

1 **Motor learning in reaching tasks leads to homogenization**
2 **of task space error distribution**

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8 ***Abstract:***

9 A human arm, up to the wrist, is often modelled as a redundant 7 degree-of-freedom serial
10 robot. Despite its inherent nonlinearity, we can perform point-to-point reaching tasks
11 reasonably fast and with reasonable accuracy in the presence of external disturbances and
12 noise. In this work, we take a closer look at the task space error during point-to-point reaching
13 tasks and learning during an external force-field perturbation. From experiments and
14 quantitative data, we confirm a directional dependence of the peak task space error with certain
15 directions showing larger errors than others at the start of a force-field perturbation, and the
16 larger errors are reduced with repeated trials implying learning. The analysis of the
17 experimental data further shows that a) the distribution of the peak error is made more uniform
18 across directions with trials and the error magnitude and distribution approaches the value when
19 no perturbation is applied, b) the redundancy present in the human arm is used more in the
20 direction of the larger error, and c) homogenization of the error distribution is not seen when
21 the reaching task is performed with the non-dominant hand. The results support the hypothesis
22 that not only magnitude of task space error, but the directional dependence is reduced during
23 motor learning and the workspace is homogenized possibly to increase the control efficiency
24 and accuracy in point-to-point reaching tasks. The results also imply that redundancy in the
25 arm is used to homogenize the workspace, and additionally since the bio-mechanically similar
26 dominant and non-dominant arms show different behaviours, the homogenizing is actively
27 done in the central nervous system.

28 ***Significance:***

29 The human arm is capable of executing point-to-point reaching tasks reasonably accurately and
30 quickly everywhere in its workspace. This is despite the inherent nonlinearities in the
31 mechanics and the sensorimotor system. In this work, we show that motor learning enables
32 homogenization of the task space error thus overcoming the nonlinearities and leading to
33 simpler internal models and control of the arm movement. It is shown, across subjects, that the
34 redundancy present in the arm is used to homogenize the task space. It is further shown, across
35 subjects, that the homogenization is not an artifact of the biomechanics of the arm and is
36 actively performed in the central nervous system since homogenization is not seen in the non-
37 dominant hand.

38

39 ***Introduction:***

40 Movement kinematics during reaching movements are reasonably homogenous despite the
41 presence of inhomogeneous biomechanics. This suggests that internal models which mediate
42 sensorimotor transformations could play a role in mitigating such inhomegenities Click or tap
43 here to enter text. while planning and executing movements (Mussa-Ivaldi et al. 1985). One
44 such signature of such inhomegenities is the observeddirection dependence of error in
45 reaching tasks was done by (Gordon et al. 1994). who showd that the variability in the
46 direction errors was larger along the axis of movement of the forearm than in the perpendicular
47 direction, since the former requires greater forces to overcome the higher inertial load of the
48 relatively larger shoulder movement. The dependence of task space error is also well-known
49 in the robotics community -- in a serial robot with rotary joints, the position and orientation of
50 an end-effector (hand) can be related to the joint variables in terms of trigonometric functions.
51 As a consequence, the map between the hand velocities/error and the joint rates is nonlinear
52 and the Jacobian matrix for a serial robot is not constant (Salisbury and Craig 1982). One of
53 the consequence is that at any location in the workspace of a serial robot, the velocity
54 distribution is an ellipse for 2D motion (an ellipsoid for 3D motion) and at different points in
55 the workspace, the shape and size of the velocity ellipse (ellipsoid) will vary (Ghosal and Roth
56 1987; Salisbury and Craig 1982) .In the robotics community, the velocity at a point in the
57 workspace is often a proxy for the task space error as higher the velocity the less is the
58 positioning accuracy. Although not explicitly mentioned and investigated in (Singh et al. 2016),
59 a careful look at the results clearly show that the task space error is not the same in all directions
60 (Singh et al. 2016) In this work, we look at the direction dependence of task space error and
61 learning along the directions of reaching tasks performed by the human arm to show how
62 internal models learn to homogenise the output.To get a better understanding of the positioning
63 error in arm movement and its dependence with direction, in this work we take a re-look at the

64 task space error in reaching tasks when the hand is subjected to a force-field perturbation. It is
65 well-known that the magnitude of the task space error decreases and the central nervous system
66 learns to adapt to the external force-field. In this work we go beyond and ask questions such as
67 is there a directional dependence of error in reaching tasks when the hand motion is subjected
68 to a force-field perturbation? Does the task space error reduction go to the level of error when
69 there is no external force-field and finally what is the nature of the error distribution in the
70 beginning and at the end of learning? We also ask the question if the redundancy in the human
71 arm is used differently in the different directions and if the observed learning is actively
72 controlled by the central nervous system.

73 ***Materials and Methods:***

74 ***Subjects:***

75 Twenty-two subjects (aged 22 ± 6 years) participated in the study. The handedness of the
76 subjects was tested by modified Edinburgh Handedness Index (Salmaso and Longoni 1985).
77 None of the subjects had any neurological diseases or chronic medication issues. All the
78 subjects were paid for participation and gave informed consents in accordance with the
79 institutional ethics committee of the Indian Institute of Science, Bangalore.

80 ***Experimental setup:***

81 All the recordings were done in a dark room with the subjects sitting on chair with backrest
82 and their chins resting on a chin rest with head locked with head bars on both sides of their
83 temple as shown schematically in figure 1A. They looked down on a semi-transparent mirror
84 on which they saw the targets while they moved a robotic arm handle on a plane below the
85 plane of the mirror -- a standard approach used to study reaching tasks (Krakauer et al. 1999;
86 Shadmehr and Mussa-Ivaldi 1994; Shadmehr et al. 2010). The targets were presented by an
87 inverted monitor (refresh rate 60 Hz) above the mirror setup which gave the impression that
88 the targets appeared below the mirror while the hand could not be seen. All the experiments
89 were performed using TEMPO/VIDEOSYNC software (Reflecting Computing, St. Louis,
90 MO) that displayed visual stimuli, sampled and stored hand position with other behavioural
91 parameters in real time. The hand positions and joint angles was recorded with a spatial
92 resolution of 0.03 inches using an electromagnetic position and orientation tracking device at
93 240 Hz (LIBERTY; Polhemus, Colchester, VT).

94 ***Experimental paradigm:***

95 In all experiment, trials are divided into three phases -- baseline trials, perturbation trials and
96 washout trials. In the baseline portion, the robot arm was free. In the second perturbation trials,
97 the robot applied a force perpendicular to the hand trajectory (discussed in detail below) and in
98 the third phase, the force applied by the robot was switched off. All subjects performed ~30
99 practice trials before performing the actual experiment. The subjects performed about 400 trials
100 per session with a typical session lasting between 2 to 3 hours. Each trial started with the
101 presentation of a fixation box at the centre of screen. When the robotic end-effector was on the
102 fixation box, then the target box was displayed. The target box was displayed 15 cm away from
103 the fixation box in any one of the 8 directions. The subject moved the robotic end-effector to
104 the target box only after the fixation box disappeared. Till the time fixation box disappeared,
105 subject did not move their hand. Auditory feedback (beep sound) was given when the subject
106 performed correctly.

107 The top of figure 1 B shows the three phases of the experiment, and the bottom figure
108 1 C shows the measured first 5 hand trajectories in the three phases for a typical subject. Data
109 similar to figure 1 C was acquired for all trials and across 22 subjects. Figure 1 D shows the
110 error at the peak velocity along the trajectory in the three phases of the experiment for a typical
111 subject. The errors are colour coded -- the blue dots, for example, shows the error when the
112 hand moves along 0 degrees. For each subject, the arm was fitted with electromagnetic trackers
113 which were used to measure joint rotation angles at each instance. The trackers were used to
114 compute the four angles shown in figure 1 E. Additionally, the (x, y) location of the end-effector
115 of the robot was also recorded. Detailed analysis of the acquired data for the 22 subjects is
116 presented in the results section.

