Evolutionary learning in the brain by heterosynaptic plasticity

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Abstract

How the brain modifies synapses to improve the performance of complicated networks remains one of the biggest mysteries in neuroscience. Existing proposals, which suppose continuous-valued synaptic weights changed according to gradient clues informed by pre- and post-synaptic activities, lack sufficient experimental support. Based on the heterosynaptic plasticity between hippocampal or cortical pyramidal neurons mediated by diffusive nitric oxide and astrocyte calcium wave as well as the flexible dendritic gating of somatostatin interneurons, here we propose that the brain learns by an evolutionary algorithm (EA), which trains biologically plausible binary-weight networks in a gradient-free manner. Our EA provides a framework to re-interpret the biological functions of dopamine, meta-plasticity of dendritic spines, memory replay, and the cooperative plasticity between the synapses within a dendritic neighborhood from a new and coherent aspect. Our EA can train neural networks to exhibit dynamics analogous to brain dynamics in cognitive tasks. Our EA manifests its broad applicability to train spiking or analog neural networks with recurrent or feedforward architecture. Our EA also demonstrates its powerful capability to train deep networks with biologically plausible binary weights in MNIST classification and Atari-game playing tasks up to performance comparable with continuous-weight networks trained by gradient-based methods. Overall, our work leads to a paradigmatically fresh understanding of the brain learning mechanism.
Introduction

Understanding the cellular-level learning mechanisms underlying brain intelligence is a grand task of neuroscience. A large body of literature has been dedicated to biologically plausible mechanisms to train neural networks. Early works have proposed Hebbian-types rules, such as spiking-time-dependent plasticity or Bienenstock–Cooper–Munro rules, according to direct experimental evidence [1]. Gradient-based algorithms, which have achieved great success in training artificial neural networks, may be biologically realized by back-propagating error signals [2, 3] or training targets [4, 5] through feedback connections, or by exploiting the correlation between the perturbed fluctuation of neuronal activities with a broadcasted scalar reward [6, 7, 8]. Recently, it is also suggested that evolution may endow each synapse with a specific set of parameters of Hebbian learning, so that the brain can learn in a manner that different synapses change in different Hebbian-type rules [9, 10].

However, the proposed learning mechanisms suffer from various shortcomings [11]. Traditional Hebbian-type rules, despite their good performance in training shallow networks [12, 13], cannot well train deep networks due to their incapability to coordinate the weight updating in different layers [14]. The proposal that error signals are back-propagated through feedback connections has not received convincing support from experiments [11]. This proposal is also hard to be compatible with the predictive-coding theory of perception [15] supported by some experiments [16, 17], which states that it is the feedforward (instead of feedback) connections that pass the error signal and the feedback connections work like a generator to reconstruct the lower-layer representations. The target propagation approach [4, 5], though consistent with the predictive-coding theory of perception [15], is biologically problematic to train recurrent neural networks [18]. The node perturbation approach [6, 7, 8] has never successfully trained large deep networks for difficult problems such as classifying natural images [11], possibly due to its sensitivity to the inaccuracy of mean subtractions of the covariant reward and neuronal activities [19]. The proposal that each synapse has an evolution-endowed specific Hebbian learning rule [9, 10] requires infeasibly huge information of over 100 trillion synapses in the brain [20] coded in DNA. Besides, biological synapses take binary (instead of continuous) weights [21], which is not addressed by the algorithms above.

The difficulties of these algorithms may root in their common assumption of locality (i.e., the change of a synapse only depends on its pre- and post-synaptic activities, or at most up to a globally available reward signal) and deep affection by gradient-based methodology. Computationally, gradient-based methods are not the only way in training neural networks [22, 23], leaving the biological implementation of many other approaches to be explored. Biologically, the locality assumption neglects the ubiquitous inter-cellular signaling pathways in the brain, rendering the
proposed cellular-level learning theory unlikely to be complete. For example, it is well known that the diffusive nitric oxide (NO) associated with a potentiated synapse can potentiate a neighboring synapse [24, 25]; the pre-synaptic release of glutamate can stimulate long-range propagating calcium waves in astrocytes, depressing synapses in a broader spatial range [26, 27]. The computational roles of such heterosynaptic plasticity remain unclear. It has been proposed that heterosynaptic plasticity may stabilize the run-away dynamics introduced by Hebbian-type rules and enhance synaptic competition [28, 29], which seems, though important, auxiliary for learning. It is unknown whether heterosynaptic plasticity implies a standalone learning rule, and if it does, how its performances are in complex tasks.

