

LinearTurboFold: Fast Folding and Alignment for RNA Homologs with Applications to Coronavirus

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As the COVID-19 outbreak spreads, there is a growing need for an efficient tool to identify conserved RNA structures as critical targets for diagnostics and therapeutics. To address this need, we present LinearTurboFold, an algorithm that scales *linearly* with sequence length, to predict conserved structures for a set of unaligned RNA homologs. LinearTurboFold uses the same iterative refinement of structures and alignments as TurboFold, but is substantially faster than previous methods and can fold full-length coronavirus genomes without constraints on base-pairing distance. It also significantly improves structure prediction accuracy and achieves comparable alignment accuracy. On SARS-CoV-2 genomes, LinearTurboFold identifies not only conserved structures but also accessible and conserved regions as potential targets for designing efficient small-molecule drugs, antisense oligonucleotides, siRNAs, CRISPR-Cas13 gRNAs and RT-PCR primers.

1 Introduction

2 RNAs play important roles in many cellular processes (1,2). To maintain their functions, secondary
3 structures of RNA homologs are conserved across evolution (3,4,5). These conserved structures
4 provide critical targets for diagnostics and treatments. Thus, there is a need for developing fast and
5 accurate computational methods to identify structurally conserved regions.

6 Commonly, conserved structures involve compensatory base pair changes, where two positions
7 in primary sequences mutate across evolution and still conserve a base pair, for instance, an AU or a
8 CG pair replaces a GC pair in homologous sequences. These compensatory changes provide strong
9 evidence for evolutionarily conserved structures (6,7,8,9,10). Meanwhile, they make it harder to align
10 sequences when structures are unknown. To solve this issue, Sankoff proposed a dynamic algorithm
11 that simultaneously predicts structures and a structural alignment for two or more sequences (11).
12 The major limitation of this approach is that the algorithm runs in $O(n^{3k})$ against k sequences with
13 the average sequence length n . Several software packages provide implementations of the Sankoff
14 algorithm (12,13,14,15,16,17) that use simplifications to reduce runtime.¹

15 As an alternative, TurboFold II (18), an extension of TurboFold (19), provides a more computa-
16 tionally efficient method to align and fold sequences. Taking multiple unaligned sequences as input,
17 TurboFold II iteratively refines alignments and structure predictions so that they conform more closely
18 to each other and converge on conserved structures. TurboFold II is significantly more accurate than
19 other methods (12,14,20,21,22) when tested on RNA families with known structures and alignments.

20 However, the cubic runtime and quadratic memory usage of TurboFold II prevent it from scaling
21 to longer sequences such as full-length SARS-CoV-2 genomes which contain $\sim 30,000$ nucleotides;
22 in fact, no joint-align-and-fold methods can scale to these genomes which are the longest among
23 RNA viruses. As a (not very principled) workaround, most existing efforts for modeling SARS-
24 CoV-2 structures (23,24,25,26,27,28) resort to local folding methods (29,30) with sliding windows
25 plus a limited pairing distance, abandoning all non-local interactions, and only consider one SARS-

¹Besides these joint-fold-and-align algorithms, there exist two alternative approaches to homologous folding: *align-then-fold* and *fold-then-align*; see Fig. S1 for details.

26 CoV-2 genome (Fig. 1B–C), ignoring homology signals. To address this challenge, we design a
27 linearized version of TurboFold II, *LinearTurboFold* (Fig. 1A), which is a global homologous folding
28 algorithm that scale linearly with sequence length. This linear runtime makes it the first joint-fold-
29 and-align algorithm to scale to full-length coronavirus genomes without any constraints on window
30 size or pairing distance, taking about 13 hours to analyze a group of 25 SARS-CoV homologs. It also
31 leads to significant improvement on secondary structure prediction accuracy as well as an alignment
32 accuracy comparable to or higher than all benchmarks.

33 Over a group of 25 SARS-CoV-2 and SARS-related homologous genomes, *LinearTurboFold* pre-
34 dictions are close to the canonical structures (31) and structures modeled with the aid of experimental
35 data (24, 25, 26) for several well-studied regions. Thanks to global rather than local folding, *Lin-*
36 *earTurboFold* discovers a long-range interaction involving 5' and 3' UTRs (~29,800 *nt* apart), which
37 is consistent with recent purely experimental work (27), and yet is out of reach for local folding meth-
38 ods used by existing studies (Fig. 1B–C). In short, our *in silico* method of folding multiple homologs
39 can achieve results similar to, and sometimes more accurate than, experimentally-guided models for
40 one genome. Moreover, *LinearTurboFold* identifies conserved structures supported by compensatory
41 mutations, which are potential targets for small molecule drugs (32) and antisense oligonucleotides
42 (ASOs) (28). We further identify regions that are (a) sequence-level conserved, (b) at least 15 *nt* long,
43 and (c) accessible (i.e., likely to be completely unpaired) as potential targets for ASOs (33), small
44 interfering RNA (siRNA) (34), CRISPR-Cas13 guide RNA (gRNA) (35) and reverse transcription
45 polymerase chain reaction (RT-PCR) primers (36).