117 ***Force-field perturbation:***

118 During the second phase, the robot applied a lateral force. The lateral force depended on the
119 instantaneous hand velocity as in equation (1)

$$120 \begin{bmatrix} F_x \\ F_y \end{bmatrix} = \begin{bmatrix} 0 & -K \\ K & 0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} \quad -- (1)$$

121 where F_x, F_y are the force components exerted by the robotic arm, \dot{x}, \dot{y} correspond to the
122 velocity of hand and K denotes the force perturbation coefficient along the two directions. The
123 force-field disturbs the hand trajectory initially and with trials, the hand trajectory tends to
124 become straighter.

125 ***Visuo-motor perturbation:***

126 During the visuomotor perturbation, the cursor movement was rotated according to equation
127 (2),

$$128 \begin{bmatrix} P_x \\ P_y \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} p_x \\ p_y \end{bmatrix} \quad -- (2)$$

129 where P_x, P_y correspond to the position of the cursor, p_x, p_y correspond to the actual position
130 of the hand and θ (45) denotes the perturbation angle about the center of workspace. This
131 perturbation led to a trajectory error that was gradually compensated over the course of many
132 trials and the hand trajectory straightened. The end-point cursor describing the movement was
133 visible throughout the movement in all experiments.

134 ***Kinematic model of arm:***

135 A 2D forward kinematics model of the human arm is assumed to have four joint rotations --
136 clavicle protraction–retraction, shoulder horizontal abduction–adduction, elbow flexion–

137 extension and wrist medial–lateral -- denoted by $\theta_1, \theta_2, \theta_3$, and θ_4 respectively (see figure 1
138 E). The Cartesian location of the hand can be written as

$$139 \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} l_1 \cos \theta_1 + l_2 \cos \theta_2 + l_3 \cos \theta_3 + l_4 \cos \theta_4 \\ l_1 \sin \theta_1 + l_2 \sin \theta_2 + l_3 \sin \theta_3 + l_4 \sin \theta_4 \end{bmatrix}$$

140 where the four joint rotations are as described above. It may be noted that the angles $\theta_i, i = 1,$
141 2, 3, 4 are absolute angles and hence the forward kinematic equations above are different from
142 typical serial robot kinematic equations with relative angles. The link lengths $l_i, i=1, 2, 3, 4$ are
143 computed from the data from the sensors placed in the arm and vary a little with different
144 subjects. To ensure that the l_i values are valid, the (x, y) obtained from above equation is
145 compared with the (x, y) values of the robotic arm handle and the l_i values are determined such
146 that the difference was less than 1.0%

147 Based on the forward kinematic model, the Jacobian matrix at any joint configuration, can be
148 obtained as

$$149 [J(\Theta)] = \begin{bmatrix} -l_1 s_1 & -l_2 s_2 & -l_3 s_3 & -l_4 s_4 \\ l_1 c_1 & l_2 c_2 & l_3 c_3 & l_4 c_4 \end{bmatrix}$$

150 where Θ is the vector $(\theta_1, \theta_2, \theta_3, \theta_4)^T$ and c_i and s_i denote cosine and sine of angle θ_i The
151 Cartesian error $(\delta x, \delta y)^T$ is related to the joint error as

$$(\delta x, \delta y)^T = [J(\Theta)](\delta \Theta) \quad \text{-- (3)}$$

153 It can be seen that the (2×4) Jacobian matrix depends on the 4 joint variables Θ and hence
154 will vary at different points in the workspace. For a unit $|\partial\Theta|$, the maximum and minimum
155 $|(\delta x, \delta y)^T|$ are the maximum and minimum singular values of $J(\Theta)$ and they occur along the
156 mutually orthogonal singular directions, and the unit circle $|\partial\Theta| = 1$ maps to an ellipse in the
157 Cartesian space. If the Jacobian matrix is independent of Θ , scaling can be used to make the

158 minimum and maximum singular values equal and thus the error distribution can be a circle
159 (Raibert and Craig 1981).

160 ***Quantifying redundancy in the arm:***

161 In a serial robot, the motion of the end-effector $|(\partial x, \partial y)^T|$ is not affected by joint motion $\partial \Theta$,
162 then the robot is said to be redundant. In the arm model, the number of joint variables is 4 while
163 the Cartesian motion is of dimension 2. This implies that there exists some Θ which do not
164 affect the Cartesian motion and there are redundant degrees of freedom in the arm. The
165 redundancy in the arm model is quantified by $N(J)$ obtained from the null space of Jacobian
166 matrix, $[J(\Theta)]$, as follows:

167 The variability in the joint variable, Θ , is obtained for the baseline trials for each of the 8
168 different directions at the maximum Cartesian velocity or at the maximum error along the
169 trajectory. The mean joint configuration across trials, $\bar{\Theta}$, was computed also at the maximum
170 Cartesian velocity. The joint configuration for the k^{th} trial Θ_k was subtracted from the mean
171 to obtain the deviation, $\Delta\Theta_k$ as

$$172 \quad \Delta\Theta_k = \Theta - \bar{\Theta}_k$$

173 To compute the null space of the Jacobian matrix, it is assumed that the mean $\bar{\Theta}$ results in the
174 nominal hand trajectory and a part of deviation $\Delta\Theta_k$ are in the null space of $[J(\bar{\Theta})]$. The vectors
175 in the null space of the Jacobian matrix, ξ_i , are obtained from

$$176 \quad [J(\bar{\Theta})]\xi_i = 0, i = 1,2$$

177 and the component of $\Delta\Theta_k$ lying in the null space spanned by $\{\xi_1, \xi_2\}$ are the inner products
178 $\langle \Delta\Theta_k, \xi_i \rangle, i = 1,2$. The sum of the two null space components for the k^{th} trial is computed
179 as

$$180 \quad (\Theta_R)_k = \sum_{i=1}^2 \langle \Delta\Theta_k, \xi_i \rangle \xi_i$$

181 and $N(J)$ is defined as

$$182 \quad N(J) = \sum_{k=1}^n \frac{(\Theta_R)_k^2}{n}$$

183 where n is the number of trials.

184 ***Quantifying learning:***

185 One of the simplest models of learning is that of a first-order process where the error is reduced
186 exponentially. The variation of peak error $e(t)$ as a function of time t , can be written as

$$187 \quad \frac{de}{dt} = -e \times \beta$$

188 where β is a parameter that describes the rate of change of error and is independent of the
189 current error. The evolution of error from the above can be written as.

$$190 \quad f(t) = a \exp(-\beta t)$$

191 where α is the initial error and β is the learning rate.

192 In our case, the error is the maximum deviation (perpendicular distance) of the hand from the
193 straight line connecting the fixation box and the target box in each of the 8 directions.

