \end{cases} \quad (14)$$

437 Importantly, the forward and backward distributions are conditionally independent, conditioned  
438 on the final value of the accumulated evidence. Given that they are independent, the posterior  
439 distribution that combines the initial and final constraints is the product of the forward and  
440 backward distributions.

$$P(a|t, \delta_R, \delta_L, \theta, a_0 = \mathcal{N}(0, \sigma_i^2), y) = f(a)b(a) \quad (15)$$

441 We approximated the backward distribution as a mixture distribution over a grid of final  
442 accumulation values with spacing  $\Delta a$ . A unit of probability mass is initialized at each point in  
443 the grid and the solution is given by:

$$b(a) = \sum_{i=-\infty}^{\infty} w_i P(a|t, \delta_R, \delta_L, \theta, a_N = \mathcal{N}(B + i\Delta a, 0)) \quad (16)$$

444 The mixture weights  $w_i$  are all equal if the bin spacing is uniform. Each unit of probability mass  
445 evolves using the same solution as the forward model, but with time reversed. This solution is  
446 exact as  $\Delta a \rightarrow 0$ .

447 For tuning curve analyses we use the full posterior distribution, for the state change triggered  
448 response analyses we use the mean of the posterior. See the Supplementary Materials for a  
449 detailed discussion on the derivation and evaluation of the backward and posterior model.

450 **Microwire array recordings** Microwire array implant surgery: Four rats were implanted with  
451 microwire arrays in their left or right FOF (n= 2 in lFOF, n = 2 in rFOF) The target region was  
452 accessed by craniotomy, using standard stereotaxic techniques (centred 2 mm anterior to the  
453 bregma and 1.3mm lateral to the midline). Dura mater was removed over the entire craniotomy  
454 with a small syringe needle. The remaining pia mater, even if not usually considered to be

455 resistant to penetration, nevertheless presents a barrier to the entry of the microelectrode arrays  
456 because of the high-density arrangement of electrodes in the multi-channel electrode arrays.  
457 This dimpling phenomenon, when the electrodes are pushing the brain cortex down without  
458 penetrating, is more pronounced for arrays with larger numbers of electrodes. In addition to  
459 potentially injuring the brain tissue, dimpling is a source of error in the determination of depth  
460 measurements. Ideally, if dimpling could be eliminated, the electrodes would move in relation  
461 to the pial surface, allowing for effective and accurate electrode placement. To overcome the  
462 dimpling problem, we implemented the following procedure. After the craniotomy was made,  
463 and the dura was carefully removed over the entire craniotomy, a petroleum-based ointment  
464 (such as bacitracin ointment or sterile petroleum jelly (Puralube Vet Ointment)) was applied to  
465 the exact site of electrode implantation. The cyanoacrylate adhesive (Vetbond Tissue Adhesive)  
466 was then applied to the zone of the pia surrounding the penetration area. This procedure  
467 fastens the pia mater to the overlying bone and the resulting surface tension prevents the brain  
468 from compressing under the advancing electrodes. Once the polymerization of cyanoacrylate  
469 adhesive was complete, over a period of few minutes, the petroleum ointment at the target site  
470 was removed, and the 32-electrode microwire array (Tucker-Davis Technologies) was inserted  
471 by slowly advancing a Narishige hydraulic micromanipulator. After inserting the array(s), the  
472 remaining exposed cortex was covered with biocompatible silicone (kwik-sil), and the microwire  
473 array was secured to the skull with C&B Metabond and dental acrylic.

474 During a ten-day recovery period, rats had unlimited access to water and food. Recording  
475 sessions in the apparatus began thereafter, using Neuralynx acquisition systems. Once rats had  
476 recovered from surgery, recording sessions were performed in a behavioral chamber outfitted  
477 with a 32 channel recording system (Neuralynx). Spiking data was acquired using a bandpass  
478 filter between 600 and 6000 Hz and a spike detection threshold of 30 microV.

479 For array recordings, clusters were manually cut (Spikesort 3D, Neuralynx), and both single-  
480 and multi-units were considered.

481 **Tetrode recordings** Tetrode drives were 3d printed from custom designs (design files available  
482 upon request) on a Form2 3d printer in tough resin. Each drive consists of a drive body, a  
483 cone and cap to protect the drive body, and four bundles of 8 tetrodes in glass tubes. Each  
484 bundle was glued together and to a cannula. Each cannula was attached to a screw using dental  
485 cement, and cured with UV light. Each wire from each tetrode was fed through a unique channel  
486 in a 128 channel Electrode Interface Board (SpikeGadgets) and pinned with a gold pin. After  
487 loading all tetrodes, trimming, and building of the drive, the day-of or night before the surgery,  
488 we electroplated the drive in gold using a nanoZ impedance tester (White Matter LLC) and  
489 measured impedences.

490 Tetrode drive implant surgery proceeded as described for microwire arrays, except we did not  
491 need to vetbond the brain surface because each tetrode bundle produced very little dimpling. A  
492 silver wire and skull screw were used to ground the drive. Drives were secured with metabond  
493 and acrylic until secure. Tetrodes were advanced 0.1mm into the brain.

494 During a seven-day recovery period, animals had unlimited access to water and food. Animals  
495 were then returned to training and water restriction. To acclimate animals to the weight of the  
496 wireless apparatus, every three days, we replaced the cap on the implant with a new cap 3g  
497 heavier than the previous cap. If animals' behavioral performance or weight dropped, or if we  
498 noticed any excess tilting of the head from the weight, we returned the animal to the previous  
499 weight and waited an additional 2 days before moving to the next weight. This process was  
500 repeated until the animals were behaving well with caps weighing 27g.

501 Once animals were acclimated to the weight, recordings could begin. Tetrodes were advanced  
502 0.25mm at a time, at least 20 hours before recording. For each recording session, the animal's  
503 cap was replaced with a 500mAh lithium battery, 128Gb Sandisk extreme plus SD card, a 160-  
504 pin Amphenol Lynx connector, and datalogger (SpikeGadgets). At the end of each session, the



505 datalogger, SD card, and battery were removed and the 27g cap replaced.

506 The tetrode recordings were automatically clustered using Kilosort2<sup>29</sup>. Automatically de-  
507 termined clusters were manually curated using the Phy GUI ([https://github.com/kwikteam/](https://github.com/kwikteam/phy)  
508 [phy](https://github.com/kwikteam/phy)).

509 **Electrophysiological analysis** We computed the firing rates for all neurons aligned to the  
510 time of stimulus onset (when the rat first broke the center port IR beam triggering playback of  
511 the stimulus) and to movement (when the rat first stopped breaking the center port IR beam to  
512 make its choice). Firing rates were computed by binning spikes into 25ms bins and smoothing  
513 them with a casual gaussian filter with a standard deviation of 100ms. Stimulus onset aligned  
514 firing rates were masked on each trial after the movement and movement aligned firing rates  
515 were masked prior to stimulus onset. Firing rates for example cells were averaged over trials  
516 conditioned on choice and outcome.