46 *LinearTurboFold* is a general technique that can also be applied to other RNA viruses (e.g., in-
47 fluenza, Ebola, HIV, Zika, etc.) for full-length genome studies.

48 Results

49 The framework of *LinearTurboFold* has two major aspects (Fig. 1A): linearized structure-aware pair-
50 wise alignment estimation (module 1); and linearized homolog-aware structure prediction (module 2).
51 *LinearTurboFold* iteratively refines alignments and structure predictions, specifically, updating pair-

52 wise alignment probabilities by incorporating predicted base-pairing probabilities (from module 2) to
53 form structural alignments, and modifying base-pairing probabilities for each sequence by integrat-
54 ing the structural information from homologous sequences via the estimated alignment probabilities
55 (from module 1) to detect conserved structures. After several iterations, LinearTurboFold generates
56 the final multiple sequence alignment (MSA) based on the latest pairwise alignment probabilities
57 (module 3) and predicts secondary structures using the latest pairing probabilities (module 4).

58 LinearTurboFold achieves linear time regarding sequence length with two major linearized mod-
59 ules: our recent work LinearPartition (37) (Fig. 1A module 2), which approximates the RNA partition
60 function (38) and base pairing probabilities in linear time, and a novel algorithm LinearAlignment
61 (module 1). LinearAlignment aligns two sequences by Hidden Markov Model (HMM) in linear
62 time by applying the same beam search heuristic (39) used by LinearPartition. Finally, LinearTur-
63 boFold assembles the secondary structure from the final base pairing probabilities using an accurate
64 and linear-time method named ThreshKnot (40) (module 4). LinearTurboFold also integrates a linear-
65 time stochastic sampling algorithm named LinearSampling (41) (module 5), which can independently
66 sample structures according to the homolog-aware partition functions and then calculate the probabil-
67 ity of being unpaired for regions, which is an important property in siRNA sequence design (34). So
68 overall, the end-to-end runtime of LinearTurboFold scales linearly with sequence length (see **Meth-**
69 **ods** for more details).

70 Scalability and Accuracy

71 To evaluate the efficiency of LinearTurboFold against the sequence length, we collected a dataset con-
72 sisting of seven families of RNAs with sequence length ranging from 210 *nt* to 30,000 *nt*, including
73 five families from the RNAstralign dataset plus 23S ribosomal RNA, HIV and SARS-CoV genomes,
74 and each family has five homologous sequences (see **Methods** for more details). Fig. 2A compares the
75 running times of LinearTurboFold with TurboFold II and two Sankoff-style simultaneous folding and
76 alignment algorithms, LocARNA and MXSCARNA. Clearly, LinearTurboFold scales linearly with
77 sequence length n , and is substantially faster than other benchmarks which scale superlinearly. The

78 linearization in LinearTurboFold brought orders of magnitude speedup over the cubic-time TurboFold
79 II, taking only 12 minutes on the HIV family (average length 9,686 *nt*) while TurboFold II takes 3.1
80 days ($372\times$ speedup). More importantly, LinearTurboFold takes only 40 minutes on five SARS-CoV
81 sequences while all other benchmarks fail to scale. Regarding the memory usage (Fig. 2B), Lin-
82 earTurboFold costs linear memory space with sequence length, while other benchmarks use quadratic
83 or more memory. In Fig. 2C–D, we also demonstrate that the runtime and memory usage against the
84 number of homologs ($k = 5 \sim 20$), using homologs of 16S rRNAs about 1,500 *nt* in length. The
85 apparent complexity against the group size of LinearTurboFold is higher than TurboFold II because
86 the cubic-time partition function calculation, which dominates the runtime of TurboFold II, has been
87 linearized in LinearTurboFold by LinearPartition (Fig. S5C).

88 We next compare the accuracies of predicted secondary structures and MSAs between LinearTur-
89 boFold and several benchmark methods. Besides Sankoff-style LocARNA and MXSCARNA, we
90 also consider three types of negative controls: (a) single sequence folding (partition function-based):
91 Vienna RNAfold (30) (-p mode) and LinearPartition; (b) sequence-only alignment: MAFFT (21) and
92 LinearAlignment (a standalone version without structural information); and (c) an align-then-fold
93 method that predicts consensus structures from MSAs (Fig. S1): MAFFT + RNAalifold (20).

94 For secondary structure prediction, LinearTurboFold, TurboFold II and LocARNA achieve higher
95 F1 scores than single sequence folding methods (Vienna RNAfold and LinearPartition) (Fig. 2E),
96 which demonstrates folding with homology information performs better than folding sequences sepa-
97 rately. Overall, LinearTurboFold performs significantly better than all the other benchmarks on struc-
98 ture prediction. For the accuracy of MSAs (Fig. 2F), the structural alignments from LinearTurboFold
99 obtain higher accuracies than sequence-only alignments (LinearAlignment and MAFFT) on all four
100 families, especially for families with low sequence identity. On average, LinearTurboFold performs
101 comparably with TurboFold II and significantly better than other benchmarks on alignments. We also
102 note that the structure prediction accuracy of the align-then-fold approach (MAFFT + RNAalifold)
103 depends heavily on the alignment accuracy, and is the worst when the sequence identity is low (e.g.,
104 SRP RNA) and the best when the sequence identity is high (e.g., 16S rRNA) (Fig. 2E–F).