194 Additionally, instead of a continuous function of time, the error is for each trial, and we have

$$195 \quad e(n+1) = a \exp(-\beta n) \quad \text{-- (4)}$$

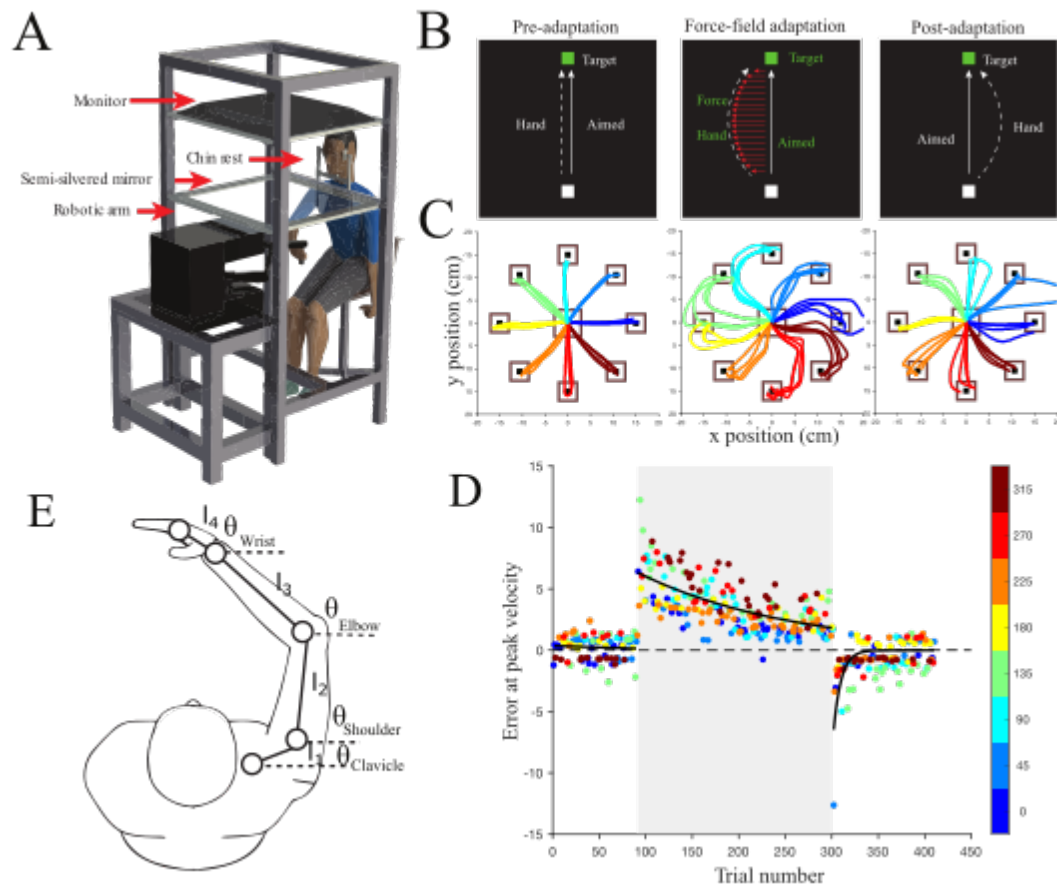
196 where n denotes the trial number.

197 ***Markov Chain Monte Carlo Method:***

198 During the experiments, the subjects performed approximately 200 trials in the 8 directions.
199 The error in the 8 directions is sorted into columns and on an average, there are 25 data points
200 for each direction -- they vary between 17 and 39 due to the random nature of the chosen
201 target directions. A typical table of error in centimetres in each trial along each of the 8
202 directions is shown in Table 1 for the subject whose data is shown in figure 1 D.

203 To obtain α and β for each of the directions, we used a Markov Chain Monte Carlo
204 (MCMC) approach (Suess and Trumbo 2010). The main motivation for using a MCMC method
205 is that it is known to be more robust in comparison to other nonlinear curve fitting schemes. It
206 also provides the distribution of the parameters for a chosen prior distribution -- chosen as same
207 uniform distribution for all directions and subjects in this work-- giving a much better insight
208 and confidence to the values obtained for α and β . The values of α and β obtained using
209 MCMC for the subject above along direction 90 degree (column 4 in Table above) is shown in
210 Table 2. The values of α and β obtained using the well-known Levenberg-Marquardt (LM)
211 nonlinear curve fitting scheme (Nocedal and Wright 2006) is also shown below for comparison,
212 and the numbers obtained from both the methods are in good agreement. It may be mentioned
213 that the R language was used to implement the MCMC and Levenberg-Marquardt algorithms.

214 The Table 2 shows the values of α and β obtained using the LM and the MCMC
215 approaches which indicate that mean values of α and β obtained are consistent. The main
216 advantage of MCMC over LM is that we also obtain the distribution of α and β which gives a
217 much better confidence to the obtained numbers. We have used MCMC for all 22 subjects and
218 for all 8 directions.



219

220 Figure 1: Experiment setup and design: (A) Subjects made point-to-point reaching movements
 221 using robotic arm. (B) Subjects made point-to-point reaching movements to visual targets in 1
 222 out of 8 directions 15 cm away from the central start point in each trial. Experiments were
 223 divided into three epochs -- a pre-adaptation, an adaptation to an external novel force field and
 224 post-adaptation epochs (left, centre and right panels). (C) First five trials of pre-adaptation,
 225 force field adaptation, and post-adaptation trials from a subject showing baseline variability,
 226 adaptation, and washout effects. (D) Error at peak velocity in pre-adaptation, adaptation, and
 227 post-adaptation showing the progression of adaptation for a subject. Errors in each of the eight
 228 directions are color-coded. Fitted exponential (black line) significantly accounts for most of
 229 the progression of errors across trials during adaptation. (E) Arm model and tracker positions
 230 to measure joint rotation angles.

231 **Results:**

232 We trained 12 subjects to learn point-to-point reaching movements using their dominant hand,
233 along 8 directions, in a force-field which was set using the force-field perturbation equation
234 (1). In this experiment the perturbation was proportional to the velocity of the hand but
235 perpendicular to the hand movement direction. We used an experimental setup shown in figure
236 1 A. The data for the three phases – a pre-adaptation baseline period, followed by a force field
237 perturbation (dynamic perturbation) and finally a post-adaptation phase when the perturbation
238 was removed – figure 1 B for a representative subject. The data consists of the location of the
239 end point and joint angles while subjects reached/pointed to the target during the three phases.

240 Figure 1 C shows the trajectories for the first five trials in the three phases for the
241 representative subject. The peak error was calculated as the perpendicular distance of the hand
242 trajectory at peak velocity from the straight line joining the central fixation box to the target
243 location. Overall, the pattern of trajectories are consistent with previous work showing that
244 while typical movements follow a straight trajectory in the baseline condition, they show strong
245 curved trajectories in the presence of a force-field. The curved trajectories gradually become
246 straighter with practice over the course of about two hundred trials with the error decreasing as
247 shown in figure 1 D. In addition, as a consequence of motor learning, subjects showed a
248 washout effect (post adaptation) where errors in trajectory invert in direction when the learned
249 force field is turned off in the post adaptation period. This washout error converges to baseline
250 levels.

251 We assume that learning is a first-order process where the error is reduced exponentially,
252 and the variation of peak error is as shown in equation (4). To compute the learning in force--
253 field perturbation trials, the errors were fitted with an exponential fit using the Levenberg-
254 Marquardt algorithm, and for the representative subject $\beta = 0.0064 \pm 0.0006$ Std. error, $t =$

255 11.51 and $Pr (> |t|) < 2e - 16$ and $a = 6.41 \pm 0.292$ cm Std. error, $t = 21.92$ and
256 $Pr (> |t|) < 2e - 16$ (see Figure 1D).

