

1 Loop-extrusion and polymer phase-separation can co- 2 exist at the single-molecule level to shape chromatin 3 folding

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14

15 Abstract

16 Loop-extrusion and phase-separation have been proposed as mechanisms that shape
17 chromosome large-scale spatial organization. It is unclear, however, how they perform
18 relative to each other in explaining chromatin architecture data and whether they compete
19 or co-exist at the single-molecule level. Here, we compare models of polymer physics based
20 on loop-extrusion and phase-separation, as well as models where both mechanisms act
21 simultaneously in a single molecule, against multiplexed FISH data available in human loci in
22 IMR90 and HCT116 cells. We find that the different models recapitulate bulk Hi-C and average
23 microscopy data. Single-molecule chromatin conformations are also well captured, especially
24 by phase-separation based models that better reflect the experimentally reported
25 segregation in globules of the considered genomic loci and their cell-to-cell structural
26 variability. Such a variability is consistent with two main concurrent causes: single-cell
27 epigenetic heterogeneity and an intrinsic thermodynamic conformational degeneracy of
28 folding. Overall, the model combining loop-extrusion and polymer phase-separation provides
29 a very good description of the data, particularly higher-order contacts, showing that the two
30 mechanisms can co-exist in shaping chromatin architecture in single cells.

31

32 INTRODUCTION

33 To understand the machinery that in the nucleus of cells establishes at large scales the 3-
34 dimensional (3D) architecture of chromatin¹⁻¹⁴, encompassing DNA loops¹⁵, Topologically
35 Associated Domains (TADs)^{16,17} and other structures^{13,18}, different physical mechanisms have
36 been proposed and investigated via models relying solely on fundamental physical
37 processes¹⁹⁻⁴⁵ and via computational approaches⁴⁶⁻⁵⁹. However, it remains unclear how well
38 different mechanisms capture folding at the single molecule level, how they compare against
39 each other in explaining experimental data and whether they compete or co-exist in
40 determining the structure of chromosomes. Here, we explore two recently discussed classes
41 of models that focus on two distinct physical mechanisms, respectively loop-extrusion and
42 polymer phase-separation, that we compare against single-molecule super-resolution
43 microscopy⁶ and bulk Hi-C data^{15,60} available in human loci in IMR90 and HCT116 cells.

44
45 Loop-extrusion and phase-separation based polymer models of chromosomes reflect two
46 classical, yet distinct scenarios of molecular biology to explain the formation of DNA
47 contacts⁶¹. The first class considers the picture where physical proximity between distal sites
48 is established by molecular motors that bind to DNA and extrude a loop^{19,20,31,39,40}. This is an
49 out-of-equilibrium, active physical process that involves energy, e.g., ATP, consumption. The
50 model envisages that those loop-extruding complexes stochastically bind to a polymer chain
51 and extrude loops until encountering another motor, an anchor site or unbinding from the
52 chain. While the polymer becomes compacted in a linear array of loops, specific contacts are
53 established between the motor anchor sites where extrusion halts, hence defining
54 boundaries between subsequent chromatin regions. Experimental evidence indicates that
55 Cohesin and Condensin can be components of the motor complex, while properly oriented
56 CTCF sites can act as anchor points⁴⁰. Computer simulations have shown that such a model
57 can explain with good accuracy loops and TADs visible in bulk Hi-C contact maps in interphase
58 as well as, for example, in mitotic chromosomes^{19,20,31,39,40}. Variants of such a model have
59 been also developed where chromatin loops are formed by thermal random sliding of DNA
60 into an extruding molecule³¹ or by, e.g., transcription-induced supercoiling³⁹.

61
62 The second class of polymer models^{21-30,32-38,41-45} considers another classical scenario where
63 physical proximity between distal DNA sites results from interactions mediated, for instance,
64 by diffusing cognate bridging molecules, such as Transcription Factors, or from direct
65 interactions produced, e.g., by DNA bound histone molecules. In the Strings and Binders (SBS)
66 model^{42,44}, for example, a chromatin filament is represented as a self-avoiding chain of beads,
67 along which are located different types of binding sites for cognate diffusing binders that can
68 bridge those sites. The binding sites have been correlated to different molecular and
69 epigenetic factors, ranging from active and poised Pol-II to eu- and heterochromatin
70 sites^{21,27,28,45}. The steady-state 3D conformations of the system are determined by the laws
71 of physics and fall in different structural classes corresponding to its thermodynamics phases.
72 In the SBS model, for instance, upon increasing the concentration or affinity of binders, the
73 system undergoes a polymer phase-separation transition from a coil, i.e., randomly folded, to
74 a globular state, where distinct globules self-assemble along the chain by the interactions of
75 cognate binding sites^{24,27,35,44}. Polymer physics explains that thermodynamic phases are
76 independent of the specific origin of the interactions - e.g., direct or mediated by diffusing
77 factors - so different models can belong to the same universality class⁶². For that reason, the

78 thermodynamic phases of, say, the SBS model also occur in models with direct chromatin
79 interactions. Those phase transitions result in structural changes of the chain that
80 spontaneously establish contact or segregation of specific, distal sites, such as genes and their
81 regulators. Such a class of models has been shown to explain Hi-C, SPRITE, GAM and
82 microscopy contact data across the genome, from the sub-TAD to chromosomal scales^{21–30,32–}
83 ^{38,41–45}, also at the single molecule level^{35,38}.