517 Cells were considered active if their average stimulus onset aligned firing rate was greater  
518 than 1 Hz during the time from 1 second prior to the stimulus onset to the time of movement  
519 onset. Cells were considered pre-movement side-selective if the spike counts during the period  
520 between stimulus onset and movement were different on trials that resulted in a left versus a  
521 right choice ( 2-tailed t-test,  $p < .05$ ). The side with the higher firing rate is referred to as the  
522 cell's preferred side.

523 A population-average PSTH was computed by averaging over all trials from all pre-movement  
524 side-selective cells conditioned on final state duration and whether the trial ended in a choice to  
525 the cell's preferred side.

526 We analyzed the timecourse of choice selectivity by computing the area under the receiver  
527 operating characteristic curve (AUC) at each 25ms time bin in for the smoothed firing rates in  
528 left choice versus right choice trials. To compute significance, we performed a permutation test  
529 where the left/right choice labels were permuted relative to the firing rates across trials. For  
530 visualization purposes, we sorted cells by latency to reach 8 significant 25ms time bins in a row  
531 (2-tailed permutation test, 250 permutations,  $p < .05$ ).

**Evidence tuning curves** We compute evidence tuning curves using a method based on the  
one used in Hanks et al.<sup>6</sup>. First, the posterior accumulation distribution

$$P(a|t, \delta_R, \delta_L, \theta, a_0 = \mathcal{N}(0, \sigma_i^2), y) \quad (17)$$

532 for each trial is computed. To simplify notation, we will refer to this distribution as  $P(a|\theta, y)$ .  
533 The joint distribution of  $P(a|\theta, y)$ , the firing rate  $r$ , and time  $t$ , which we will call  $P(r, a, t)$   
534 is computed by binning time, accumulation values, and firing rates. For each trial and each  
535 timepoint, the probability mass in each accumulation value bin in  $P(a|\theta, y)$  is added to the bin  
536 in  $P(r, a, t)$  associated with that timepoint and that firing rate. Because the shortest trials are  
537 500ms, not all trials contribute to each time point, so each time bin is normalized according to  
538 the number of trials that contribute to it. Each neuron's firing rate was divided into 100 bins  
539 spanning the minimum to the maximum firing rate of the neuron. Time was binned into 25ms  
540 bins. Accumulation value bin size was divided into 10 bins with width set to 1.625 except the  
541 last bin which was larger to capture the tails of the distribution.

542 The expected firing rate as a function of accumulation value  $a$  and time  $t$  is computed by  
543 marginalizing with respect to the firing rate  $E[r(a, t)] = \sum_i r_i P(r_i, a, t)$ . The expected difference  
544 from the average firing rate is computed by computing the average firing rate as a function of  
545 time  $E[r|t] = \sum_j r_j \sum_j P(r_j, a_j, t)$  and subtracting this from average expected firing rate at each  
546 timepoint  $E[\Delta r|a, t] = E[r|a, t] - E[r|t]$ . Average accumulation value tuning over the trial is  
547 computed by averaging over time  $E[\Delta r|a] = \frac{1}{N} \sum_i E[\Delta r|a, t_i]$  where  $N$  is the number of time  
548 bins. The same procedure is used to compute a map of z-scored firing rates. And these maps  
549 are averaged across pre-movement side-selective cells to produce a population average.

550 Rank 1 approximations of the  $E[\Delta r|a, t]$  are computed using the singular value decomposi-  
551 tion. The approximation is equal to the outer product of the first left singular vector  $u_1$  and the  
552 first right singular vector  $v_1$ , scaled by the first singular value  $s_1$ . These terms are rearranged  
553 to give the outer product of a firing rate modulation,  $\hat{m}(t) = u_1 s_1 \text{range}(v_1)$  and a tuning curve  
554  $\hat{f}(a) = v_1 / \text{range}(v_1)$ . Scaling by  $\text{range}(v_1)$  gives  $\hat{f}(a)$  unit scale and  $\hat{m}(t)$  units of spikes/s. Our  
555 complete tuning curve approximation becomes:

$$r(a, t) \approx E[r(t)] + \hat{m}(t) * \hat{f}(a). \quad (18)$$

556 The variance explained by this approximation is given by the ratio between the first singular  
557 value and the sum of all singular values:

$$\frac{s_1}{\sum_{i=1}^N s_i}. \quad (19)$$

558 **State change triggered response** On each trial, we computed a residual firing rate by sub-  
559 tracting the average firing rate at each time point during the trial. We then aligned these residual  
560 firing rates to either the model-predicted state changes or the generative environmental state  
561 changes. We masked firing rates before the preceding state change and after the following state  
562 change when applicable. We computed the mean of these residual firing rates for visualization.  
563 To test for significant discrimination of state, we compared  $d'$  in the real data to a permutation  
564 distribution created by permuting state labels across state changes (2-tailed permutation test,  
565 250 permutations,  $p < .05$ ).

566 We defined model-predicted state changes as time points where the running average of the  
567 mean of the posterior accumulator value crossed 0. The running average was computed over 100  
568 bins of 1ms. To avoid introducing noisy state changes, we excluded state changes from the first  
569 and last 200ms of the trial. We also excluded state changes that did not meet two change strength  
570 criteria designed to identify state changes that were immediately reversed. The first, was based  
571 on the average value of the posterior mean in the 100ms before the change compared to the  
572 100ms after the change. State changes were excluded if these strength values were inconsistent  
573 with the direction of the identified state change. The second state change strength was based on  
574 the slope of the running average of the posterior mean at the time of the change. If the sign of  
575 the slope was inconsistent with the sign of the state following the state change, this means that  
576 the accumulation value immediately returned back to the previous state. We excluded these  
577 state changes. Our results were robust to variations in the state change inclusion criteria.

578 **State change triggered tuning** State change triggered tuning maps  $E[\Delta r(a, t - t_c)]$  were  
579 computed using the tuning curve methods described above, but using time relative to state  
580 changes instead of stimulus onset. Firing rates were masked before and after the preceding and  
581 following state changes as described above. Data was also masked in the 300ms around the  
582 state change where the accumulated value distribution is too narrow to estimate tuning. Rank  
583 1 approximations and population tuning maps were computed as above.

## 584 Author contributions

585 AP, and AEH designed the study. AEH managed rat training and care. AEH, and EJD recorded  
586 the neural data. AP, and TB analyzed the neural data. AP, AEH, and TB wrote the manuscript.  
587 AEH, and CB oversaw all aspects of the project.