105 **Highly Conserved Structures in SARS-CoV-2 and SARS-related Betacoron-** 106 **aviruses**

107 RNA sequences with conserved secondary structures play vital biological roles and provide potential
108 targets. The current COVID-19 outbreak raises an emergent requirement of identifying potential
109 targets for diagnostics and therapeutics. Given the strong scalability and high accuracy, we used
110 LinearTurboFold on a group of full-length SARS-CoV-2 and SARS-related (SARSr) genomes to
111 obtain global structures and identify highly conserved structural regions.

112 We used a greedy algorithm to select the 16 most diverse genomes from all the valid SARS-CoV-
113 2 genomes submitted to the Global Initiative on Sharing Avian Influenza Data (GISAID) (42) up to
114 December 2020 (Methods). We further extended the group by adding 9 SARS-related homologous
115 genomes (5 human SARS-CoV-1 and 4 bat coronaviruses). In total, we built a dataset of 25 full-
116 length genomes consisting of 16 SARS-CoV-2 and 9 SARS-related sequences (Tab. S2). The average
117 pairwise sequence identities of the 16 SARS-CoV-2 and the total 25 genomes are 99.9% and 89.6%,
118 respectively. LinearTurboFold takes about 13 hours and 43 GB on the 25 genomes.

119 To evaluate the reliability of LinearTurboFold predictions, we first compare them with the Hus-
120 ton *et al.*'s SHAPE-guided models (24) for regions with well-characterized structures across betacoro-
121 naviruses. For the extended 5' and 3' untranslated regions (UTRs), LinearTurboFold's predictions are
122 close to the SHAPE-guided structures (Fig. 3A–B), i.e., both identify the stem-loops (SLs) 1–2 and
123 4–7 in the extended 5' UTR, and the bulged stem-loop (BSL), SL1, and a long bulge stem for the
124 hypervariable region (HVR) including the stem-loop II-like motif (S2M) in the 3' UTR. Interestingly,
125 in our model, the high unpaired probability of the stem in the SL4b indicates the possibility of being
126 single-stranded as an alternative structure, which is supported by experimental studies (28, 25). In
127 addition, the compensatory mutations LinearTurboFold found in UTRs strongly support the evolu-
128 tionary conservation of structures (Fig. 3A).

129 The most important difference between LinearTurboFold's prediction and Huston *et al.*'s experimentally-
130 guided model is that LinearTurboFold discovers an end-to-end interaction (29.8 kilobases apart) be-
131 tween the 5' UTR (SL3, 60-82 nt) and the 3' UTR (final region, 29845-29868 nt), which fold locally

132 by themselves in Huston *et al.*'s model. Interestingly, this 5'-3' interaction matches *exactly* with the
133 one discovered by the purely experimental work of Ziv *et al.* (43) using the COMRADES technique
134 to capture long-range base-pairing interactions (Fig. 3C). These end-to-end interactions have been
135 well established by theoretical and experimental studies (44, 45, 46) to be common in natural RNAs,
136 but are far beyond the reaches of local folding methods used in existing studies on SARS-CoV-2 sec-
137 ondary structures (24, 25, 26, 27). By contrast, LinearTurboFold predicts secondary structures globally
138 without any limit on window size or base-pairing distance, enabling it to discover long-distance in-
139 teractions across the whole genome. The similarity between our predictions and the experimental
140 work shows that our *in silico* method of folding multiple homologs can achieve results similar to,
141 if not more accurate than, those experimentally-guided single-genome prediction. We also observed
142 that LinearPartition, as a single sequence folding method, can also predict a long-range interaction
143 between 5' and 3' UTRs, but it involves SL2 instead of SL3 of the 5' UTR (Fig. 3A), which indi-
144 cates that the homologous information assists to adjust the positions of base pairs to be conserved in
145 LinearTurboFold. Additionally, the align-then-fold approach (MAFFT + RNAalifold) fails to predict
146 such long-range interactions (Fig. S6B).

147 The frameshifting stimulation element (FSE) is another well-characterized region. For an ex-
148 tended FSE region, the LinearTurboFold prediction consists of two substructures (Fig. 4A): the 5'
149 part includes an attenuator hairpin and a stem, which are connected by a long internal loop (16 nt)
150 including the slippery site, and the 3' part includes three stem loops. We observe that our predicted
151 structure of the 5' part is consistent with experimentally-guided models (24, 25, 27) (Fig. 4B-D). In
152 the attenuator hairpin, the small internal loop motif (UU) was previously selected as a small molecule
153 binder which stabilizes the folded state of the attenuator hairpin and impairs frameshifting (32). For
154 the long internal loop including the slippery site, we will show in the next section that it is both highly
155 accessible and conserved (Fig. 5), which makes it a perfect candidate for drug design. For the 3'
156 gion of the FSE, LinearTurboFold successfully predicts stems 1-2 (but misses stem 3) of the canonical
157 three-stem pseudoknot (31) (Fig. 4E). Our prediction is closer to the canonical structure compared to
158 the experimentally-guided models (24, 25, 27) (Fig. 4B-D); one such model (Fig. 4B) identified the