Here, we unify a broad spectrum of cellular-level experimental evidence (including heterosynaptic potentiation mediated by NO [24, 25], heterosynaptic depression mediated by astrocyte calcium wave [26, 27], dendritic gating [30], binary synapses [21], dopamine gating of synaptic plasticity [31], meta-plasticity [32, 33], memory replay [34], cooperative plasticity between the synapses within a dendritic neighborhood [35, 36, 37], etc.) into a coherent picture, and show that the heterosynaptic plasticity of the synapses between hippocampal or cortical pyramidal neurons implies an evolutionary algorithm (EA) [22, 23]. Our EA trains recurrent neural networks to exhibit dynamics analogous to brain dynamics in experimental observation [38]. Our EA has broad applicability; it can train both spiking and analog neural networks with both feedforward and recurrent network architectures. Our EA also has powerful capability; it can train deep networks with biologically plausible binary weights in MNIST classification and Atari-game playing tasks up to performance comparable with continuous-weight networks trained by gradient-based algorithms. Therefore, EA is a biologically-supported, broadly applicable and powerful brain learning paradigm.

**Results**

**Network architecture**

In traditional computational models of neuroscience, two neurons are connected by a single connection (Figure 1b). This contrasts with the reality that in the brain two pyramidal neurons are connected by synapses on multiple dendrites (Figure 1c) [39]. These dendritic routes can be independently gated by dendrite-inhibiting somatostatin (SST) interneurons [30], which are controlled by a hub area, e.g., prefrontal cortex (PFC) or mediodorsal thalamus (MD) [40], through long-distance corticocortical or thalamocortical connections to vasoactive intestinal peptide (VIP) interneurons that inhibit SST interneurons [41, 42]. Parvalbumin (PV) interneurons inhibit pyramidal neurons near the soma [30].
Figure 1: The network architecture. (a) Schematic of the evolutionary algorithm. Each generation of agents (squares) are procreated by multiple parental agents (yellow squares) in the previous generation with high fitness (black arrows and brackets). The one with the highest fitness (red circle) is the elite and becomes an agent of the next generation without any mutation (blue arrows and circles), a technique called elitism. (b) Traditional models suppose that two neurons are connected by a single connection. (c) Our model supposes that two pyramidal neurons are connected through multiple dendritic routes, which are gated by MD or PFC (network A) through SST and VIP interneurons. Only synapses (red) between pyramidal neurons are plastic in our model, all the other synapses to and from PV, SST and VIP interneurons remain fixed. (d) Dendritic routes labeled $k$ ($k = 1, 2, 3$) in network $N$ are gated by neurons labeled $k$ in network $A$. Dopamine, which represents the evaluation of the output of network $N$, are released dispersedly to network $N$. (e) Neurons in network $A$ are activated alternately one at a time. At time $t_k$ ($k = 1, 2, 3$), only neuron $k$ in network $A$ is activated (white square) with others silent (black squares), so that the dendritic routes labeled $k$ in network $N$ are gated on (white circles) with others gated off (black circles). The sub-network connected by all the $k$th dendritic routes in network $N$ is regarded as the $k$th agent in evolutionary algorithm (lower subplots). The 3rd agent stimulates a high level of dopamine (red text in the upper-right subplot), is therefore the parental agent; the other two agents stimulate a low level of dopamine, are therefore non-parental agents. (f) The plasticity rule. The synaptic efficacy $w_{ji}^{\text{non-parental}}$ between the $j$th and $i$th pyramidal neurons in a non-parental agent becomes the synaptic efficacy between the same pair of pyramidal neurons in a parental agent plus a small value $\epsilon$. 