84
85 It is unclear, however, how loop-extrusion and polymer phase-separation perform relative to
86 each other in capturing chromatin folding and whether they compete or co-exist in
87 establishing chromosome architecture. Here, we implemented different versions of those
88 models to benchmark their structural predictions at the single-molecule level against
89 independent multiplexed FISH data⁶. We simulated first a simple loop-extrusion (LE) model²⁰
90 of the considered loci. Next, we developed an extended LE (eLE) model whose anchor site
91 genomic locations are optimized to best fit experimental contact data. Additionally, to mimic
92 epigenetic differences of single cells, in the model the anchor sites can differ across single
93 molecules²⁹. We also considered the SBS model of the studied loci³⁵ and, finally, we
94 introduced a model combining eLE and SBS (the LE+SBS model), i.e., a model where in a single
95 molecule both LE and SBS mechanisms act simultaneously. We find that both loop-extrusion
96 and phase-separation based models can explain well ensemble-averaged microscopy and
97 bulk Hi-C data, albeit the simple LE model is only partially effective. Our single-molecule
98 analyses show that both types of models do capture the main features of single-cell chromatin
99 conformations and higher-order contacts. Yet, phase-separation based models better reflect
100 the experimentally reported segregation in globules of the considered genomic loci and their
101 cell-to-cell structural variability. Such a variability results from two main concurrent sources:
102 the intrinsic thermodynamic degeneracy of polymer folding and single-cell epigenetic
103 heterogeneity. Consistent with such a picture, the LE+SBS model turns out to provide overall
104 an excellent description of all the different datasets and to have the least discrepancy with
105 microscopy triple contact data, supporting the view that loop-extrusion and phase-separation
106 can co-exist at the single-molecule level in determining chromatin architecture.

107

108 **RESULTS**

109 **Polymer models of the studied loci**

110 We implemented the polymer models of two 2Mb wide loci in human IMR90 and HCT116
111 cells where, as stated, single-cell super-resolution microscopy data⁶ are available at 30kb
112 resolution (**Fig. 1a**, **Suppl. Fig. 1a**). To assess the role of the different ingredients of the
113 models, we developed distinct versions that we compared against single-cell data.

114

115 First, we implemented a simple LE model²⁰, where loop-extruding motors stochastically bind
116 to a polymer bead chain and extrude loops until encountering anchor points with opposite
117 orientation or another motor or unbinding from the chain (**Fig. 1b** and **Methods**). The position
118 and orientation of the anchor points are identified by the FIMO standard motif finding
119 analysis⁶³ based on the peaks of CTCF ChIP-seq data from ENCODE⁶⁴. While the motors can
120 stochastically bind to and unbind from the chain, the anchor sites are fixed and equal in all
121 single-molecule computer simulations. Their anchoring strength is set to 100%, i.e., when an
122 extruder arrives at an anchor point it remains blocked at that position, yet we checked that
123 the overall results do not change for strengths in the range down to 60% (**Methods**). This

124 model is hereafter referred to as the LE model. To explore the potential of the loop-extrusion
125 mechanism beyond such a minimal implementation, we also considered a more refined
126 version where, to mimic epigenetic differences across single cells, each anchor site is present,
127 with a given probability, only in a subset of model single-molecules²⁹ (**Fig. 1c, Suppl. Fig. 1b**
128 and **Methods**). Additionally, to best reproduce population-averaged Hi-C and microscopy
129 distance data, we searched for the optimal genomic location and probability of the motor
130 anchor sites, independently of CTCF tracks (**Methods**). In the considered loci, most of those
131 optimal sites coincide with FIMO CTCF peaks (**Fig. 1c, Suppl. Fig. 1b**), but not all, and
132 conversely many FIMO CTCF peaks are not included as model anchor sites. The probability to
133 be present in a model single-molecule is found to be different for different anchor sites,
134 ranging from roughly 50% to 100% (**Fig. 1c, Suppl. Fig. 1b**), values consistent with current
135 estimates of cell epigenetic heterogeneity⁶⁵. Finally, to better fit the features of Hi-C and
136 microscopy data, such as TADs and globules (see below), the beads of the polymer chain are
137 subject to a self-interaction produced by unspecific bridging molecules. Such a variant of the
138 LE model is hereafter named the extended LE (in short, eLE).

139
140 Next, in the considered loci we implemented the SBS model³⁵ whereby chromatin is
141 represented as a self-avoiding chain of beads, in a thermal bath, with specific binding sites for
142 cognate diffusing molecular binders (**Fig. 1d, Suppl. Fig. 1c** and **Methods**). The location and
143 types of binding sites are different for the different loci and are inferred via a machine learning
144 procedure based on the PRISMR method, which takes as input only Hi-C data^{28,35}. The model
145 of the HCT116 locus has four binding site types and the model of the IMR90 locus has seven
146 types, each visually represented by a different color along the chain (**Fig. 1d, Suppl. Fig. 1c**).
147 The binding site types have been shown to correlate with different combinations of chromatin
148 architectural factors, such as CTCF/Cohesin, H3K4me3 or H3K4me1³⁵. As mentioned above,
149 the equilibrium 3D conformations of the SBS model fall in structural classes corresponding to
150 its thermodynamics phases⁶²: upon increasing binder concentration or affinity above a
151 threshold value, the system undergoes a phase transition from a coil (i.e., randomly folded)
152 to a polymer phase-separated state where distinct, compact globules self-assemble along the
153 chain in correspondence of its different, prevailing binding domains (i.e., locally enriched
154 colors)³⁵. The intrinsic thermodynamic degeneracy of the states of the model results in a
155 broad variety of 3D single-molecule conformations³⁵ (**Methods**). We also developed a variant
156 of the SBS model where cognate DNA sites have direct physical interactions, rather than
157 mediated by binders, and our overall findings remain unchanged (**Suppl. Fig. 2, Methods**) as
158 expected from Statistical Mechanics⁶².