159 pseudoknot (stem 3) but with an open stem 2. Note that all these experimentally-guided models for
160 the FSE region were estimated for specific local regions. As a result, the models are sensitive to the
161 context and region boundaries (27, 24, 47) (see Fig. S7D–F for alternative structures of Fig. 4B–D
162 with different regions). LinearTurboFold, by contrast, does not suffer from this problem by virtue of
163 global folding without local windows. Besides SARS-CoV-2, we notice that the estimated structure
164 of the SARS-CoV-1 reference sequence (Fig. 4F) from LinearTurboFold is similar to SARS-CoV-2
165 (Fig. 4A), which is consistent with the observation that the structure of the FSE region is highly con-
166 served among betacoronaviruses (31). Finally, as negative controls, both the single sequence folding
167 algorithm (LinearPartition in Fig. 4G) and the align-then-fold method (RNAalifold in Fig. S7G) pre-
168 dict quite different structures compared with the LinearTurboFold prediction (Fig. 4A) (39%/61% of
169 pairs from the LinearTurboFold model are not found by LinearPartition/RNAalifold, respectively).

170 In addition to the well-studied UTRs and FSE regions, LinearTurboFold discovers 50 conserved
171 structures with identical structures among 25 genomes, and 26 regions are novel compared to pre-
172 vious studies (23, 24) (Fig. 4H and Tab. S4), which might be potential targets for small-molecule
173 drugs (32) and antisense oligonucleotides (28, 48). LinearTurboFold also recovers fully conserved
174 base pairs with compensatory mutations (Tab. S3), which imply highly conserved structural regions
175 whose functions might not have been explored.

176 **Highly Accessible and Conserved Regions in SARS-CoV-2 and SARS-related** 177 **Betacoronaviruses**

178 Studies show that the siRNA silencing efficiency, ASOs inhibitory efficacy, CRISPR-Cas13 knock-
179 down efficiency and RT-PCR testing efficiency all correlate with the target *accessibility* (34, 35, 36, 49),
180 which is the probability of a target site being fully unpaired. To get unstructured regions, Ran-
181 gan *et al.* (23) imposed a threshold on unpaired probabilities of each position, which is not a truly cor-
182 rect method because the unpaired probabilities are dependent. By contrast, the widely-used stochastic
183 sampling algorithm (50, 41) builds a representative ensemble of structures by sampling independent
184 secondary structures according to their probabilities in the Boltzmann distribution. Thus the acces-

185 sibility for a region can be approximated as the fraction of sampled structures in which the region
186 is single-stranded. LinearTurboFold utilized LinearSampling (41) to generate 10,000 independent
187 structures for each genome according to the modified partition functions after the iterative refinement
188 (Fig. 1A module 5), and calculated accessibilities for regions at least 15 nt long. We then identify
189 *accessible regions* with at least 0.5 accessibility among all 16 SARS-CoV-2 genomes (Fig. 5A–B).

190 In addition to accessibility, sequence conservation is another critical aspect for efficient therapeutic
191 and diagnostic target sites. We further identify *accessible and conserved regions* that are not only
192 structurally accessible among SARS-CoV-2 genomes, but also fully conserved among SARS-CoV-2
193 genomes with at most one mutation at each position across SARS-related genomes (Fig. 5C). These
194 regions are less likely to accumulate mutations in the future. Finally, we identified 35 accessible and
195 conserved regions (Fig. 5G and Tab. S5). Because the nucleotide content and specificity are also key
196 factors influencing siRNA efficient (57), we searched BLAST against the human mRNA dataset for
197 these regions and calculated the GC content (Tab. S5). Among these regions, region 16 corresponds
198 to the internal loop containing the slippery site in the extended FSE region, and it is conserved at
199 both structural and sequence levels (Fig. 5D and 5H). Region 29 in the ORF3a gene is fully con-
200 served among all the 25 genomes with average accessibility 0.936 (Fig. 5D). Besides SARS-CoV-2
201 genomes, the SARS-related genomes such as the SARS-CoV-1 reference sequence (NC_004718.3)
202 and a bat coronavirus (BCoV, MG772934.1) also form similar structures around the slippery site
203 (Fig. 5A). To investigate if the the mutations are sensitive to the sampled SARS-CoV-2 genomes, we
204 further checked the conservation of these regions among a dataset including 257,672 valid genomes
205 submitted to GISAID up to December 2020, and most of these regions are still highly conserved²
206 (Tab. S5). The mutations of new lineages of SARS-CoV-2 in South African, Brazil³ and India⁴ are
207 outside of these predicted regions, which implies that the sequence conservation constraint imposed
208 on SARS-related genomes is helpful in selecting evolutionarily conserved regions.

209 We also designed a negative control by analyzing the SARS-CoV-2 reference sequence alone,

²the fraction of valid genomes in which the whole region is identical.