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## 673 **Supplementary Materials**

674 The supplementary materials contains extended figures, control analyses, and in-depth method  
675 discussions.

### 676 1. Individual Rat Behavior

#### 677 1.1. Model Parameters

#### 678 1.2. Psychometric Curves

#### 679 1.3. Chronometric Curves

#### 680 1.4. Reverse Correlation Kernels

### 681 2. Electrophysiology, Table of Rats

### 682 3. Posterior Accumulation Model

#### 683 3.1. Mathematical Details

#### 684 3.2. Toy Model Example

#### 685 3.3. Numerical Validation

### 686 4. Switch Triggered Averages

## Supplementary Materials - Individual Rat Behavior

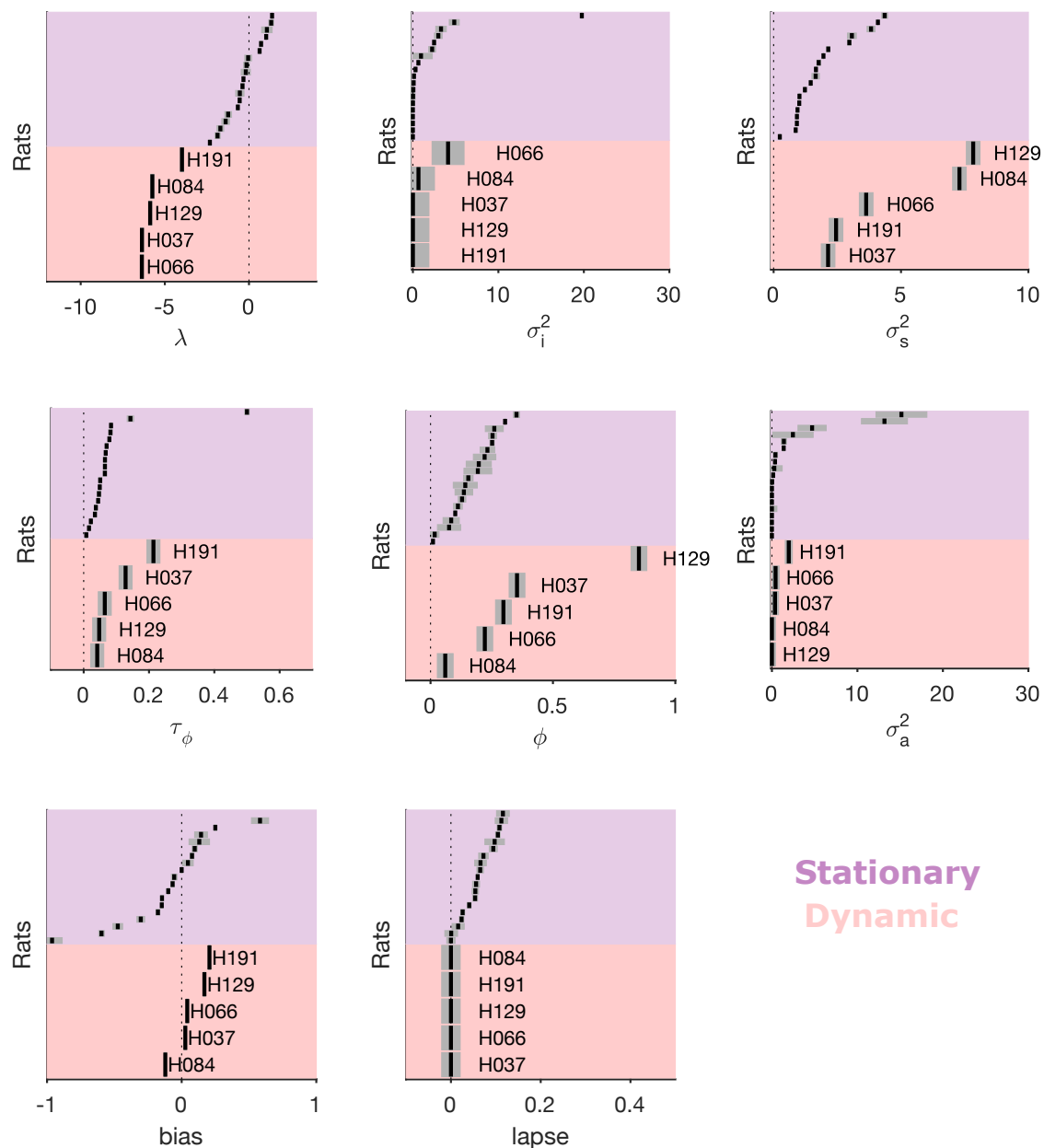


Figure S1: **Accumulation Model Parameters.** Best fit model parameters and 95% confidence intervals for each rat in this study. In addition the model parameter fits reported in Brunton et al.<sup>5</sup> for 19 rats in a stationary environment are included for comparison.