159
160 Finally, to check whether active mechanisms, such as loop-extrusion, and passive
161 mechanisms, such as thermodynamic polymer phase-separation, could coexist to shape
162 chromatin architecture in the studied loci, we implemented a polymer model combining the
163 above described eLE and SBS models, i.e., a model where both mechanisms act
164 simultaneously in each single molecule (named the LE+SBS model, **Fig. 1e, Suppl. Fig. 1d** and
165 **Methods**). For each of the considered models, an ensemble of 3D conformations was
166 obtained via Molecular Dynamics simulations in the steady state^{29,35} (**Methods**). In all the
167 considered cases the model unit length scale was mapped into physical units by equating the
168 median gyration radius to its corresponding experimental counterpart^{6,35} (**Suppl. Fig. 3,**
169 **Methods**).

170 **Both loop-extrusion and phase-separation based models recapitulate average microscopy**
171 **and Hi-C data**

172 To benchmark the different models, we focused first on how they recapitulate population-
173 averaged experimental data by comparing their median distance and contact maps against,
174 respectively, multiplexed FISH⁶ and bulk Hi-C data^{15,60}.

175
176 In our IMR90 case study locus, we found that the models all capture the global patterns visible
177 in the median distance matrix⁶ (**Fig. 2a, Methods**). To have a quantitative measure of
178 similarity, we computed the genomic distance-corrected Pearson correlation coefficient, r' ,
179 between model and experiment. The LE has the lowest r' ($r'=0.19$), while the eLE has $r'=0.49$,
180 highlighting a markedly improved similarity to the experiment. The data appear to be better
181 captured by the SBS and by the LE+SBS models, as signaled by their higher correlations
182 ($r'=0.77$ and $r'=0.70$, respectively). Analogous results are found by comparing the model
183 contact matrices against Hi-C data¹⁵ (**Fig. 2b, Methods**): LE has the lowest correlation
184 ($r'=0.24$), eLE has $r'=0.57$, while SBS and LE+SBS models comparatively better reproduce Hi-C
185 contact patterns ($r'=0.74$ and $r'=0.72$, respectively). We also considered other measures of
186 similarity, such as the simple Pearson correlation (**Suppl. Table I**), which provided analogous
187 results.

188
189 Next, we focused on the relative distances of specific, interesting pairs of sites in the IMR90
190 locus (**Suppl. Table II**). We considered: (i) a pair of sites (green, **Fig. 2c** and **Suppl. Table II**)
191 located 0.3Mb apart from each other within the same TAD, having a strong interaction; (ii) a
192 pair of sites (red), located 0.7Mb away in different sub-TADs, having a strong loop contact in
193 the median distance matrix; and (iii) a pair of 1.1Mb distant sites (yellow) from different TADs,
194 separated by a strong TAD boundary. Albeit the genomic separation of the red pair is twice
195 as large than the separation of the green, those pairs have a similar average distance in the
196 experiment, close to 400nm, whereas the boundary separated yellow pair is more than
197 800nm apart (**Fig. 2c, Suppl. Table II**). We found that the different models all recapitulate
198 those values (**Fig. 2c, Suppl. Table II**) and, interestingly, the LE+SBS model is overall the closest
199 to the experiment across those specific pairs of sites. Additionally, we checked that the
200 distance distributions derived from the models are all similar to the corresponding
201 microscopy distance distributions (**Suppl. Fig. 4**). We stress, however, that the specific values
202 of those distances can depend on the minute details of the models, such as the shape of the
203 interaction potential or loop-extruder size, so the agreement could be further improved.

204
205 To assess how well distinct models capture different aspects of chromatin folding, we also
206 computed the probability to find a TAD boundary at a given genomic location and the average
207 separation score⁶ along the locus in single-molecule conformations (**Fig. 2d,e, Methods**). In
208 the IMR90 case study locus, we found that the boundary probability and the boundary
209 strength averaged over all genomic positions are similar across the different models and very
210 close to the experimental values (**Suppl. Fig. 5**). The boundary probability as a function of the
211 genomic coordinates of the locus, however, is better captured by the eLE model, which has
212 the highest Pearson correlation with experimental data ($r=0.83$, **Fig. 2d**), while the LE has the
213 lowest correlation ($r=0.31$). The SBS and LE+SBS models also provide a good fit to the data,
214 having respectively $r=0.63$ and $r=0.65$. We also found that all the models provide a good
215 overall description of the average separation score along the locus (**Fig. 2e**): the LE has the

216 lowest correlation to the corresponding experimental data ($r=0.51$), the eLE has $r=0.74$, the
217 SBS $r=0.79$ and the LE+SBS model $r=0.82$.

218

219 Our analysis of the HCT116 locus returned a very similar picture about the performance of
220 the different models to describe average distance and Hi-C data (**Suppl. Fig. 6a, b**) as well as
221 TAD boundary probabilities and separation scores (**Suppl. Fig. 6c, d and Suppl. Fig. 7**).

222

223 Taken together, our results show that both loop-extrusion and phase-separation based
224 models are consistent with ensemble-averaged microscopy and bulk Hi-C data. While the LE
225 model is only partially effective, the eLE, which incorporates a single-molecule variability of
226 optimized anchor sites, works well in the description of those data and is the best to
227 recapitulate the TAD boundary probability function. However, polymer models including
228 globule phase-separation mechanisms (SBS and LE+SBS) have overall higher correlation
229 values with average microscopy distance and Hi-C contact data, and better capture some local
230 features of chromatin folding, such as the separation score.

231 **The models are overall consistent with chromatin structure at the single-molecule level**

232 To compare how the different models describe chromatin structure at the single-molecule
233 level we took advantage of the mentioned super-resolution microscopy data⁶ and of the
234 ensemble of polymer 3D conformations produced via our computer simulations.