³<https://www.cdc.gov/coronavirus/2019-ncov/more/science-and-research/scientific-brief-emerging-variants.html>

⁴<https://www.cdc.gov/coronavirus/2019-ncov/variants/variant-info.html>

210 which can also obtain some accessible regions. However, these regions are not structurally conserved
211 among the other 15 SARS-CoV-2 genomes, resulting in vastly different accessibilities, except for one
212 region in the M gene (Tab. S6). The reason behind this is that even with a high sequence identity (over
213 99.9%), single sequence folding algorithms still predict greatly dissimilar structures for the SARS-
214 CoV-2 genomes (Fig. 5E–F). Both regions (in nsp11 and N genes) are fully conserved among the
215 16 SARS-CoV-2 genomes, yet they still fold into vastly different structures due to mutations outside
216 the regions; as a result, the accessibilities are either low (nsp11) or in a wide range (N) (Fig. 5D).
217 Conversely, addressing this by folding each sequence with proclivity of base pairing inferred from all
218 homologous sequences, LinearTurboFold structure predictions are more consistent with each other
219 and thus can detect conserved structures (Fig. 5A–B).

220 Summary

221 We have presented LinearTurboFold, an end-to-end linear-time algorithm for structural alignment and
222 conserved structure prediction of RNA homologs, which is the first joint-fold-and-align algorithm to
223 scale to full-length SARS-CoV-2 genomes without imposing any constraints on base-pairing distance.
224 We also demonstrate that LinearTurboFold leads to significant improvement on secondary structure
225 prediction accuracy as well as an alignment accuracy comparable to or higher than all benchmarks.

226 Unlike existing work using local folding workarounds, LinearTurboFold enables unprecedented
227 global structural analysis on the SARS-CoV-2 genomes; in particular, it can capture long-range in-
228 teractions, especially the one between 5' and 3' UTRs across the whole genome, which matches
229 perfectly with a recent purely experiment work. Over a group of 25 SARS-CoV-2 and SARS-related
230 homologs, LinearTurboFold identifies not only conserved structures supported by compensatory mu-
231 tations and experimental studies, but also accessible and conserved regions as vital targets for design-
232 ing efficient small-molecule drugs, siRNAs, ASOs, CRISPR-Cas13 gRNAs and RT-PCR primers.
233 LinearTurboFold is widely applicable to the analysis of other RNA viruses (influenza, Ebola, HIV,
234 Zika, etc.) and full-length genome analysis.