$w_{ji}^{\text{non-parental}} \leftarrow w_{ji}^{\text{parental}} + \epsilon$
This neural substance provides a natural basis for implementing EA. EA is an optimization algorithm based on the evolution of a population of agents [22, 23] (Figure 1a). Each generation of agents are procreated by the parental agents (i.e., the agents with high fitness, or in other words, better task performance) in the previous generation through mutation or cross-over; elite agents (i.e., the agents with the highest fitness) become agents in the next generation without mutation, a technique called elitism.

The implementation of EA in brain learning is illustrated in Figure 1d, e, where we suppose that the dendrites of pyramidal neurons in a network \( N \) representing hippocampus or neocortex are gated by a network \( A \) representing PFC or MD. If neuron \( k \) in network \( A \) is active, all the dendrites labeled \( k \) in network \( N \) are gated on, with the others gated off. Neurons in network \( A \) are alternatively activated one at a time, such that dendrites with different labels are gated-on alternatively (Figure 1e, upper subplots). With different dendrites gated on, the network \( N \) gives different outputs in response to the same stimulus, leading to different rewards, either from the real world or estimated by the brain [43]. These reward signals stimulate different levels of neuromodulators, such as dopamine and serotonin, informed dispersedly to the whole network \( N \) to guide synaptic plasticity [31, 44, 45] (Figure 1d). The sub-network of the network \( N \) connected by the dendrites labeled \( k \) is regarded as the \( k \)th agent in EA. Dopamine level stimulated by an agent indicates the fitness of that agents, such that high (or low) dopamine level indicates parental (or non-parental) agents (Figure 1e, lower subplots).

In the next subsection, we will show that the heterosynaptic plasticity between pyramidal neurons is equivalent to a learning rule in which the synapse \( w_{ij}^{\text{non-parental}} \) between the \( j \)th and \( i \)th pyramidal neurons on a non-parental dendrite becomes approximately the synapse \( w_{ij}^{\text{parental}} \) on a parental dendrite (Figure 1f). In this way, non-parental agents are updated to be descendants of the parental agent with some mutations (represented by \( \epsilon \) in Figure 1f) caused by the stochastic nature of the nervous system [46, 47]. All the synapses to and from inhibitory interneurons are kept fixed in our model (Figure 1c). Note that we only consider mutation in this EA, and do not consider cross-over, which however can easily be included for further research (see Discussion).

**Basic learning process**

We consider a delayed reward process widely in nature or the laboratory: neuronal activities result in eligibility traces in synapses, according to which sparse and delayed rewards then guide synaptic plasticity. Eligibility traces are molecular states in synapses, which can change synaptic efficacies under a high dopamine level, but have no effect under a low dopamine level [48]. The above plasticity rule (Figure 1f) can be realized by heterosynaptic plasticity.
through the following cellular-level mechanisms. To illustrate the mechanisms, we consider two pre-synaptic neurons collected to one post-synaptic neuron through three dendrites \((a, b, c)\) with different synapse efficacies (represented by the boutons of different sizes in Figure 2); at a certain time step, the \(c\) dendrites is gated on (see Figure 2).

(1) If the synapse on a gated-on dendrite has large efficacy (e.g., synapse \(c_1\) from neuron 1 in Figure 2a), then a pre-synaptic spike has a high probability of stimulating a post-synaptic spike. This pairing of pre- and post-synaptic spikes produces diffusive messengers such as nitric oxide (NO) [49, 24], which generates potentiation eligibility traces in neighboring synapses (synapses \(a_1\) and \(b_1\) in Figure 2a) on gated-off dendrites in the condition of the simultaneously arriving spikes from neuron 1 [25, 24, 50]. This heterosynaptic potentiation is subject to a spatial scope comparable with the scale of the dendritic arbor of a pyramidal neuron \((\sim 150 \mu m\) in hippocampal CA1 [25, 24]). Synapses from neuron 2 in Figure 2a (i.e., synapses \(a_2, b_2\) and \(c_2\)) cannot be potentiated by the diffusive NO without simultaneous pre-synaptic spikes.