235

236 First, we checked how well each model represents single-cell chromatin conformations by
237 performing an all-against-all comparison of single-molecule imaged and model 3D structures.
238 We used a method^{35,66} whereby each 3D conformation from microscopy data is univocally
239 associated to a corresponding model structure (for each considered type of model) by
240 searching for the least root mean square deviation (RMSD) of their coordinates (**Fig. 3a, Suppl.**
241 **Fig. 10a and Methods**). To test the statistical significance of the association, we compared
242 the RMSD distribution of the best-matching experiment-model pairs against a simple control
243 case where the RMSD distribution is computed between random pairs of imaged structures.
244 We verified that for each of the considered polymer models the RMSD distribution of the
245 best-matching pairs is statistically different from the control in both the IMR90 and HCT116
246 loci (**Fig. 3b and Suppl. Fig. 8a**, two-sided Mann–Whitney test p -value = 0). Quantitatively, in
247 the IMR90 locus we found, consistently across the models, that less than 5% of the former
248 distribution is above the first decile of the control (**Fig. 3c**) and, in particular, the SBS model
249 performs slightly better than the others. The analysis of the models of the HCT116 locus
250 returned similar results (**Suppl. Fig. 8b**). As an additional test, we also considered a more
251 stringent control where the RMSD is computed only between pairs of imaged structures
252 having overall similar distance matrices, i.e., with a corresponding genomic distance-
253 corrected correlation larger than 0.5 (i.e., with $r' > 0.5$, see below), and we found analogous
254 results (**Suppl. Fig. 9**). Hence, the model conformations best matching the experimental
255 structures have a statistically significant RMSD distribution and provide a non-trivial
256 description of chromatin molecules in single cells (**Fig. 3a, Suppl. Fig. 10a**).

257

258 Next, we tested whether the variability of the ensemble of model single-molecule structures
259 reflects the experimentally observed variability of chromatin single-cell conformations⁶. In
260 the IMR90 locus, for example, the distribution of r' correlations between pairs of
261 experimental single structure distance matrices has an average $r'=0.23$ and a variance equal

262 to 0.18 (**Fig. 3d**), showing that while the imaged structures are broadly varying they have also
263 a significant degree of similarity^{6,35}. For each model, we computed the corresponding
264 distribution of r' correlations between all model single-molecule distance matrices and we
265 compared it with the experimental one (**Fig. 3d** and **Methods**). Interestingly, the r'
266 distributions of the different models have all a shape similar to the experiment and a similar
267 variance, yet they have different average values (**Fig. 3d**). The LE and eLE model average r'
268 ($r'=0.06$ and $r'=0.04$ respectively) is significantly lower than the experimental value, showing
269 that their single-molecule structures have a lower degree of similarity with each other than
270 single-cell imaged chromatin conformations. The LE+SBS model has an average $r'=0.14$, while
271 the SBS model has $r'=0.23$, which is equal to the microscopy value (**Fig. 3d**). In fact, the r'
272 distribution of the SBS model is statistically indistinguishable from the experimental
273 distribution (two-sided Mann–Whitney test p value = 0.362), while the other models are
274 statistically different ($p < 0.001$). Additionally, we verified that analogous results are found if
275 the experiment-experiment r' distribution is compared to the distribution of r' correlations
276 between experiment and model single-molecule distance matrices (**Suppl. Fig. 11a**). We
277 stress, again, that those correlation measures can depend on the minute details employed to
278 construct the models and the agreement with the experiment could be further improved.
279 Finally, the analysis of the HCT116 locus returns very similar results to those of the IMR90
280 locus (**Suppl. Fig.s 10b, 11b**).

281
282 In summary, consistent with our findings on bulk data, our single-molecule analyses support
283 the view that the different polymer models all provide a non-trivial description of single-cell
284 chromatin conformations. While both loop-extrusion and phase-separation based models
285 capture the main features of chromatin single-molecules, in the studied loci we find that the
286 latter models better reflect the microscopy observed single-molecule globular structure and
287 variability. In particular, our analysis shows that chromatin structure variability across single
288 cells results from two main distinct, yet concurrent sources: on the one hand from the intrinsic
289 degeneracy of folding that we find in all the considered models, and on the other hand from
290 the differences of anchoring points (or, analogously, binding sites) in single-molecules,
291 representing the epigenetic heterogeneity of single cells.

292 **The models well reproduce microscopy triple contact data**

293 To assess how well the different models capture higher-order contacts, we investigated their
294 predicted average triplet contact probability matrix, which we compared to microscopy data⁶
295 (**Methods**). We focused on triplets formed by six different genomic viewpoints roughly
296 equally spaced along the IMR90 locus that correspond to some main TAD boundaries and
297 loops of the pairwise median distance matrix (**Fig. 4** and **Suppl. Fig. 12**). In our analysis, by
298 definition, a triplet is formed if three genomic sites have all their pairwise distances below a
299 threshold value. The triplet probability depends on such a threshold, but we checked that the
300 measured values are proportionally conserved if the threshold is varied around 150nm in a
301 range from 100 to 200nm (**Methods**).

302
303 Microscopy data reveal that triplets are typically compartmentalized in the studied loci and
304 restricted to the TAD encompassing each of the selected viewpoints (**Fig. 4a-c** and **Suppl. Fig.**
305 **12**), showing that TADs tend to create local environments where also multiple contacts
306 become enriched. The different polymer models do capture experimental triplet patterns
307 across all the considered viewpoints. To quantitatively assess the similarity between

308 experiment and model predicted triplets, we computed the mean relative squared difference
309 (MRSD) between the corresponding entries of the two matrices over the studied viewpoints
310 (**Fig. 4d, Methods**). To set a reference, we also considered the triplets formed in a random
311 control made of self-avoiding-walk (SAW) polymer chains having the same number of beads
312 and gyration radius (i.e., linear size) as the real images of the locus (**Methods**). Our analysis
313 shows that the LE model has an MRSD with the experiment that is one third of the random
314 control value, yet it has the largest discrepancy with the experiment compared to the other
315 considered models, whose MRSD is at least one order of magnitude smaller than the control.
316 Interestingly, the LE+SBS model has the lowest distance from the experiment and its MRSD is
317 statistically different from both the LE, the SBS and control case (**Fig. 4d**, two-sided Welch's t-
318 test $p < 0.001$), whereas it is statistically equal to the eLE MRSD (two-sided Welch's t-test
319 $p = 0.097$).