687

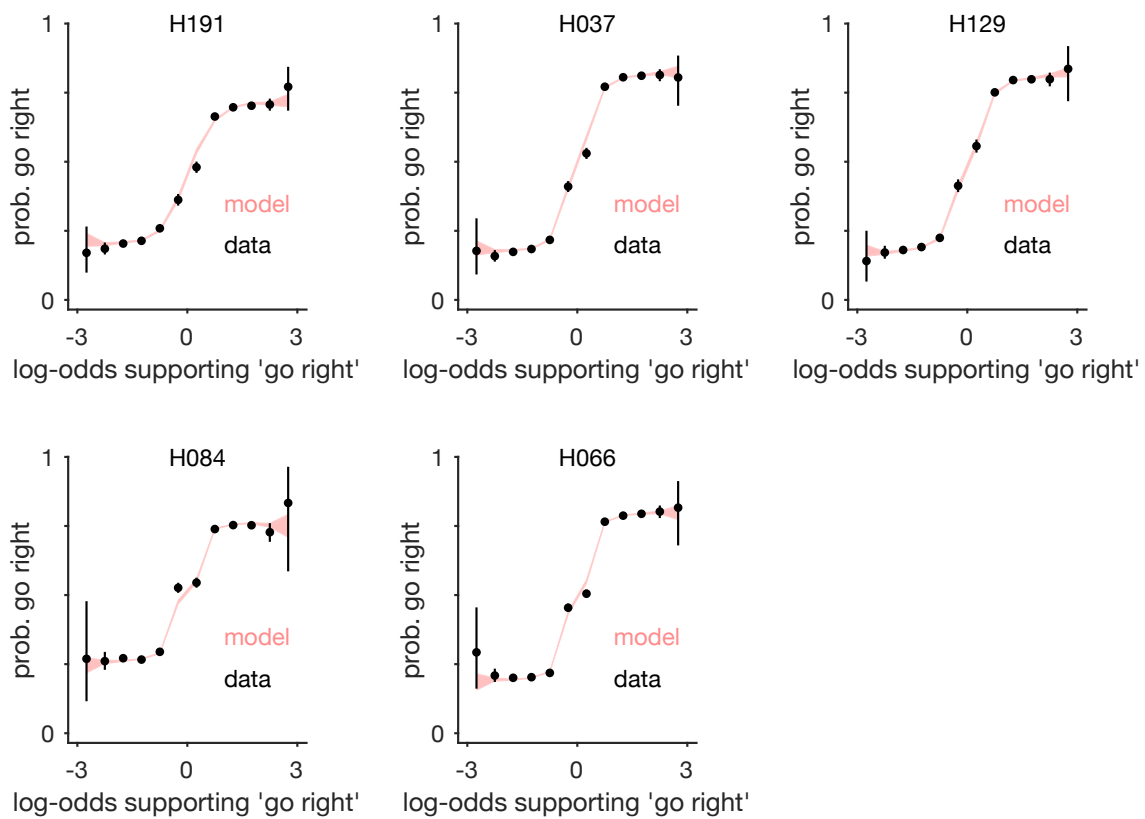


Figure S2: **All rats' psychometric curves.** Each plot shows the probability of a right choice given an ideal observer's log-odds supporting a go-right choice.

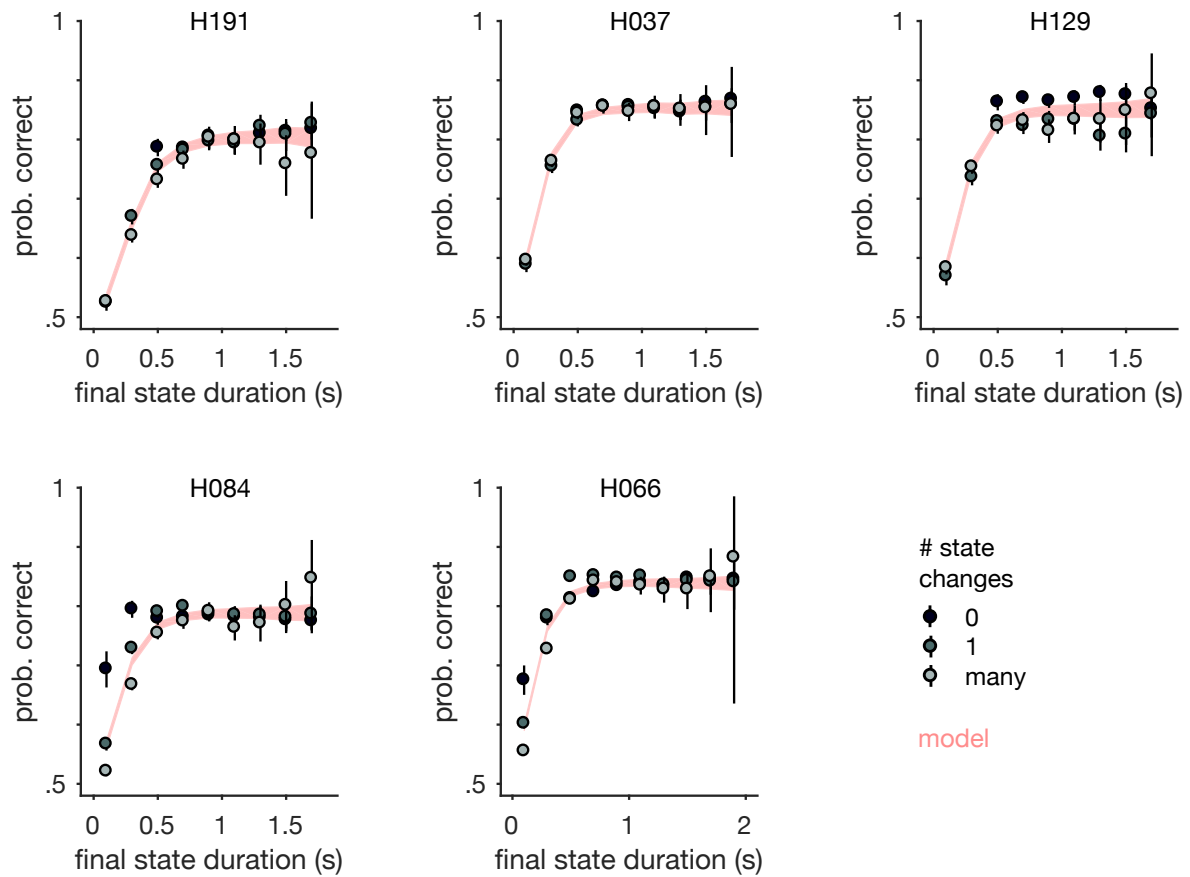


Figure S3: **All rats' final state chronometric curves.** Each plot shows the probability the rat made the correct choice as a function of the final state duration, and the total number of state changes in the trial. The final state duration was defined as the time from the last environmental state change to the end of the stimulus period. The best fit model prediction averaged over the total number of state switches is shown in pink.