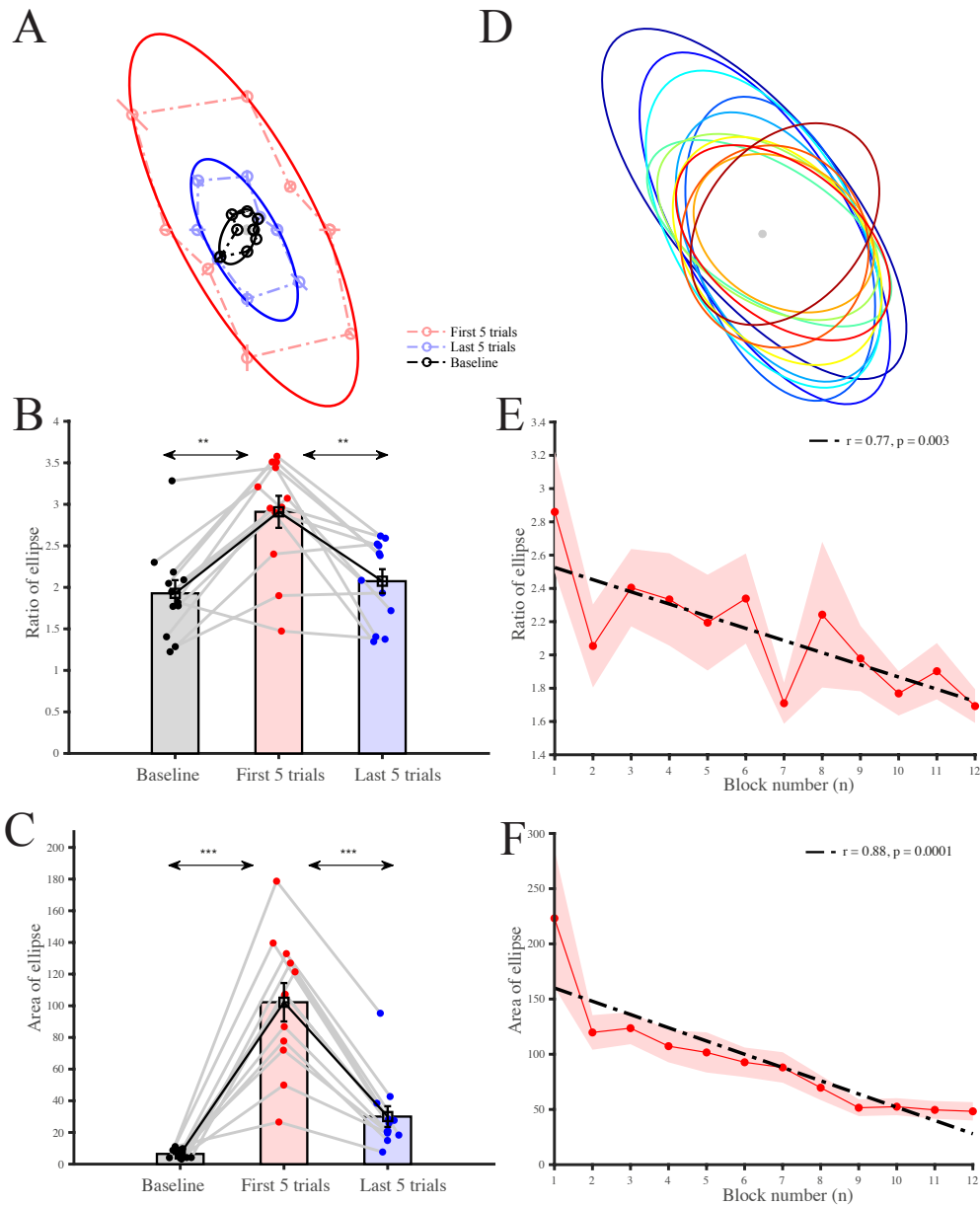
257 To obtain a more geometric view, we plot the mean error and variation in the baseline,
258 first 5 trials and last 5 trials under force-fled perturbation in the 8 directions. An ellipse is fitted
259 (motivated by the velocity distribution at a point seen in a robot -- (see text after equation (3))
260 with the mean error in the 8 directions for each of the three data sets. The mean error is denoted
261 by a “circle” and the “line” through the “circle” denote the variation with trials. Figure 2 A
262 shows the error distribution along 8 directions for the representative subject data shown in
263 figure 1. The size of the error ellipse decreases from the first 5 to the last 5 trials -- the errors
264 are large when the lateral force is applied and as the trials progress, the error decreases,
265 indicating that the subject learns to adapt to the external lateral force. It can be observed that
266 the error ellipse in the baseline (when no external force is applied) is the smallest.

267 Figure 2 B and C shows the bar plot of the ratio of the major to the minor axis and the
268 area of the ellipse for all the subjects. The ratio is large in the first 5 trials and the ratio in the
269 last 5 trials approaches the value in the baseline. The mean ratio in baseline period (mean =
270 1.93 ± 0.55) was significantly less than the mean ratio at the starting of perturbation (mean =
271 2.91 ± 0.67) (figure 2 B; $p = 1.44e-4$, $t(11) = 5.67$). There was also significant difference in
272 the mean ratio between the starting of perturbation and end of perturbation (figure 2 B; mean
273 = 2.07 ± 0.50 ; $p = 0.002$, $t(11) = 4.16$). Figure 2 D shows the evolution of the error ellipse
274 with trails for subject.

275 We also computed the area of the ellipse, which is a proxy for overall learning rate,
276 across directions. The area in the baseline period (mean = 6.44 ± 2.68) was significantly less
277 than the mean area at the starting of perturbation (mean = 102.23 ± 42.02) (figure 2 C; $p =$
278 $8.25e-06$, $t(11) = 7.80$). There was good difference in the mean ellipse area between the starting

279 of perturbation and the end of perturbation (figure 2 C; mean = 30.02 ± 22.68 ; $p = 1.82e-06$, t
280 (11) = 9.13). However, the area of the ellipse is larger in the last 5 trials as compared to the
281 baseline in all the 12 subjects. The area of the ellipse gradually decreases with practice over
282 the course of about two hundred trials as shown in figure 2 F.

283 Overall, the main findings are that the maximum error due to the external force-field is
284 very large when it is applied and due to learning the error decreases with trials. This can be
285 seen from the straightening of the curved trajectories as shown in figure 1 D for a subject and
286 quantitatively in the plot of area of the ellipse for 12 subjects shown in figure 2 D. Secondly,
287 as shown in figure 2 E, the ratio of the major to minor axis of the ellipse decreases with trials
288 and the error distribution tends to become circular -- it is not a perfect circle which would imply
289 a ratio of 1.0. The learning results shown by the 12 subjects is not only in terms of reduction
290 of error magnitude but also making the error distribution more uniform -- this is termed as
291 homogenization of the task space error distribution.



292

293 Figure 2: Homogenization of workspace in the dominant arm: (A) Representative subject
294 directional error ellipse for the baseline (black), starting of perturbation (red) and end of
295 perturbation (blue). (B) Comparison of directional error ellipse eccentricity in baseline (black),
296 starting of perturbation (red) and end of perturbation (blue) reveal higher eccentricity in the
297 starting of perturbation. (C) Comparison of directional error ellipse area in baseline (black),
298 starting of perturbation (red) and end of perturbation (blue) reveal higher area in the starting of
299 perturbation. (D) Representative subject directional error ellipse progression towards a more
300 circular ellipse. The ratio gradually decreases with practice over the course of about two
301 hundred trials (E) Regression across subjects showing the progression of change in ellipse
302 eccentricity indicative of homogenization of error distribution. (F) Regression across subjects
303 showing the progression of change in ellipse area indicative of learning.