(2) If the synapse on a gated-on dendrite has small efficacy (e.g., synapse \(c_2\) in Figure 2b), then a pre-synaptic spike has a low probability of stimulating a post-synaptic spike, such that the generated potentiation eligibility traces are low in the neighboring synapses sharing the same pre-synaptic neurons (i.e., synapses \(a_2\) and \(b_2\)).

(3) Pre-synaptic release of glutamate stimulates astrocytes to spread calcium waves [51, 27], which induces depression eligibility traces in synapses spatially widespread [52, 53, 26] (at least 300–500 \(\mu m\) in hippocampal CA1 [54, 26]), see Figure 2a, b.

The mechanisms above are best established in the hippocampus [28] (also see the references above), and have also been observed in the neocortex [29, 50, 53, 55, 56]. Collectively, if a synapse on a gated-on dendrite has high (or low) efficacy, it will induce high (or low) potentiation eligibility traces in the neighboring synapses from the same pre-synaptic neuron on gated-off dendrites; the depression eligibility traces on different synapses are similar (Figure 2c).

If the delayed reward is small so that the dopamine level is low (i.e., the gated-on dendrite is non-parental in EA, see Figure 1e), no synaptic change will happen. If the delayed reward is large so that the dopamine level is high (i.e., the gated-on dendrite is parental, see Figure 1e), the synapses will be updated [31] according to the following rules (Figure 2d): synapses with large potentiation eligibility traces will be potentiated, while synapses with small potentiation eligibility traces will be depressed by the depression eligibility trace. As biological synapses take binary efficacies [21], potentiation (or depression) only works in synapses with low (or high) efficacies, whereas those already with high (or low) efficacies remain unchanged (Figure 2d). As a result, synapses on non-parental dendrites
are updated towards those on parental dendrites connecting the same pair of pre- and post-synaptic neurons (i.e., synapses $a_1$ and $b_1$ are updated towards $c_1$, whereas $a_2$ and $b_2$ towards $c_2$), while those on parental dendrites (i.e., synapses $c_1$ and $c_2$) remain unchanged (Figure 2d), fulfilling the requirement of the EA (Figure 1f).

Generally speaking, the above arguments still hold if we regard each neuron in the model as a population of closely neighboring pyramidal neurons. First, closely neighboring pyramidal neurons have high firing synchrony [57], so the synchronous pre-synaptic firing required for potentiation still holds if neuron 1 in Figure 2a represents a population of closely neighboring neurons. Second, both the heterosynaptic potentiation and depression mechanisms (Figure 2a, b) are mediated by inter-cellular pathways, therefore still hold for synapses on different post-synaptic neurons spatially nearby [50, 25, 26]. Third, the synapses from a population of neighboring pyramidal neurons with high synchrony spatially cluster on the same compartment of a post-synaptic dendrite [58] with correlated synaptic efficacies [59, 36, 37], therefore can be considered as a big synapse taking binary values.

**Multiple parents and elitism**

In EA of computer science, each generation usually has multiple parents and at least one elite [22, 23] (Figure 1a). The plasticity process illustrated in Figure 2, however, corresponds to the case that a generation has only one parent, because the agents at any time step are the descendants of a single parent (Figure 3a). Here we show that both multiple parents and elitism can be implemented in our model by introducing a meta-plasticity mechanism. In this mechanism, if a synapse with large (or small) efficacy on a gate-on dendrite receives potentiation (or depression) signals under high dopamine level, the efficacy of this synapse will get stabler at the large (or small) level, harder to be switched in future learning processes (Figure 3b). Under this mechanism, if agent $k$ is parental, the synapses on the dendrites with label $k$ will get stabilized, so that if another agent $m$ induces a high dopamine level afterward (i.e., agent $m$ is also parental), the synapses with label $k$ will remain unchanged, instead of being updated towards the synapses with label $m$. Therefore, if agent $k$ is activated again later, a high dopamine level may still be induced, so that non-parental agents (whose synapses are not stabilized) can still be updated toward agent $k$, becoming descendants of agent $k$ (Figure 3d). In this way, the agents at any time step are the descendants of multiple parents. Despite the stability, agent $k$ may also gradually lose its synaptic configuration after a sufficiently long time. The parental agent that induces the highest dopamine level (i.e., the elite) has the highest stability, and therefore will be maintained for the longest time, fulfilling the elitism technique in EA (Figure 3d).