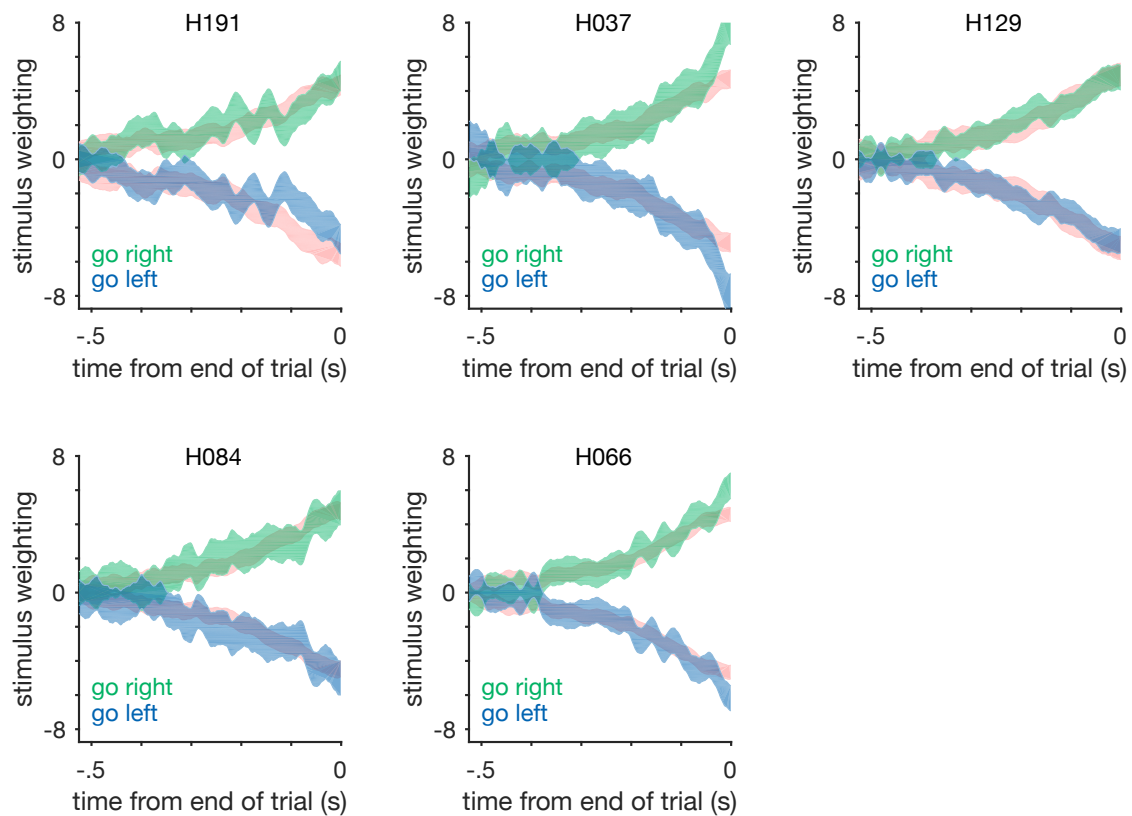


Figure S4: **All rats' psychophysical reverse correlation kernels.** Each plot shows the reverse correlation kernel for the rat (blue/green) and the best fit model (pink).

688 **Supplementary Materials - Table of Electrophysiology De-**  
689 **tails**

Rat	Implant Location	Recording Type
H191	blFOF	tetrode
H037	rFOF	array
H129	lFOF	array
H084	lFOF	array
H066	rFOF	array

690  
691 **Table 1.** List of rats used in the study, and the recording location  
692

## Supplementary Materials - Posterior Accumulation Model

The accumulation model has two components, the forward model and the backward model. When these components are combined it creates the posterior model used for analyzing neural data. The forward model is used to estimate a set of model parameters  $\theta$ . It assumes the initial distribution of accumulation values is gaussian distributed with mean zero and variance  $\sigma_i^2 \in \theta$ ,

$$P(a|t=0) = \mathcal{N}(\mu_0 = 0, \sigma_i^2). \quad (20)$$

So we can write the full forward model as:

$$f(a) = P(a|t, \delta_R, \delta_L, \theta, a_0 = \mathcal{N}(\mu_0 = 0, \sigma_i^2)) \quad (21)$$

At each moment in the trial, the forward model distribution of accumulation values  $f(a)$  is Gaussian distributed with mean  $\mu$  and variance  $\sigma^2$  given by:

$$\mu(t) = \mu_0 e^{\lambda t} + \int_0^t (\delta_{R,s} \cdot C(R(s)) - \delta_{L,s} \cdot C(L(s))) ds \quad (22)$$

$$= \sum_i^{\#R_t} e^{\lambda(t-R(i))} C(R(i)) - \sum_i^{\#L_t} e^{\lambda(t-L(i))} C(L(i)) \quad (23)$$

$$\sigma^2(t) = \sigma_i^2 e^{\lambda t} + \frac{\sigma_a^2}{2\lambda} (e^{2\lambda t} - 1) + \int_0^t \sigma_s^2 (\delta_{R,s} \cdot C(R(s)) - \delta_{L,s} \cdot C(L(s))) e^{2\lambda t} ds \quad (24)$$

$$= \sigma_i^2 e^{\lambda t} + \frac{\sigma_a^2}{2\lambda} (e^{2\lambda t} - 1) + \sum_i^{\#R_t} \sigma_s^2 C(R(i)) e^{2\lambda(t-R(i))} + \sum_i^{\#L_t} \sigma_s^2 C(L(i)) e^{2\lambda(t-L(i))} \quad (25)$$

Where  $\#R_t$  is the number of right clicks on this trial up to time  $t$ , and  $R(i)$  is the time of the  $i^{th}$  right click.  $C(R(i))$  tells us the effective adaptation for that click.

Computing the posterior distribution is more complicated than the forward model. First, we find the parameter set that best explains the choices  $y$  on each trial by maximizing  $P(y|\theta)$ . Then we use the best fit parameter set to evaluate the forward model for each trial, producing a probability distribution over accumulated evidence value at each time point consistent with the initial conditions. Next, we compute the backward model  $b(a)$ . Note this is not the “backward pass distribution” discussed by Brunton et al.<sup>5</sup>. The backward model here ignores the forward model, and instead computes the probability distribution over observing accumulated evidence values at each time point consistent with the stimulus and the choice at the end of the trial  $t_N$ .

$$b(a) = \begin{cases} P(a|t, \delta_R, \delta_L, \theta, a_N = \mathcal{U}(B, \infty)), & \text{if } y = 1 \\ P(a|t, \delta_R, \delta_L, \theta, a_N = \mathcal{U}(-\infty, B)), & \text{if } y = -1. \end{cases} \quad (26)$$

Importantly, the forward and backward distributions are conditionally independent, conditioned on the final value of the accumulated evidence. Given that they are independent, the posterior distribution that combines both observations is the product of the forward and backward distributions.

$$P(a|t, \delta_R, \delta_L, \theta, a_0 = \mathcal{N}(0, \sigma_i^2), y) = f(a)b(a) \quad (27)$$

One technical wrinkle is that our analytical solution for solving the model (forward or backward) relies on initial conditions that are gaussian. The forward model assumes an initial distribution  $\mathcal{N}(0, \sigma_i^2)$ . However, for the backward model the choice data only constrains the

718 sign of  $a$  at the end of the trial, meaning our initial conditions for the backward model is a  
 719 uniform distribution over  $(B, \pm\infty)$ . Therefore, we constructed a solution by discretizing the  
 720  $a$ -value axis into small bins of width  $\Delta a$ , and solved the backward model for each bin assuming  
 721 a delta function of initial probability mass in each bin. We refer to the backward distribution  
 722 from each bin  $i$  as the delta-backward solution  $b_i(a)$ . Our entire backward distribution is the  
 723 mixture distribution over all the individual delta-backward solutions.

$$b(a) = \sum_{i=0}^{\pm\infty} w_i b_i(a) \quad (28)$$

$$b_i(a) = P(a|t, \delta_R, \delta_L, \theta, a_N = \mathcal{N}(B + i\Delta a, 0)) \quad (29)$$

724 To solve each delta-backward solution  $b_i(a)$ , we use the time-reversed solution to the forward  
 725 model. The mixture weights  $w_i$  are all equal if the bin spacing is uniform. Note that it might be  
 726 tempting to think that we need to weight each individual delta-backward solution by the forward  
 727 model's probability mass in each bin; however, this is not correct. Given that the backward model  
 728 is independent of the forward model, we want the complete backward distribution to reflect all  
 729 possible states consistent with the choice observation, which is the uniform distribution over the  
 730 correct sign of  $a$ . With a set of  $b_i(a)$  solutions, we can now combine them into the posterior  
 731 distribution,  $p(a)$ . The exact solution as  $\Delta a \rightarrow 0$  is given by:

$$p(a|t, \delta_R, \delta_L, \theta, a_0 = \mathcal{N}(0, \sigma_{initial}^2), y) \propto f(a) \sum_{i=0}^{\pm\infty} w_i b_i(a). \quad (30)$$