320
321 Taken together, our results show that both loop-extrusion and phase-separation mechanisms
322 can explain higher-order contacts. However, a model combining both mechanisms (LE+SBS)
323 turns out to have the least discrepancy with microscopy triplet data and overall provides an
324 excellent description of all the different experimental datasets considered, supporting the
325 view that loop-extrusion and phase-separation can co-exist in single-molecules in establishing
326 chromatin architecture.

327 **DISCUSSION**

328 To investigate the physical mechanisms that shape chromatin 3D large scale organization, we
329 explored via Molecular Dynamics simulations two classes of polymer models where folding is
330 based on two distinct physical processes: DNA loop-extrusion and polymer phase-separation,
331 recapitulated respectively by the LE and by the SBS models (**Fig. 1**). We assessed how they
332 perform relative to each other in capturing chromatin bulk Hi-C contact^{15,60} and single-
333 molecule microscopy data⁶ in human IMR90 and HCT116 cells, and we exploited such data to
334 establish whether those mechanisms compete or coexist in single cells.

335
336 We considered, first, a simple loop-extrusion (LE) model²⁰ of those loci (**Fig. 1b**). Next, we
337 introduced an extended version of the LE (named eLE, **Fig. 1c**), where the genomic locations
338 of the extruding motor anchor sites are optimized, independently of CTCF peaks, to best
339 reproduce population-averaged experimental data. Additionally, to mimic epigenetic
340 differences among single cells, each of those anchor sites has a specific probability to be
341 present in a model single molecule²⁹. The probability values returned by the optimization
342 search range from 50% to 100%, consistent with current estimates of cell epigenetic
343 heterogeneity⁶⁵. Interestingly, most anchor sites of the optimal eLE model are found to
344 coincide with CTCF peaks (**Fig. 1c**), but not all, and conversely many CTCF peaks are not taken
345 as anchor sites, hinting that CTCF may be combined with other signals in anchoring loop-
346 extruding motors⁶⁷. Considering the basic ingredients that inform the LE model, we find that
347 it performs well to fit experimental data. Yet, the eLE model better recapitulates average
348 microscopy and Hi-C data and higher order contacts in single-molecules.

349
350 We also considered the SBS model of the studied loci (**Fig. 1d**), i.e., a model where the
351 attraction between cognate binding sites on the polymer chain and their associated binding
352 molecules drives a micro-phase-separation of the chain in distinct globules³⁵. For
353 completeness, we checked that a model with direct interactions between binding sites (rather

354 than mediated by diffusing binders) has behaviors analogous to the SBS. Finally, we
355 introduced a model combining the molecular elements of the eLE and of the SBS (the LE+SBS
356 model) where in a single molecule both the LE and SBS mechanisms act simultaneously (**Fig.**
357 **1e**). We find that the SBS and LE+SBS models explain well bulk Hi-C and single-molecule
358 microscopy data, and reflect the experimentally reported chromatin segregation in globules
359 and its cell-to-cell structural variability more accurately than the LE or eLE models.

360
361 Importantly, a further optimization of the model fine details, such as the employed specific
362 interaction potentials (shape, depth, distance of the potential minimum, etc.) or the specific
363 nature of the modelled DNA extruding motors (size, speed, directionality, etc.), can on one
364 hand improve even more the model agreement with experiments and on the other hand
365 provide additional mechanistic information. Nevertheless, the models here investigated
366 perform well considering their simplicity (**Fig.s 2-4**). In particular, the LE+SBS model returns
367 an overall excellent description of the different datasets and the least discrepancy with
368 microscopy triplet data, showing that loop-extrusion and phase-separation can co-exist in
369 shaping the complex chromatin architecture of the studied loci. Our analyses also illustrate
370 that the experimentally observed structural variability of chromatin in single-cells is
371 consistent with two main co-existing sources of noise, i.e., the heterogeneity of single-cell
372 epigenetics and, interestingly, an intrinsic conformational degeneracy, as chromatin can
373 dynamically fold in many different conformations rather than in a single naïve structure as
374 usual proteins.

375
376 While other folding mechanisms are likely to contribute to the organisation of the genome
377 (such as heterochromatin adsorption to the lamina), one can speculate on why different
378 molecular processes could cooperate in determining chromatin folding. Beyond ensuring
379 redundancy in regulation, they appear to be more effective in implementing complementary
380 tasks. For instance, loop-extrusion is particularly suited to establish TAD borders and
381 pointwise strong loop interactions, whereas globule phase separation can better act to
382 segregate different regions and to form more stable (i.e., with lower variability) and hence
383 more reproducible regulatory structures. Additionally, while loop-extrusion requires energy
384 consumption, phase transitions are sustained by the thermal bath, and they are robust and
385 reversible processes as the system only needs, e.g., to set an above threshold concentration
386 (or affinity) of binders, with no need of fine tuning their number (or strength).