Physiologically, it is well known that memory formation, which implies stabilized synaptic changes, requires in-
Figure 2: Heterosynaptic plasticity induces EA. (a) When neuron 1 emits a spike, the large synapse $c_1$ on the gated-on dendrite stimulates a post-synaptic spike; this pairing of pre- and post-synaptic spikes produces diffusive NO, which, together with the spikes simultaneously invading synapses $a_1$ and $b_1$, induces potentiation eligibility traces in these synapses. (b) When neuron 2 emits a spike, the small synapse $c_2$ on the gated-on dendrite does not stimulate a post-synaptic spike, therefore no potentiation eligibility traces in synapses $a_2$ and $b_2$. In both panels a and b, pre-synaptic release of glutamate stimulates astrocytes to produce widely spreading calcium waves, which induce depression eligibility traces in all synapses. (c) The synapses targeted by neuron 1 have higher potentiation eligibility traces than those targeted by neuron 2, but the depression eligibility traces in these synapses are similar. (d) With a low dopamine level, no synaptic change happens. With a high dopamine level, the synapses $a_1, b_1, c_1$ (or $a_2, b_2, c_2$) with high (or low) potentiation eligibility traces tend to be potentiated (or depressed), see upper (or lower) red arrows. Synapses $a_1, c_1$ (or $b_2, c_2$) remain unchanged because they already have large (or small) efficacy.
Figure 3: Multiple parents and elitism. (a) The learning process in which each generation has a single parent and no elitism. The initial condition contains one parental agent (red square) that can induce a high dopamine level and five non-parental agents (white squares). In step 1, the red parental agents are activated (black arrow), so that all non-parental agents are updated towards the red agent, becoming its descendants (red crosses). In step 2, the activated agent cannot induce a high dopamine level, so this agent remains non-parental. In step 3, the activated agent induces a high dopamine level, so all the other agents (including the red parental agent) are updated toward the activated agent, becoming its descendants (blue crosses). (b) The synaptic stabilization mechanism. If a large (upper panel) or small (lower panel) synapse on a gated-on dendrite (white circle) receives potentiation or depression signal (upward or downward arrow) at a high dopamine level, then the synapse is stabilized at large or small efficacy. (c) The stabilization of small synapses can also be realized by stabilizing large synapses (upper panel) together with dendrite-level synaptic homeostasis (lower panel). (d) How multiple parents and elitism are fulfilled using the stabilization mechanism in panels b and c. Notice that the parental agents (red, blue, yellow and black squares) are stabilized, so that in steps 1, 3 and 4, these parental agents do not update toward the activated agent that stimulates high dopamine level. After many steps (last row), parental agents may also lose their synaptic configurations (the question marks mean we do not know the states of those agents); but the elite agent (black square), which can induce the highest dopamine level, is the stabllest one, and can last for the longest time.
hibiting SST or parvalbumin (PV) interneurons to increase the excitability of pyramidal neurons [60, 61], and that dopamine stabilizes potentiated synapses [32], which are the basis of the stabilization mechanism of large synapses (upper subplot of Figure 3b). The stabilization of small synapses (lower subplot of Figure 3b) may be related to a heterosynaptic meta-plasticity process: if a synapse is potentiated, a nearby synapse will become harder to get potentiated [62, 63], probably due to an extra-cellular mechanism related to astrocytes [33], which is similar to heterosynaptic depression (Figure 2a, b). However, the modulations of this meta-plasticity by SST interneurons and dopamine level remain to be experimentally tested.