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Acknowledgments

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Figures

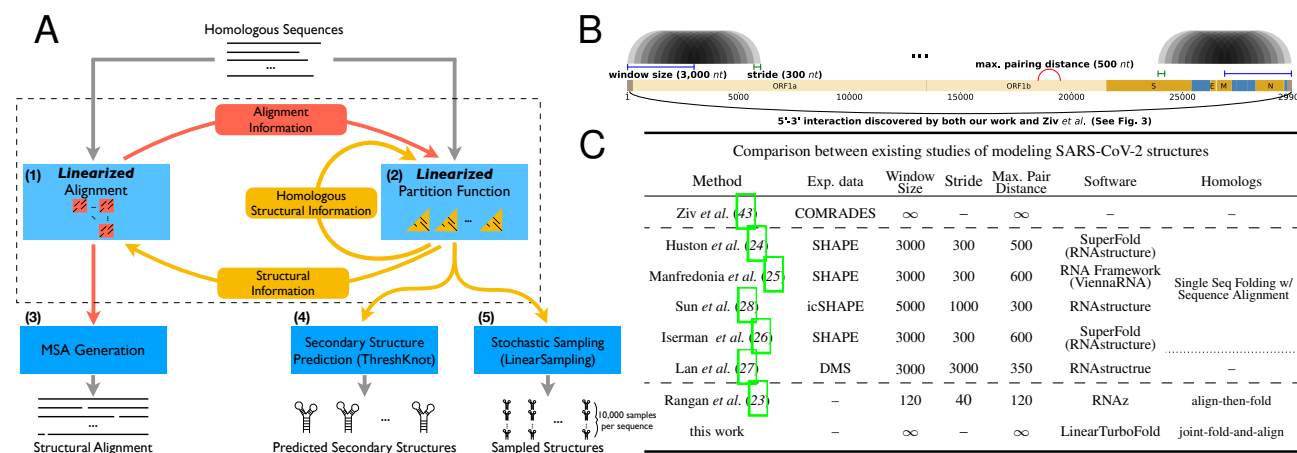


Figure 1: **A**: The LinearTurboFold framework. Like TurboFold, LinearTurboFold also takes multiple unaligned homologous sequences as input and then outputs a multiple sequence alignment and structures for each sequence, but unlike TurboFold, it employs two linearizations to ensure linear run-time: a *linearized* alignment computation (module 1) to predict posterior co-occurrence probabilities (red squares) for all pairs of sequences and a *linearized* partition function computation (module 2) to estimate base-pairing probabilities (yellow triangles) for all the sequences. These two modules take advantage of information from each other and iteratively refine predictions (see Fig. S2 for details). After several iterations, module 3 generates the final multiple sequence alignments, and module 4 predicts secondary structures. Module 5 is an optional output to stochastically sample structures. **B-C**: Most prior studies (expect for a purely experimental work by Ziv *et al.*) used local folding methods with limited window size and maximum pairing distance. **B** shows the local folding of the SARS-CoV-2 genome by Huston *et al.* Some work also used homologous sequences to identify conserved structures, but they only predicted structures for one genome and utilized sequence alignments to extract mutations. By contrast, LinearTurboFold is a global folding method without any limitations on sequence length or pairing distance, and it jointly folds and aligns homologs to obtain conserved structures. Consequently, LinearTurboFold can capture long-range interactions even across the whole genome (the long arc in **B**, Fig. 3).

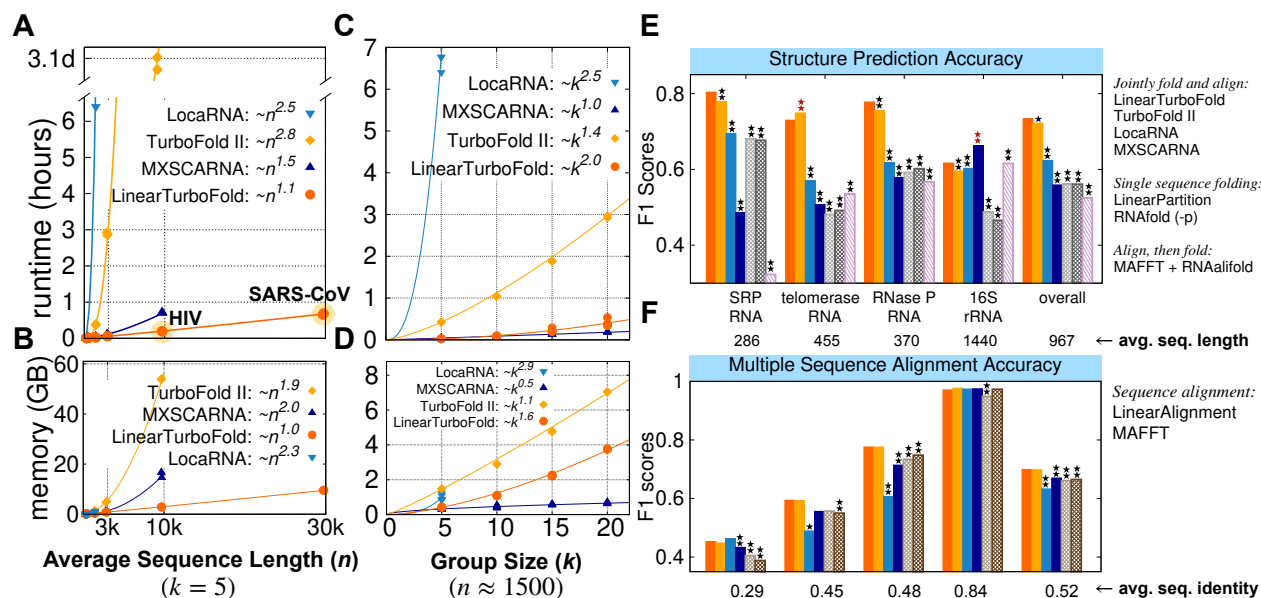


Figure 2: End-to-end Scalability and Accuracy Comparisons. **A–B**: End-to-end runtime and memory usage comparisons between benchmarks and LinearTurboFold against the sequence length. **C–D**: End-to-end runtime and memory usage comparisons against the group size. LinearTurboFold is the first joint-fold-and-align algorithm to scale to full-length coronavirus genomes (~30,000 nt) due to linear runtime. **E–F**: The F1 accuracy scores of the structure prediction and multiple sequence alignment (see Tab. [S1](#) for more details). LocARNA and MXSCARNA are Sankoff-style simultaneous folding and alignment algorithms for homologous sequences. As negative controls, LinearPartition and Vienna RNAfold predicted structures for each sequence separately; LinearAlignment and MAFFT generated sequence-level alignments; RNAalifold folded pre-aligned sequences (e.g., from MAFFT) and predicted conserved structures. Statistical significances (two-tailed permutation test) between the benchmarks and LinearTurboFold are marked with one star (★) if $p < 0.05$ or two stars (✳) if $p < 0.01$. The benchmarks whose accuracies are significantly lower than LinearTurboFold are annotated with black stars, while benchmarks higher than LinearTurboFold are marked with dark red stars. Overall, on structure prediction, LinearTurboFold achieves significantly higher accuracy than all evaluated benchmarks, and on multiple sequence alignment, it achieves accuracies comparable to TurboFold II and significantly higher than other methods (See Tab. [S1](#) for detailed accuracies).