387

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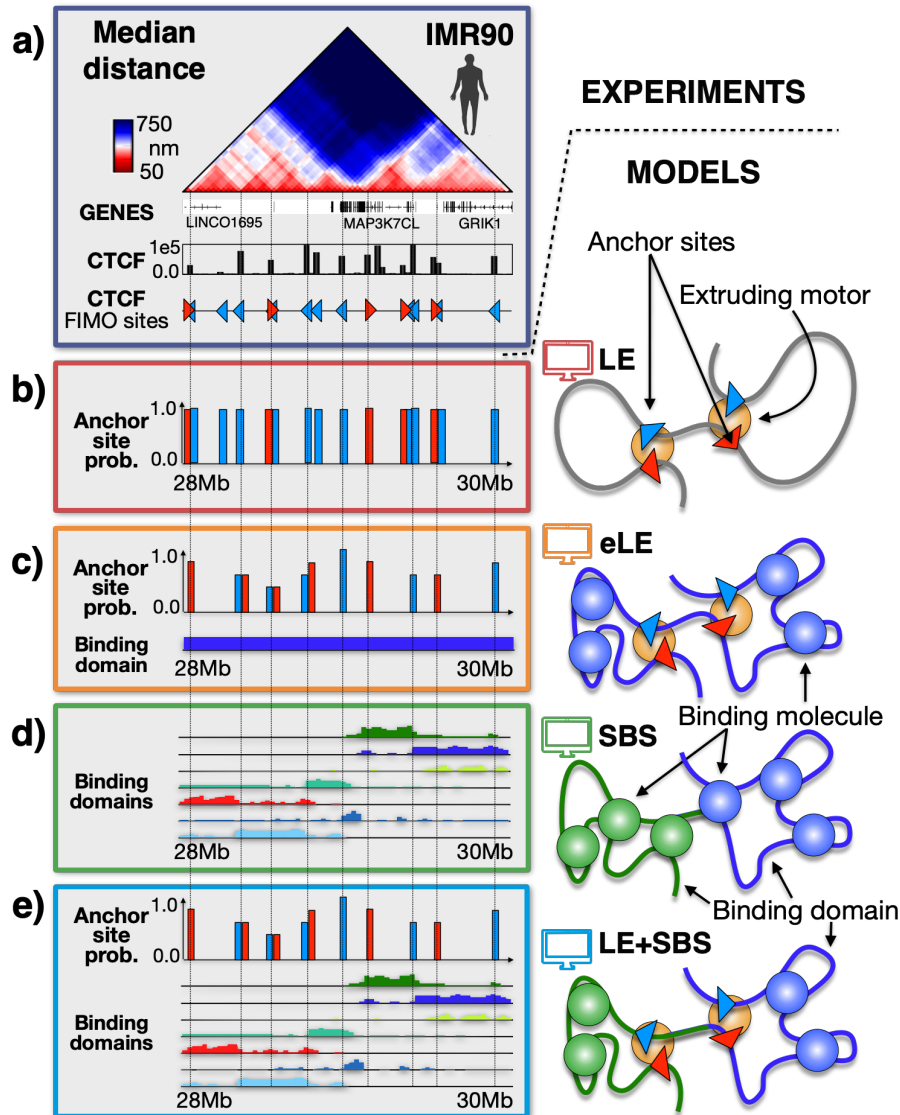
560 **ACKNOWLEDGEMENTS**

561 M.N. acknowledges support from the National Institutes of Health Common Fund 4D
562 Nucleome Program grant 1U54DK107977-01 and 1UM1HG011585-01, EU H2020 Marie Curie
563 ITN n.813282, CINECA ISCRA ID HP10CYFPS5 and HP10CRTY8P, Einstein BIH Fellowship Award
564 (EVF-BIH-2016 and 2019), Regione Campania SATIN Project 2018-2020, and computer
565 resources from INFN, CINECA, ENEA CRESCO/ENEAGRID (Iannone et al. 2019) and
566 *Scope/ReCAS/Ibisco* at the University of Naples.

567 **AUTHOR CONTRIBUTIONS**

568 M.N., M.C and E.I. designed the project. M.N., M.C., E.I. and A.M.C. developed the modelling.
569 M.C., E.I. and A.A. ran computer simulations and performed data analyses with help from
570 A.M.C., S.B. and A.E.. M.N. and M.C. wrote the manuscript with input from all the authors.