732 To clarify notation, we now write the initial variance of the distribution as  $\sigma_{initial}^2$  to distinguish  
 733 this parameter from the series of grid solutions indexed by “ $i$ ”. In practice we truncate the  
 734 infinite series at a suitable extrema value of  $a$ , and use a finite bin spacing  $\Delta a = 1$ . On each  
 735 trial, the extent of the grid of delta solutions was determined by finding the accumulation value  
 736 where less than  $1e - 4$  probability mass of the posterior model lay beyond that point at the  
 737 end of the trial. Given that  $f(a)$ , and  $b_i(a)$  will be gaussian, let  $p_i(a) = f(a)b_i(a)$ , which is also  
 738 gaussian. This lets us write the posterior distribution as the sum of many delta-posterior modes.

$$p(a|t, \delta_R, \delta_L, \theta, a_0 = \mathcal{N}(0, \sigma_{initial}^2), y) \propto \sum_{i=0}^{\pm\infty} w_i p_i(a). \quad (31)$$

### 739 Toy Model Example

740 To illustrate the posterior model, consider a simple random walk process. On each time step  
 741 there is a  $1/3$  probability of staying in place,  $1/3$  probability of taking a step of size  $+1$ , and  
 742 a  $1/3$  probability of taking a step of size  $-1$ . We start at  $a(t = 0) = 0$ , and then observe the  
 743 process ten time steps later,  $t = 10$ . We now want to compute the forward model, and the  
 744 posterior distribution for this data.

745 Using a binomial distribution we can analytically compute the forward model, which describes  
 746 the probability of observing the process given the initial conditions and an elapsed duration (Fig.  
 747 S5A). To compute a backward model, we must define the final value. For example, if we let  
 748 the final value of the random walk be  $a(t = 10) = 2$ , then we can again use the binomial  
 749 distribution to compute the distribution of possible values at earlier time steps (Fig. S5B). We  
 750 can alternatively let the final value be the sign of the random walk process and compute the  
 751 full backwards distribution. Combining the independent forward and backwards distributions,  
 752 we can predict the posterior distribution, which is all possible states the random walk could be  
 753 in given these two observations (Fig. S5E). We can check this against a particle simulation by  
 754 sampling 2,000,000 trajectories from the random walk process to get the forward distribution  
 755 (Fig. S5C). To get the posterior distribution we filter our samples for trajectories that ended

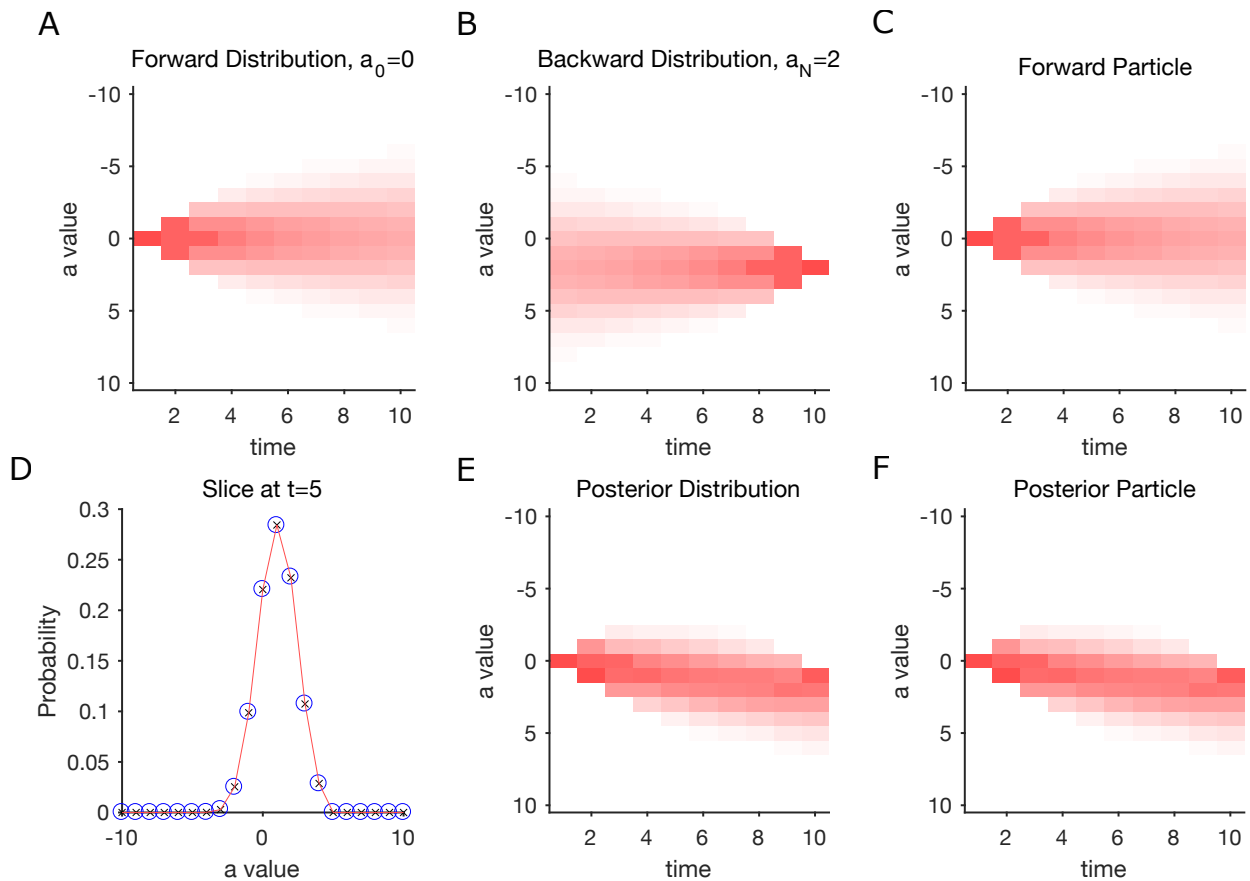


Figure S5: **Toy Model Demonstration** (A) Forward model analytical solution for the toy model. (B) Delta-Backward model analytical solution for final accumulation value of  $a_N = 2$ . (C) Distribution of the forward model from sampling. (D) Comparing the distributions from E (red) and F (blue) at  $t = 5$ . (E) Posterior model analytical solution for initial conditions  $a_0 = 0$  and final “choice”  $a > 0$ . (F) Posterior model solution from sampling.