304 ***Active learning of a force-field perturbation:***

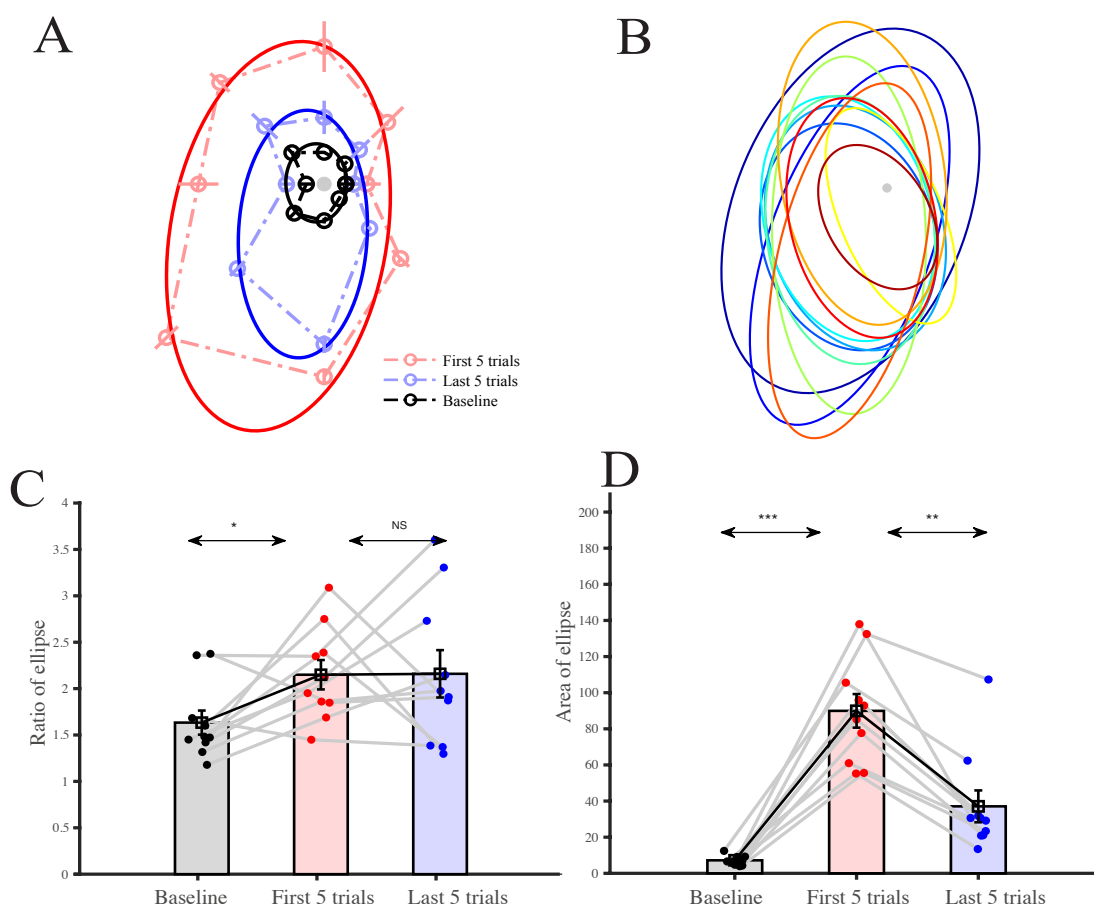
305 Differences in the ratio of the major to minor axis may reflect a difference in the intrinsic
306 biomechanics of the human arm. In contrast, differences in the ratio may also reflect the effect
307 of neural control that assists in homogenizing the workspace of the human arm. To assess this,
308 we tested and compared the ratio of the major to minor axis of the ellipse across direction in
309 dominant and non-dominant hand in 10 subjects, thereby normalizing any differences in the
310 biomechanics.

311 Similar to the dominant hand, the figure 3 A shows the measured maximum peak error
312 in the baseline where no perturbation is applied, in the first 5 and last 5 trials with force-field
313 perturbation in each of the 8 directions with the non-dominant hand for a typical subject. Again,
314 the mean error is denoted by a “circle” and the “line” through the “circle” denote the variation
315 with trials. We fit ellipses through the mean error along the 8 directions. This is shown in figure
316 3 A for the baseline, the first 5 and the last 5 trials. It can be seen that as trials progress the ratio
317 of major to minor axis in non-dominant hand do not decrease as seen in the dominant hand (see
318 figure 2 E). The mean ratio in the baseline period (mean = 1.63 ± 0.41) was significantly less
319 than the mean ratio at the starting of perturbation (mean = 2.15 ± 0.50) (see figure 3 C; $p =$
320 0.03 , $t(9) = 2.53$). There was no difference in the mean ratio between the starting of
321 perturbation and end of perturbation (see figure 3 C; mean = 2.16 ± 0.81 ; $p = 0.97$, $t(9) =$
322 0.03).

323 The mean ellipse area in baseline period (mean = 7.19 ± 2.66) was significantly less
324 than the mean ellipse area at the starting of perturbation (mean = 89.94 ± 29.43) (see figure 3
325 D; $p = 5.83e-06$, $t(9) = 9.43$). There was significant difference in the mean direction ellipse
326 area between the starting of perturbation and end of perturbation (see figure 3 D; mean = 37.07

327 ± 27.90 ; $p = 1.39e-04$, $t(9) = 6.31$). The ellipse area gradually become smaller with practice
328 over the course of about two hundred trials which implies some learning is taking place.

329 Taken together these findings indicates that the difference in ratio of the major to the
330 minor axis between the dominant and non-dominant hand or the capability of homogenization
331 of the error distribution maybe a consequence of the different mechanisms in the dominant
332 hand and non-dominant hand -- most likely due to active neural control.