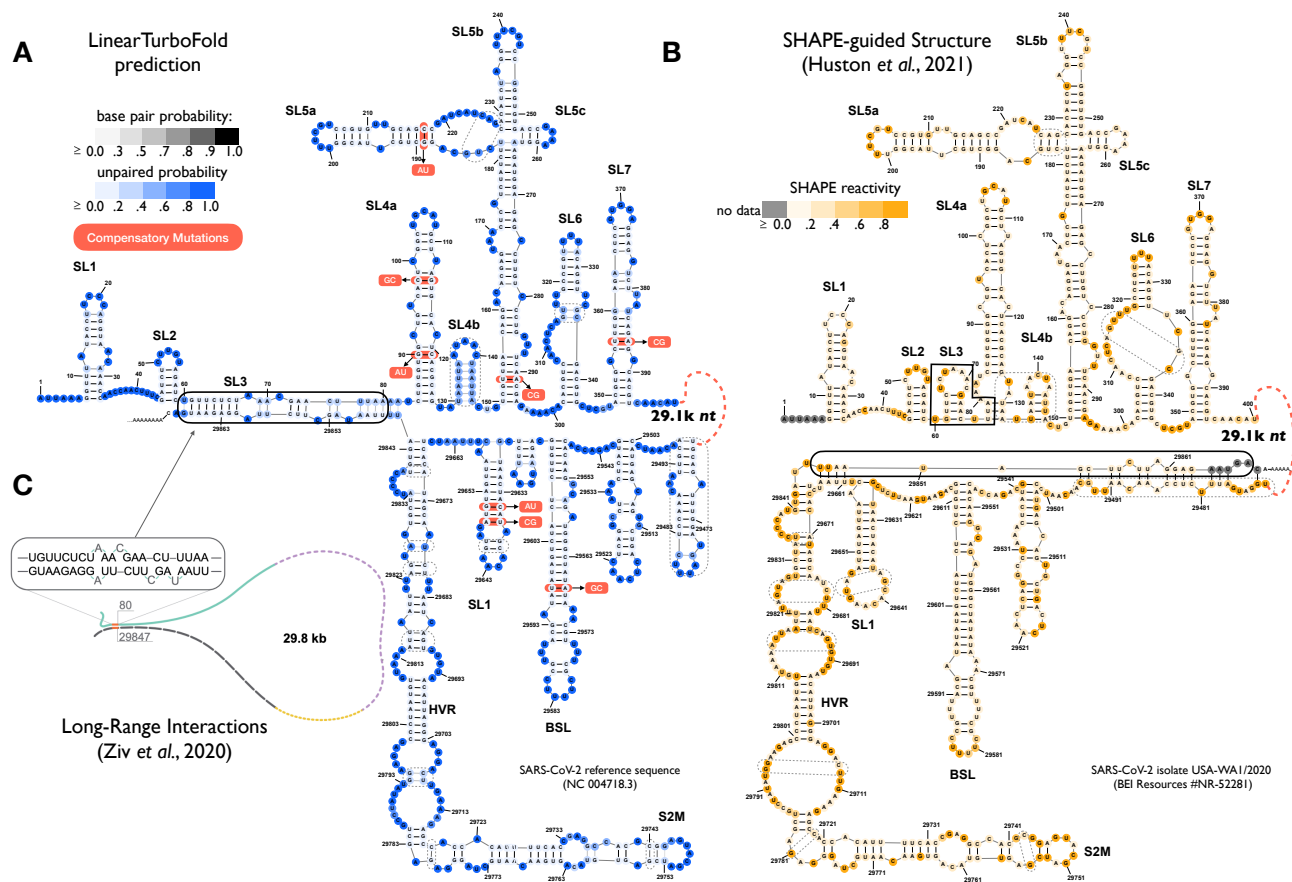


Figure 3: Secondary structures predictions of SARS-CoV-2 extended 5' and 3' UTRs. **A**: LinearTurboFold prediction. The nucleotides and base pairs are colored by unpaired probabilities and base-pairing probabilities, respectively. The compensatory mutations extracted by LinearTurboFold are annotated with alternative pairs in red boxes (see Tab. S3 for more fully conserved pairs with co-variational changes). **B**: SHAPE-guided model by Huston *et al.* (24) (window size 3000 nt sliding by 300 nt with maximum pairing distance 500 nt). The nucleotides are colored by SHAPE reactivities. Dash boxes circle the different structures between **A** and **B**. Our model is close to Huston *et al.*'s, but the major difference is that LinearTurboFold predicts the end-to-end pairs involving 5' and 3' UTRs (solid box in **A**), which is *exactly* the same interaction detected by Ziv *et al.* using the COMRADES experimental technique (43) (**C**). Such long-range interactions cannot be captured by the local folding methods used by prior experimentally-guided models (Fig. 1B). The similarity between models **A** and **B** as well as the exact agreement between **A** and **C** show that our *in silico* method of folding multiple homologs can achieve results similar to, if not more accurate than, experimentally-guided single-genome prediction. As negative controls (Fig. S6), the align-then-fold (RNAalifold) method cannot predict such long-range interactions. Although the single sequence folding algorithm (LinearParti-18 tion) predicts a long-range 5'-3' interaction, the positions are not the same as the LinearTurboFold