756 up with positive value (Fig. S5F). We can compare these two distributions by taking a slice in  
 757 time (Fig. S5D). We can now move on to the accumulation model, which has the same basic  
 758 random walk structure, but with a few more bells and whistles.

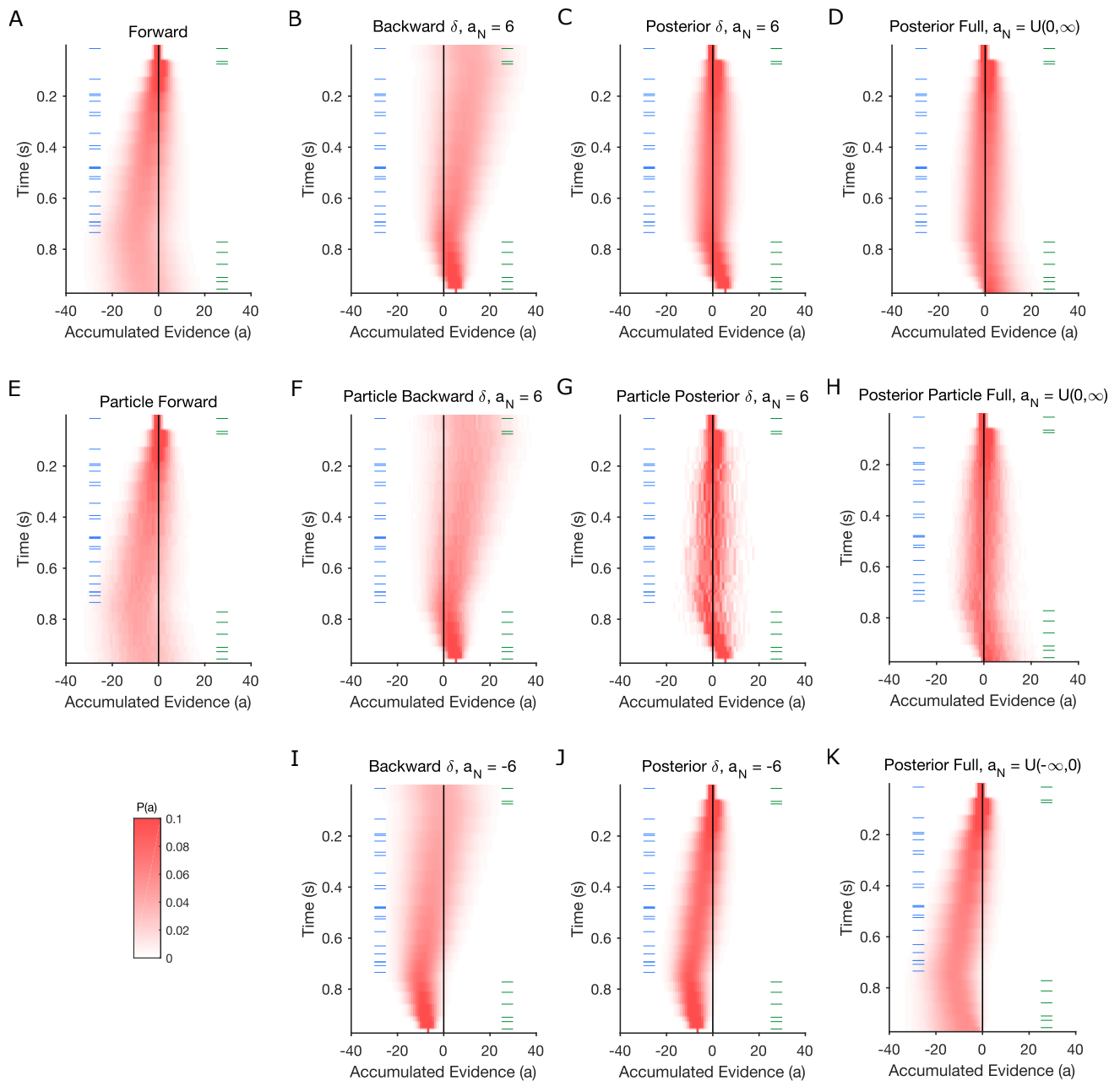
### 759 Verification of Accumulation Model Posterior Solution

760 To illustrate the posterior model solution on the full accumulation model we can again compare  
 761 the analytical solution to sampled trajectories, this time for an example trial. Each trajectory  
 762 has a unique noise realization.

763 For both the model and sample trajectories, we have four distributions (Fig. S6). The forward  
 764 distribution showing the predicted trajectories given the initial conditions. The backwards-delta  
 765 distribution showing the possible trajectories that result in a single final accumulation value. The  
 766 posterior-delta distribution showing the possible trajectories that start at the initial condition  
 767 and result in a single final accumulation value. Finally, the full posterior distribution that shows  
 768 the possible accumulation values that start at the initial condition and result in the appropriate  
 769 choice or sign of the accumulation value. In this example trial we consider a trial where there is  
 770 change in state at 750ms, and we evaluate a left choice ( $a < B$ ) at  $t = 1$ s. The entire solution was  
 771 computed using 51 backward-delta solutions on a grid from  $(-50, B)$  with  $\Delta i = 1$ . We can also  
 772 examine slices through the posterior distribution at various time points to confirm agreement  
 773 between the trajectories and the analytical solution (Fig. S7).

774 A few notes on the advantages of the analytical solution. First, the analytical model offers

775 a large increase in accuracy of the model over previous numerical approaches. Second, the  
776 analytical model is much faster to fit and evaluate. Second, we can compute the posterior  
777 distribution for only a subset of all time points without computing the solution for all time  
778 points. This fact allows for very rapid computation of the posterior distribution.