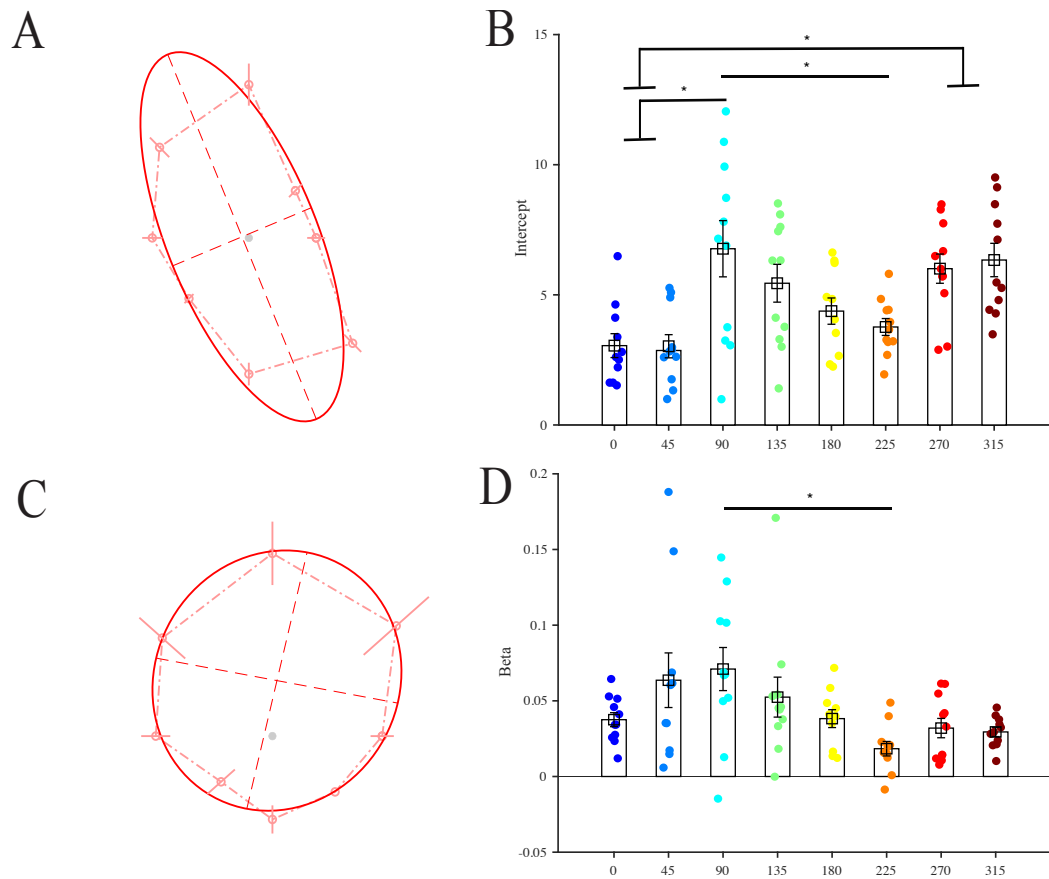


333

334 Figure 3: Homogenization of workspace in non-dominant arm: (A) Representative subject
335 directional error ellipse for the baseline (black), starting of perturbation (red) and end of
336 perturbation (blue). (B) Representative subject directional error ellipse progression. (C)
337 Comparison of directional error ellipse eccentricity in baseline (black), starting of perturbation
338 (red) and end of perturbation (blue) reveal higher eccentricity in the starting of perturbation.
339 (D) Comparison of directional error ellipse area in baseline (black), starting of perturbation
340 (red) and end of perturbation (blue) reveal higher area in the starting of perturbation.

341 ***Maximum error, learning rate and redundancy along directions:***

342 As mentioned earlier, the learning rate β and the maximum error α see equation (4) for all the
343 200 trials (along all directions taken together) for a particular subject was 6.40 and 0.006.
344 Similar results were obtained for all the 12 subjects. To investigate the variation of α and β
345 along each direction, the force-field perturbation trail data are sorted along the 8 directions.
346 This is shown for a typical representative subject is shown in Table 1-- the variation from 25
347 trials along each direction is due to the random nature of the target presented. To obtain α and
348 β in each of the 8 directions, we used the robust MCMC algorithm. The mean α and β values
349 across 12 subjects is shown in figure 4 A and C and obtained values in Tables 3 and 4 (in Table
350 3 and 4, the values of α and β for subject 9 is not considered in the analysis since the results
351 obtained from MCMC and LM differ significantly.), respectively. Figure 4 of B and D shown
352 the bar plots of α and β across subjects. There is statistical difference in α value in each of the
353 directions ($F(7,79) = 5.54, p = 3.24e-5$, see figure 4 C) indicative that some directions have
354 higher initial errors. Furthermore, there is a clear statistical difference in learning rate along
355 90° and 225° ($F(7,78) = 3.11, p = 0.006$, see figure 4 D). These are also very close to the major
356 and minor axis of the ellipse of error distribution.



357

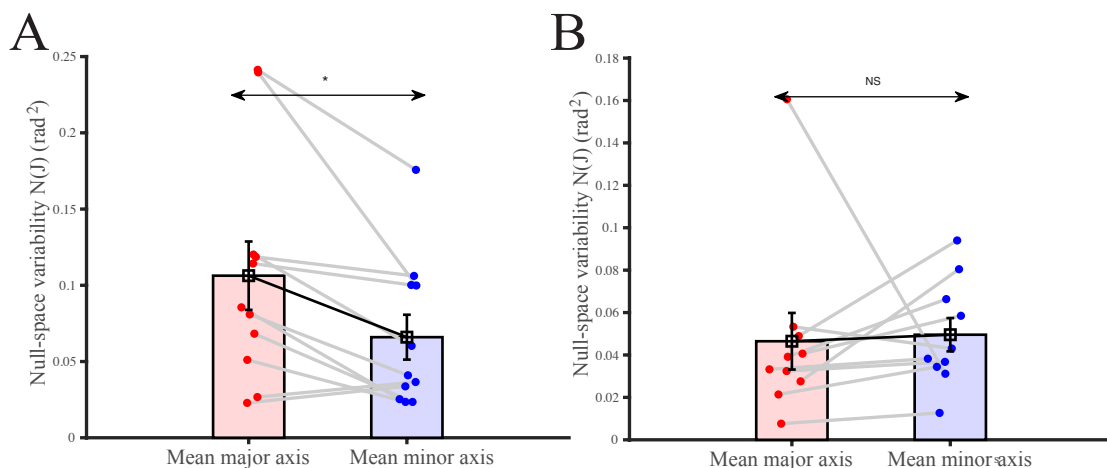
358 Figure 4: Learning is greater for more difficult directions leading to homogenization. Difficult
359 direction in adaptation: (A) Intercept mean \pm se across subjects showing the ratio of the major
360 to minor axis of the ellipse indicative of some direction has higher initial errors. (B)
361 Comparison of intercept in different directional. (C) learning rate mean \pm se across subjects
362 showing the learning rate in different directions. (D) Comparison of learning rate in different
363 directions.

364

365 As mentioned earlier, the human hand is known to be redundant. In the model shown in section
366 material, a four degree-of-freedom model was assumed for the human hand performing planar
367 point-to-point reaching motions and hence, the human hand is assumed to possess two
368 redundant degrees of freedom. In order to test whether redundancy could aid in homogenization
369 of error distribution due to a force-field perturbation, we computed the null space of the
370 Jacobian $N(J)$, representing the use of the redundancy in the four degree-of-freedom arm

371 model. Figure 5 shows a bar plot of $N(J)$ for the dominant and non-dominant hand for all
372 subjects in the baseline period. Consistent with our hypothesis, $N(J)$ was lesser along the minor
373 axis (mean = 0.06 ± 0.05) compared to the major axis (mean = 0.11 ± 0.07 , $p = 0.006$, $t(9) =$
374 3.49),) for the dominant hand. For the non-dominant hand, there was no difference in $N(J)$
375 values along the major and minor axis.

376 We have earlier observed that during learning homogenization of the error takes place,
377 i.e., the ratio of the major to minor axis of the direction error ellipse decreases with trials. Taken
378 together we can suggest that redundancy may also play a role in making error distribution more
379 uniform, i.e., use of redundancy leads to homogenization of error distribution in point-to-point
380 reaching tasks.