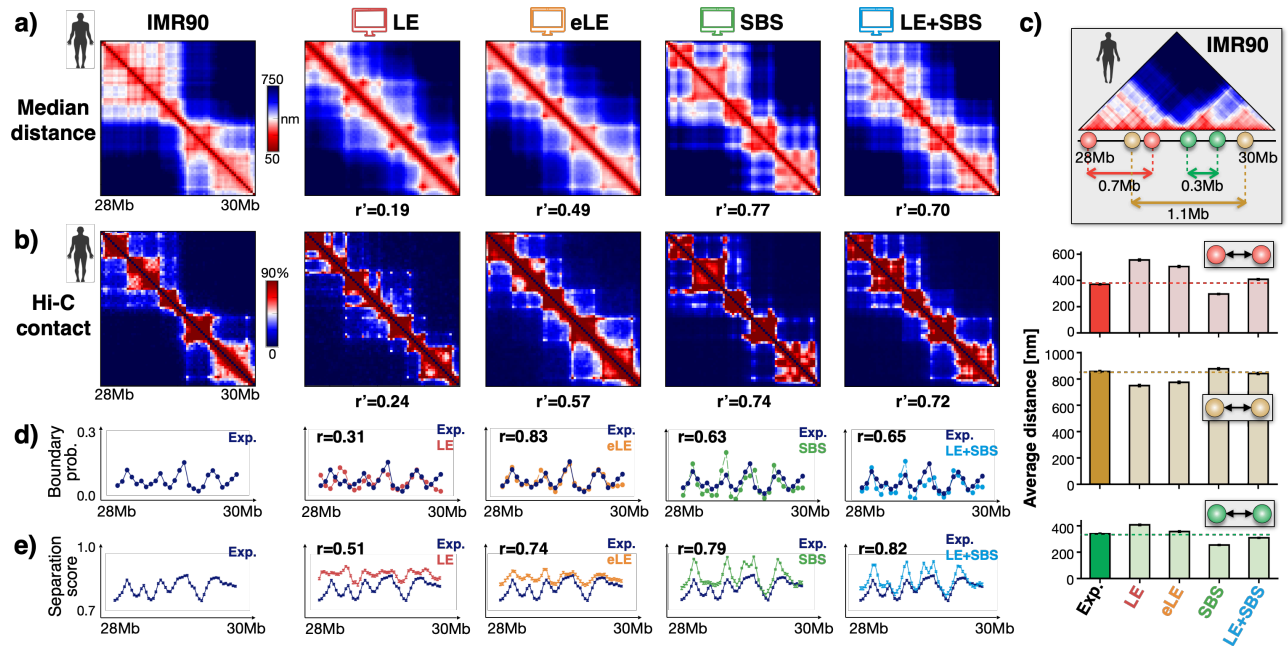
571 FIGURE CAPTIONS



572 **Figure 1**

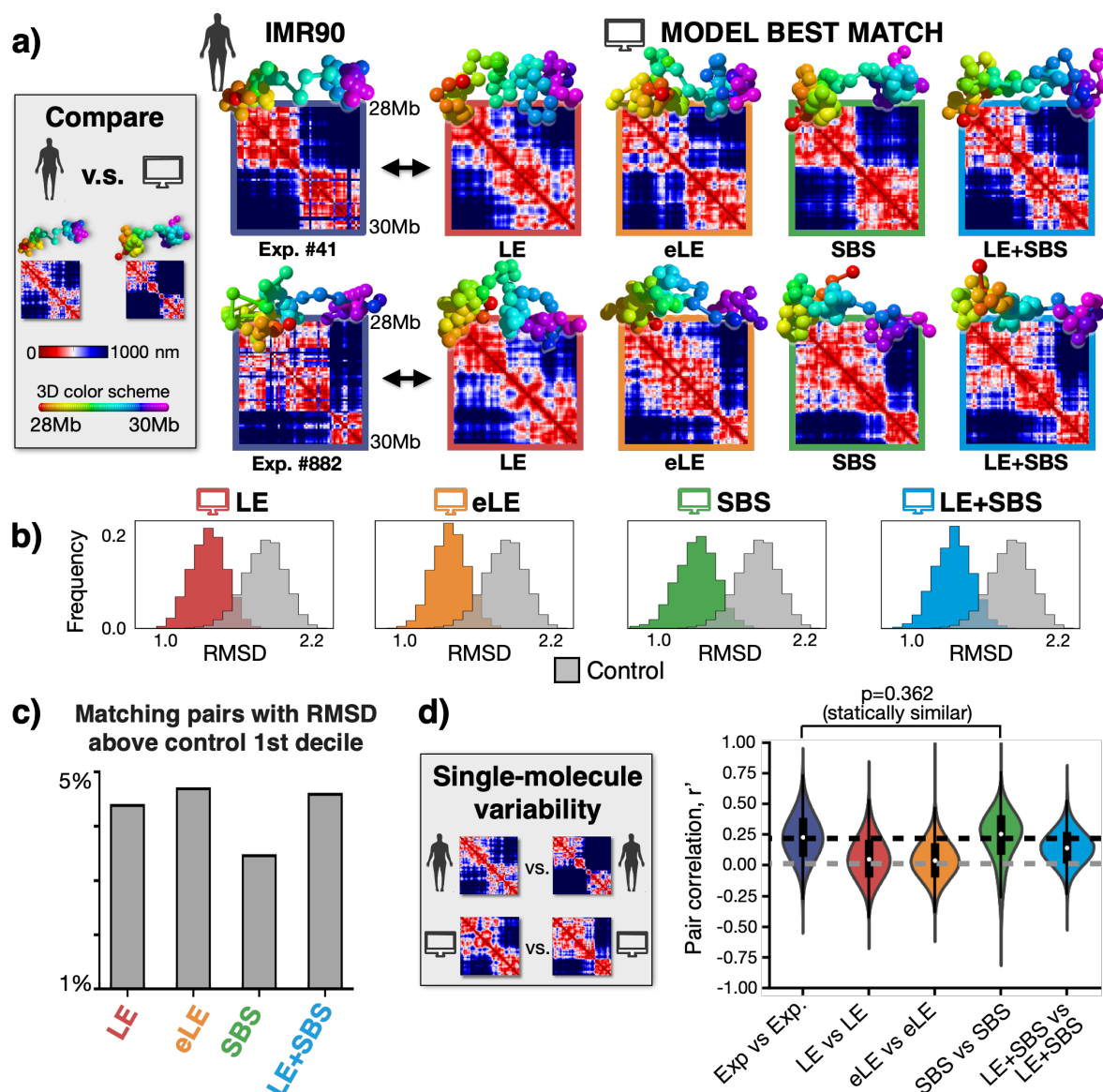
573 **Scheme of the investigated polymer models.** We used Molecular Dynamics simulations to
 574 investigate polymer models where folding is based on two different physical processes: (i)
 575 DNA loop-extrusion and (ii) polymer phase-separation, recapitulated respectively by the
 576 LE^{19,20} and by the SBS models^{42,44}. **a)** Microscopy median distance⁶ and ENCODE⁶⁴ CTCF data
 577 are shown for the studied 2Mb wide locus in human IMR90 cells. **b)** We considered a simple
 578 Loop-Extrusion (LE) model²⁰ where active motors extrude polymer loops until encountering
 579 another motor or CTCF anchor points with opposite orientation, which are fixed and equal in
 580 all single-molecule simulations. **c)** We also considered an extended version of the LE (eLE)
 581 whose anchor site locations are optimized, independently of CTCF, to best reproduce Hi-C and
 582 average microscopy data. To represent the epigenetic heterogeneity of single cells, those