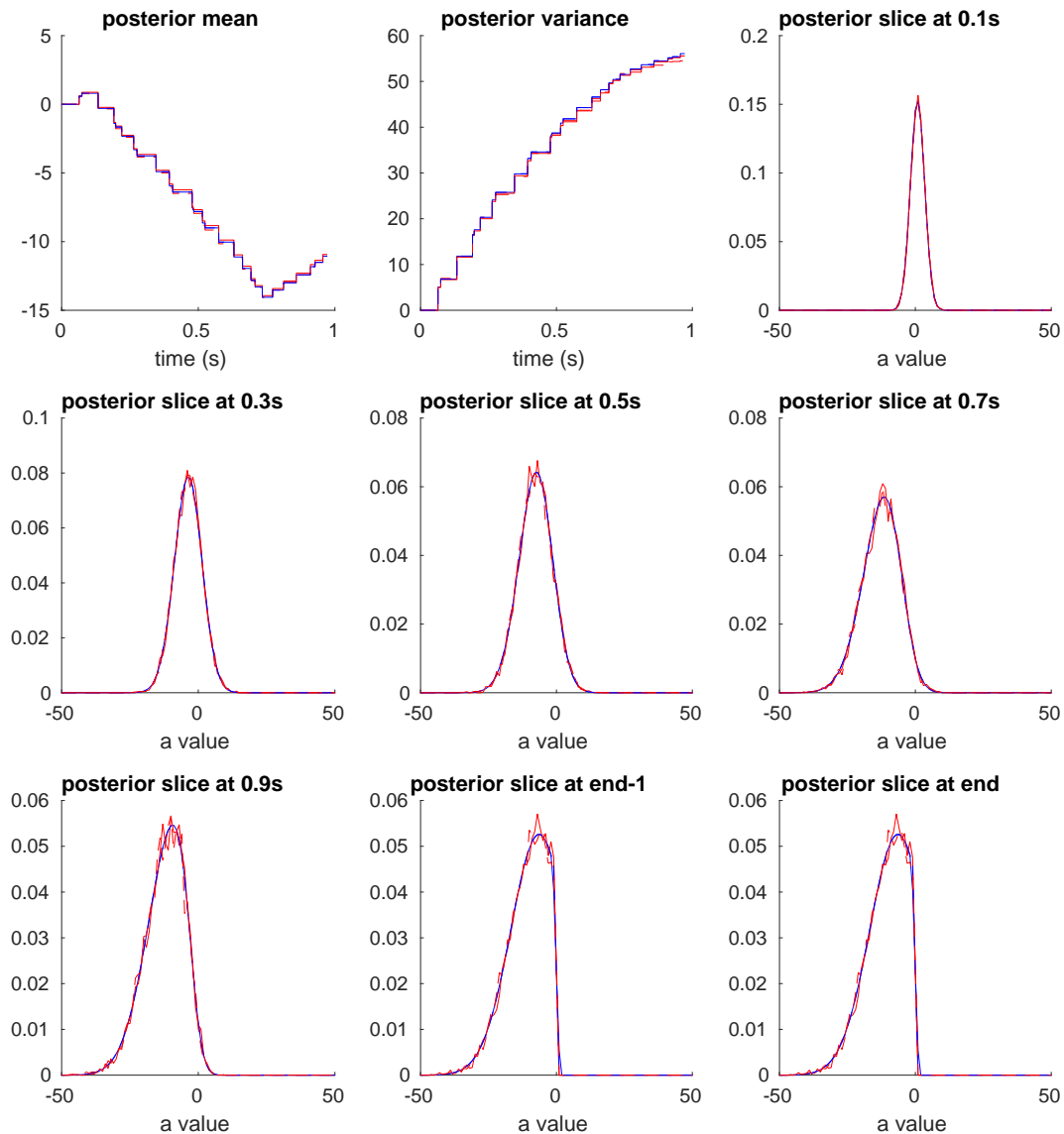


Figure S7: **Posterior Model Validation.** Comparison of the posterior model distribution computed from the analytical solution (blue) and sampled trajectories (red) for an example trial. This figure shows the same distribution as Fig. S6K (Red). (A) Mean of posterior over time. (B) Variance of posterior over time. (C-I) Posterior comparisons at specific time points.



## 779 Supplementary Materials - Switch Triggered Averages

780 To determine the times of changes of mind, we used the mean of the posterior distribution  
781  $P(a|t, \delta_R, \delta_L, \theta, y)$  on each trial. These trajectories have several sources of noise that complicated  
782 our analysis. First, they have sharp discontinuities at the times of each stimulus click. This  
783 sometimes resulted in the mean trajectory repeatedly crossing the decision boundary in a short  
784 period of time. It is unlikely that each of these crossings represented a true change of mind,  
785 since the subject was likely in a general state of indecision (Fig S8A). To resolve this issue we  
786 smoothed the mean trajectories with a 100 ms running average. This smoothing resolved the  
787 flickering changes of mind from individual stimuli (Fig S8B). However, this reveals a second issue.  
788 There are changes of mind that briefly cross the decision boundary, or still oscillate around the  
789 decision boundary. To detect and remove these time points we estimated the local slope of the  
790 smoothed trajectory, and filtered out changes of mind where the local slope had an inconsistent  
791 sign with the direction of the change of mind (Fig S8C). We excluded any changes of mind that  
792 occurred in the first or last 200ms of the trial.

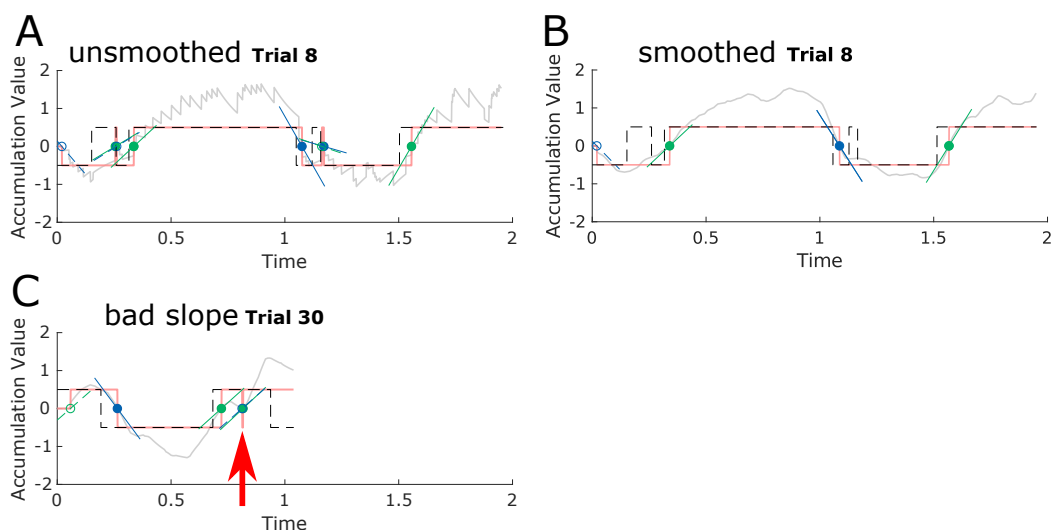


Figure S8: **Analysis of smoothing model mean and weak and strong state switches** (A) The mean trajectory of the posterior model is shown in gray, along with the categorical decision taken from the sign of the model accumulation value in pink. The generative state (dashed black traces) is overlaid on the model state indicator. Each change of mind is marked with a circle. Changes of mind to the ‘go right’ state are in green, and changes of mind to the ‘go left’ state in blue. The local slope is marked with a colored line. The initial decision at the start of the trial was excluded (empty circle). (B) The mean trajectory was smoothed, removing many spurious changes of mind when the trajectory flickered near the decision boundary. (C) In this example trial around 750ms, the mean trajectory is generally moving to the ‘go right’ (positive accumulation value) state, but briefly returns to the go-left state. This creates a situation where there are multiple changes of mind at the same time, with an incongruent slope (red arrow).