381

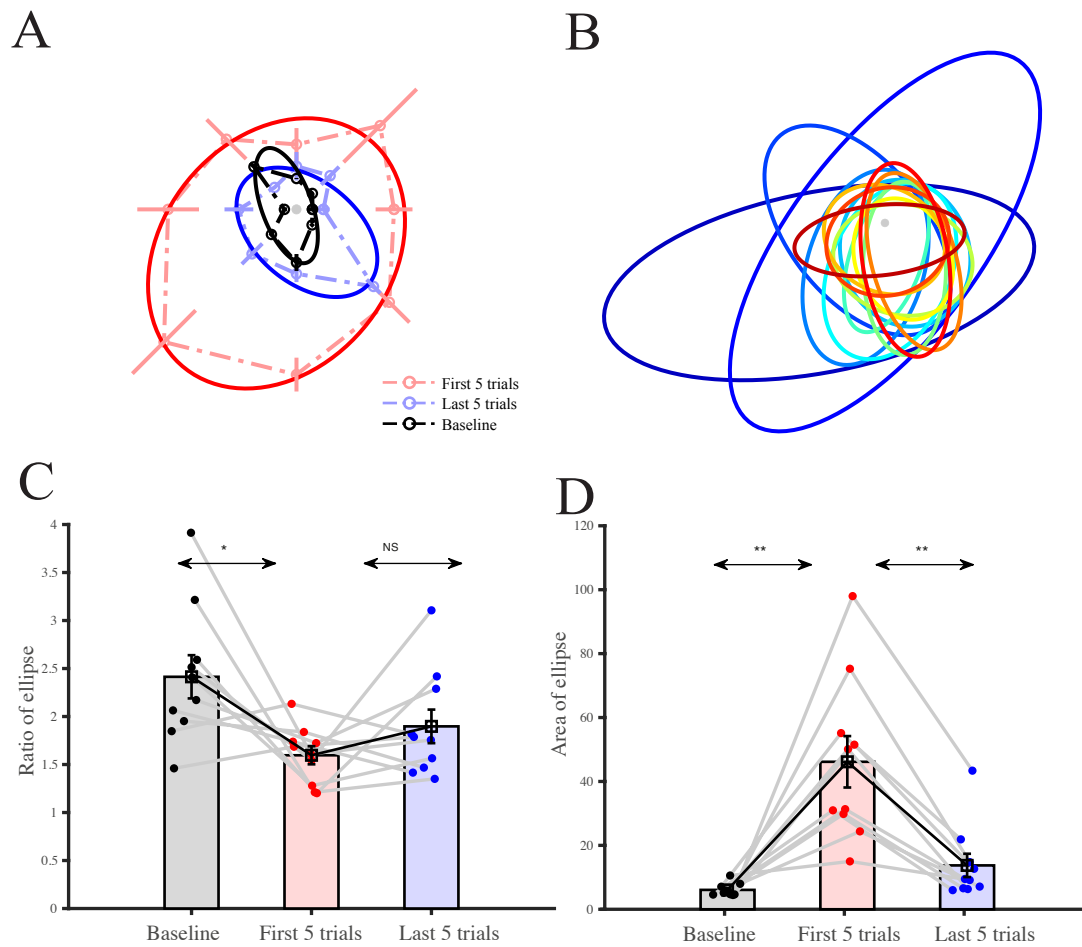
382 Figure 5: Null-space variability in the major axis and minor axis in direction ellipse. (A)
383 Comparison of dominant hand null-space variability along major axis (red), and minor axis
384 (blue). (B) Comparison of non-dominant hand null-space variability along major axis (red), and
385 minor axis (blue).

386 ***Learning of a visuo-motor perturbation:***

387 To test whether homogenization is a property related specifically to learning of the newly
388 imposed biomechanics or is a more general feature of motor learning we next examined 10
389 subjects while they learnt point-to-point reaching movements with the visuo-motor
390 perturbation, equation (2), along 8 directions, where in each case, the cursor was rotated by 45°
391 from the hand trajectory. Similar to the dominant hand, figure 6 A shows the measured
392 maximum peak error in the baseline where no perturbation is applied, in the first 5 and last 5
393 trials with visuo-motor perturbation in each of the 8 directions with the dominant hand for a
394 typical subject. Again, the mean error is denoted by a “circle” and the “line” through the
395 “circle” denote the variation with trials. We fit ellipses through the mean error along the 8
396 directions. This is shown in figure 6 A for the baseline, the first 5 and the last 5 trials. Unlike
397 in the dynamics condition, as trials progress the ratio of major to minor axis in visuo-motor
398 perturbation and did decrease for the dominant hand (see figure 2 E). The mean ratio in the
399 baseline period (mean = 2.41 ± 0.71) was different than mean ratio at the starting of
400 perturbation (mean = 1.59 ± 0.29) (see figure 6 C; $p = 0.018$, $t(9) = 2.88$). There was also no
401 difference in the mean ratio between the starting of perturbation and end of perturbation (see
402 figure 6 C; mean = 1.89 ± 0.55 ; $p = 0.18$, $t(9) = 1.46$).

403 The mean ellipse area in baseline period (mean = 6.06 ± 1.95) was significantly less
404 than the mean ellipse area at the starting of perturbation (mean = 46.15 ± 25.44) (see figure 6
405 D; $p = 0.001$, $t(9) = 4.76$). There was, however, a significant difference in the mean direction
406 ellipse area between the starting of perturbation and end of perturbation (see figure 6 D; mean
407 = 13.73 ± 11.52 ; $p = 2.06e-4$, $t(9) = 5.98$). The ellipse area gradually become smaller with
408 practice over the course of about two hundred trials as shown in figure 6 D which implies

409 learning. occurs in the absence of homogenization of the error distribution in the visuo-motor
410 perturbation condition.



411

412 Figure 6: Learning in the absence of homogenization of workspace during visuomotor
413 adaptation: (A) Representative subject directional error ellipse for the baseline (black), starting
414 of perturbation (red) and end of perturbation (blue). (B) Representative subject directional error
415 ellipse progression. (C) Comparison of directional error ellipse ratio in baseline (black),
416 starting of perturbation (red) and end of perturbation (blue) reveal a higher ratio in the starting
417 of perturbation. (D) Comparison of directional error ellipse area in baseline (black), starting of
418 perturbation (red) and end of perturbation (blue) reveal a larger area in the starting of
419 perturbation indicating learning

420

421 ***Discussion:***

422 In this study, we have presented variation in task space error along directions in point-to-point
423 reaching tasks. We demonstrated that the motor learning is homogenizing the workspace of the
424 human arm possibly to increase the efficiency and accuracy. In addition, our results also
425 showed a significantly larger use of redundancy along the directions with more error (or major
426 axis) versus the direction with less errors (or the minor axis) of error ellipse. The data revealed
427 novel direction specificity of motor learning not reported earlier to the best of our knowledge.
428 What is more significant is the observation that this anisotropy in errors distribution is reduced
429 with trials and the error distribution becomes more circular. This work also indicates that the
430 redundancy in the arm is used to homogenize the error distribution. Although not known where
431 in the central nervous system the resolution of redundancy is performed and how, this works
432 provides a possible reason on how the redundancy in the arm is used for reaching tasks.

433

434 ***Homogenization and Linearization of Control:***

435 In control theory, it is well known that it is significantly easier to design controllers for a linear
436 system to achieve a desired accuracy. More specifically, for a robot to follow a desired path
437 with desired accuracy, extensive effort has gone into designing controllers starting from the
438 simplest proportional, integral and derivative (PID) to sophisticated model-based control
439 (Craig 2004). A robot can be considered to be linear up to first order if the Jacobian matrix
440 relating the end-point velocity to the joint rates is constant and for such a Jacobian matrix, the
441 error distribution is circular or isotropic. From control theory, it has been argued that such a
442 “linear” robot would be easier to control and would achieve better accuracy -- such robots
443 containing only sliding joints and with no rotary joints was attempted in the earlier days of
444 robotics but was given up due to the issues in friction at the sliding joints. This work indicates
445 that the learning leads to approximate homogenization of the error distribution, i.e., makes the

446 arm more ``linear'' even though it contains rotary joints which makes the Jacobian matrix a
447 function of the location and inherently nonlinear. A possible consequence of homogenization
448 of the error distribution would be that the overhead on the central nervous system in terms of
449 control during reaching tasks is reduced.