583 anchor sites have a finite probability to be present in a model single molecule²⁹. **d)** In the
584 Strings and Binders (SBS) model³⁵ a chromatin filament is represented as a self-avoiding chain
585 of beads including different types of binding sites (colors) for diffusing cognate binders that
586 can bridge those sites, hence driving a micro-phase-separation of the chain in distinct
587 globules. The binding site locations are determined by the PRISMR method and correlate with
588 different combinations of epigenetic factors including, but not limited to, CTCF and
589 cohesin^{28,35}. **e)** We also considered a polymer model (LE+SBS) where in a single molecule both
590 the eLE and SBS mechanisms act simultaneously.
591



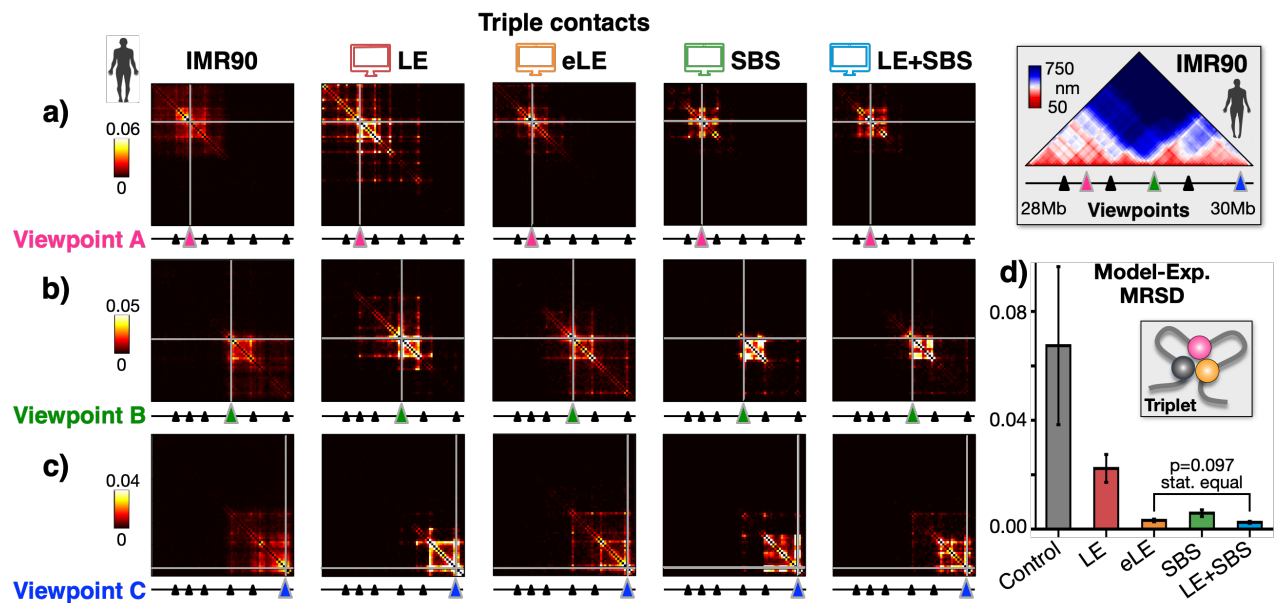
592 **Figure 2**

593 **Both loop-extrusion and phase-separation based models recapitulate bulk Hi-C and average**
 594 **microscopy data.** **a)** In-silico median distance and **b)** average contact data are compared to
 595 microscopy⁶ and Hi-C¹⁵ data (left) in the IMR90 locus. The different models have high genomic
 596 distance-corrected Pearson correlations (r') with the experiments, particularly the eLE, SBS
 597 and LE+SBS models. **c)** The model derived average distances are reported for specific pairs of
 598 sites separated by TAD boundaries (yellow) or connected in loops within a TAD (green) or
 599 across a TAD boundary (red). **d)** The average single-molecule genomic boundary probability
 600 and **e)** the separation score are also well recapitulated by the models.
 601



602 **Figure 3**

603 **Single-cell chromatin conformations are well captured by the models, especially by phase-**
 604 **separation based ones.** **a)** Microscopy single-cell chromatin structures of the IMR90 locus⁶
 605 (left) are associated to a best matching single-molecule conformation in each model via the
 606 minimum RMSD criterion. Here two examples are shown. **b)** For each of the considered
 607 polymer models, the RMSD distribution of the best-matching experiment-model pairs is
 608 statistically different from a control RMSD distribution made of random pairs of experimental
 609 structures (two-sided Mann–Whitney test p -value = 0). **c)** Less than 5% of best matching pairs
 610 have an RMSD above the 1st decile of the control distribution. **d)** The variability of microscopy
 611 single-molecule structures is measured by the distribution of r' correlations between pairs of
 612 distance matrices and is compared to the variability of in-silico structures. The r' distribution
 613 of the SBS model is statistically indistinguishable from the experimental one (two-sided
 614 Mann–Whitney test p -value = 0.362).
 615



616 **Figure 4**

617 **Triple contact data are well described by the models, especially by the eLE and the LE+SBS.**
618 Triple contact probability maps are shown in microscopy data⁶ (left) and in the models from
619 three different viewpoints (**a**), **b**), **c**), more viewpoints in **Suppl. Fig. 12**). **d**) The mean relative
620 squared difference (MRSD) between imaging and model triplet contact maps is the lowest in
621 the LE+SBS model, which is statistically equivalent to the eLE model (two-sided Welch's t-test
622 $p=0.097$). Their MRSDs are, instead, statistically different from both the LE, the SBS and the
623 control (two-sided Welch's t-test $p<0.001$). The control is made of randomly folded self-
624 avoiding polymer chains with same number of beads and size than the experimental
625 structures.
626