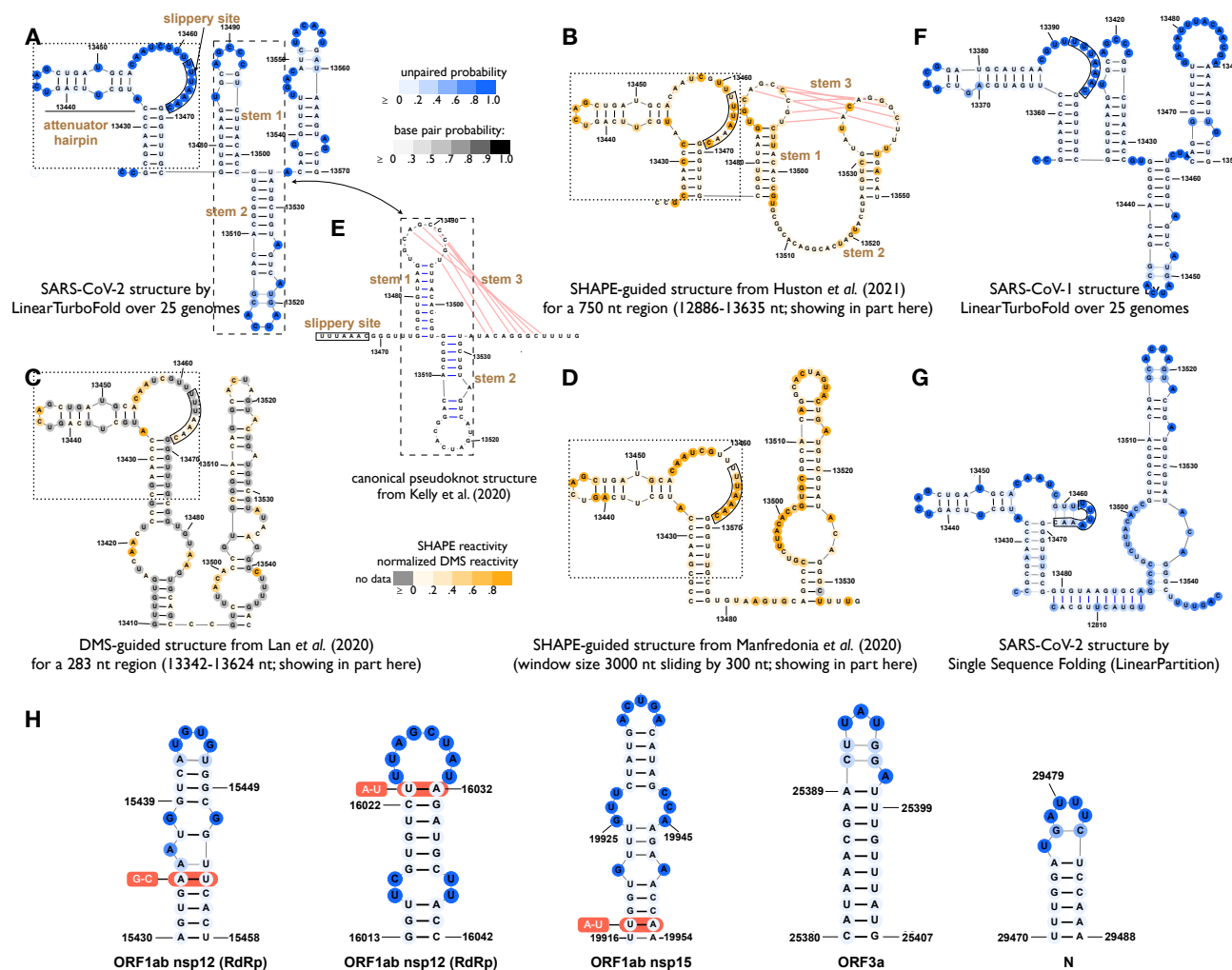


Figure 4: **A–D**: Secondary structure predictions of SARS-CoV-2 extended frameshifting stimulation element (FSE) region (13425–13545 nt). **A**: LinearTurboFold prediction. **B–D**: Experimentally-guided DMS-predictions from the literature (24, 27, 25), which are sensitive to the context and region boundaries due to the use of local folding methods (Fig. S7). **E**: The canonical pseudoknot structure by the comparative analysis between SARS-CoV-1 and SARS-CoV-2 genomes (31). For the 5' region of the FSE shown in dotted boxes (attenuator hairpin, internal loop with slippery site, and a stem), the LinearTurboFold prediction (A) is consistent with B–D; for the 3' region of the FSE shown in dashed boxes, our prediction (predicting stems 1–2 but missing 3) is closer to the canonical structure in E compared to B–D. **F**: LinearTurboFold prediction on SARS-CoV-1. **G**: Single sequence folding algorithm (LinearPartition) prediction on SARS-CoV-2, which is quite different from LinearTurboFold's. As another negative control, the align-then-fold method (RNAalifold) predicts a rather dissimilar structure (Fig. S7G). **H**: Five examples from 59 fully conserved structures among 25 genomes (see Tab. S4 for details), 26 of which are novel compared with prior work (23, 24).