450

451 ***Homogenization and Biomechanics:***

452 Our results indicate that some directions appear to be easier to learn and some are more
453 difficult. In Howard et al. 2013 (see also Singh et al. 2016), error plots in 8 directions indicate
454 some dependence on direction but this is not brought out clearly. In this work, we can clearly
455 see that the errors are significantly larger in direction of the major axis and significantly smaller
456 in the direction of the minor axis -- the mean (across subjects) major axis is observed to be
457 around 111.68° , and the minor axis is around 201.68° . It is known from biomechanics of human
458 arm (or dynamics of serial robots) that the inertia of the arm (as seen from shoulder) and the
459 effort required is largest when the arm is moving in the direction of the major axis and likewise
460 the inertia seen, and the effort required is smaller when the arm is moving along the minor axis
461 and a mechanistic view of more/less error along direction of more/less effort is consistent with
462 this observation.

463 The task space positioning error for a robot, in the presence of external disturbances
464 and noise, is also related to the stiffness (or impedance) of the robotic arm. The impedance of
465 the robot arm is typically determined by the control scheme (Hogan 1984; Raibert and Craig
466 1981) and controller gains, and the larger the stiffness the less is the positioning error. In
467 reaching tasks by human arms, position control involves increase in limb impedance (Franklin
468 et al. 2007; Wong et al. 2009). In (Wong et al. 2009), the authors claim that the limb stiffness
469 is modulated to achieve accuracy requirement in the absence of destabilizing force, and they
470 observe a modest increase in limb stiffness perpendicular to the direction of motion when more

471 accuracy is required while moving along a narrow track. In (Franklin et al. 2007) the authors
472 state that the end-point stiffness, primarily due to the co-activation of bi-articular muscles,
473 approximately aligns with the direction of instability in the environment with the stiffness
474 ellipse rotating to align with the direction of instability. In this work, we proceed further and
475 show that learning results in homogenization, possibly through changes in the stiffness
476 properties as well as changes in the internal model of the arm.

477

478 ***Homogenization is controlled by the CNS:***

479 A second major observation in this study is that bio-mechanically similar dominant and non-
480 dominant arms showed different homogenizing behaviours. Both the dominant and the non-
481 dominant hand shows directional dependency of error and both show that the size of the error
482 ellipse decreases with trials. This indicates that learning is present in both arms. However, the
483 homogenization of error distribution is not seen in the non-dominant hand and this indicates
484 that homogenizing is being actively done by the central nervous system -- the homogenizing
485 of the workspace seen in the dominant hand provides a natural explanation of why learning
486 might be more potent in the dominant hand and reaffirms the belief that homogenizing not only
487 reflects the bio-mechanical characteristics of the arm but may reflect active control from brain
488 or the central nervous system. In a related set of experiments with visuo-motor rotation, all
489 directions show the same error distribution (see figure 4). This is consistent with our study as
490 learning a visuo-motor rotation involves learning of an internal kinematic model and unlikely
491 requires the homogenization since the inhomogeneities are relate to dynamics.

492 ***Homogenization by feedback control internal models:***

493 As mentioned previously, movements associated with larger inertia will have greater muscle
494 loads and as a consequence of signal dependent noise will generated larger errors. In this
495 context, it is interesting that the distribution of errors in the baseline condition is low and
496 homogenous despite inhomogeneous biomechanics. Such uniformity is likely to be a
497 consequence of kinematic feedback controllers that ensure of uniform endpoint control, as
498 well as internal models that ensure homogeneity even during the early feedforward driven
499 aspect of the trajectory. This suggests that in the baseline the brain (CNS) uses a well learnt
500 sensorimotor mappings that get transiently inactivated, exposing the underlying
501 inhomogeneous biomechanics. Although, we do not propose a mechanism on how feedback
502 gains and internal models are relearned following the perturbation, our results suggest that a form
503 of learning that is sensitive not just to the magnitude of the errors as a consequence first order
504 learning, but of the learning rates themselves that are sensitive to the direction of movement
505 that helps homogenise errors. We suggest that such directional specific learning linearizes
506 responses and facilitates generalization beyond the region of training One important issue is
507 direction generalization function – this refers to the movement direction that displays the
508 maximum degree of adaptation after learning with the degree of adaptation falling off with
509 adjacent movements. Donchin et al. (2003) argue from experiments conducted during reaching
510 tasks, subjected to a force-field disturbance, that the generalization is bi-modal, perhaps
511 reflecting basis elements that encode direction bi-modally. This work also does not discuss the
512 direction dependence of positioning error in reaching tasks. In this work, our results indicate
513 that the generalization is not bi-modal -- the distribution of error is best approximated by an
514 ellipse with some direction showing large error and some direction showing less error across
515 subjects.

516 ***Homogenization, redundancy, and learning:***

517 In previous work we showed through first and second order correlations of null space
518 variability--a proxy for joint redundancy--a possible role for joint redundancy in motor learning
519 of dynamics and kinematics. Here, we extended this correlation to study the directional
520 dependency of redundancy and its possible impact on motor learning. In congruence and
521 extension with our previous study, we observed that the redundancy exhibited a directional
522 axis that aligned with the learning axis, which could also explain the observed homogenization.
523 Furthermore, this redundancy axis was only observed for the dominant arm and not seen in the
524 non-dominant arm which suggests that this alignment maybe causal in nature. In contrast to
525 the learning of dynamics, kinematic perturbations did not produce inhomogeneities in errors
526 for redundancy to exploit. Taken together, we speculate that a greater redundancy may allow
527 better learning by increasing available options, contributing to homogenising errors across
528 directions.

529

530 ***Conclusions:***

531 This paper deals with motor learning in point-to-point reaching tasks performed by a human
532 arm. Experiments conducted with adult subjects show that maximum error along the trajectory,
533 when the arm is perturbed by a lateral force, decreases with practice and approaches the
534 situation when no lateral force is applied. In this paper, we investigate the error in the 8 different
535 directions in which the reaching task was carried out. The error distribution along the 8
536 directions is fitted with an ellipse and it is observed that, across subjects, a direction between
537 90 and 135 degrees has the largest error while a direction between 180 and 225 degrees shows
538 the least error. As the trial progresses and learning takes place, the magnitude of the error along
539 all directions reduce to a value close to the baseline trials where no lateral force is applied.
540 Moreover, it is observed that the eccentricity of the error ellipse reduces, and the error
541 distribution becomes more circular. These two observations indicate that the learning leads to
542 homogenization of the trajectory error. Similar experiments done with the non-dominant and
543 bio-mechanically similar arm, show that while there is learning (error magnitude decreasing
544 with trials), the eccentricity of the error ellipse does not reduce. This leads us to the conclusion
545 that the homogenization is a consequence of active neural control. Furthermore, analysis
546 suggest that the redundancy in the arm is used to make the error in different directions more
547 uniform and is thus a possible use of the redundancy in all human arms.

548 A typical anthropomorphic robot is known to have a nonuniform (ellipse) error
549 distribution at different locations in its workspace and in a redundant robot, the redundancy
550 can be used to make the error distribution uniform (circular) in all directions at a location.
551 Uniform error distribution is also seen in a linear system which is known to be more easily
552 controllable. The homogenization result and its analogy with a mechanical robot suggests that
553 the redundancy is used to make the human arm more linear which in turn make it easier for the
554 central nervous system to control. More work is required to obtain a better understanding where

555 the redundancy in human arm is processed and how the redundancy in the actuation system,
556 namely muscles, are resolved.

557

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560 **Conflict of Interest**

561 The authors declare no competing financial interests.

562 **Author Contributions**

563 P.S., A.G. and A.M. designed the experiments, P.S. performed the experiments, P.S. and

564 O.G. analysed the data and P.S., A.G., and A.M. wrote the paper.

565 **Data Availability**

566 Data available on request from the authors.

567 **Keywords**

568 Motor learning, Reaching tasks, Force-field perturbation, Error along directions,
569 Redundancy, Homogenization of error distribution

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