A better established process to fulfill the stabilization of small synapses (lower subplot of Figure 3b) is dendrite-level synaptic homeostasis [64, 65], which constrains the total synaptic efficacy on a dendrite (Figure 3c). With this constraint, small-efficacy synapses are hard to be potentiated if other large-efficacy synapses on the same dendrite are stabilized, realizing the stabilization of small-efficacy synapses. If we consider a single neuron model in Figures 1 and 2 as a population of closely neighboring biological pyramidal neurons (as we discussed in the previous subsection), and the information route is gated at the neuron level (instead of the dendrite level), neuron-level synaptic homeostasis [66, 67, 68] will also fulfill this task.

**EA is compatible with off-line replay**

According to the no-free-lunch theorem [69], no single learning algorithm is advantageous in every problem. Therefore, it is necessary for the brain to combine a variety of learning strategies for the best living of the animal, and it is also necessary for us to understand the suitable scenarios for EA, thereby getting insight into the functional roles of EA. Here we show that EA is compatible with off-line replay.

Consider the following on-line learning scenario which possibly happens in a waking animal (Scenario 1, Figure 4a). In a learning session, the performance of a gated-on dendritic route in response to an input stimulus is scored by the reward feedback from the real world, inducing corresponding dopamine level; learning sessions may not be immediately successive, but are separated by intervals of variable durations. This scenario is not compatible with EA. To see this, consider a situation where dendritic route 1 brings 10 scores of reward in response to stimulus 1, and brings 0 scores in response to stimulus 2; dendritic route 2 brings 6 scores of reward in response to both stimuli 1 and 2 (Figure 4c). If high dopamine level is induced only in the session corresponding to the highest reward (the session with stimulus 1 and dendritic route 1 gated-on), the synapses on dendritic route 2 will update towards those on dendritic route 1 (Figures 1f and 2). In other words, dendritic route 1 is the parental route in the on-line learning
**Figure 4:** EA is compatible with off-line replay. (a) During an on-line learning scenario, the performance of a gated-on dendritic route in response to an input stimulus is represented by the level of dopamine given at the end of each session. There are intervals of variable durations between sessions (blue text). (b) During an off-line learning scenario, the brain can quickly sweep over samples of input stimuli when a dendritic route $k$ ($k = 1, 2, 3, \cdots$) is gated on. The dopamine level represents the average reward associated with the replayed stimuli when the same dendritic route is gated on. (c) An example situation. Route 1 brings the highest reward (blue text) in response to a single stimulus, but route 2 brings the highest average reward over both stimuli.

However, this parental selection is incorrect because it is dendritic route 2 that brings more rewards on average over the two input stimuli (Figure 4c).

This problem can be solved if we suppose that the evaluation of dendritic routes happens during off-line replay (Scenario 2, Figure 4b): a gated-on dendritic route is quickly evaluated over a number of replayed inputs, and the dopamine level is released according to the average reward (estimated by the brain) associated with a number of input stimuli just replayed (Figure 4b). In this case, the dopamine level indicates the average performance of the gated-on dendritic route over a batch of input samples just replayed. If we set the batch size to be 2, the dendritic route 2 will be selected as the parental agent during the replay.

The incompatibility of EA with on-line learning lies in the very nature of EA. In gradient-descent algorithms, the updating direction of a synaptic weight is computable and equal to the average over the directions computed from individual input stimuli. Therefore, the neural network can be optimized by accumulating small updating steps in
response to individual stimuli during the on-line learning in Scenario 1. However, in EA, such synaptic updating direction cannot be computed, and relies on the comparison of different dendritic routes. This comparison depends on the performance of dendritic routes over a number of stimulus samples (as an estimation of the performance over the whole stimulus set) instead of a single stimulus, which makes the on-line learning inappropriate. The compatibility of EA with off-line replay suggests that EA plays an important role in transforming episodic memory into unconscious dexterous skills at rest or in sleep [70, 71]. This off-line replay scenario (Figure 4b) is a prediction of our model, which can be experimentally tested in the future. In the following simulations, EA is performed in an off-line manner.

**Cooperative plasticity between synapses from correlated inputs**

From Figure 2a, the simultaneous arrival of the pre-synaptic spikes from neuron 1 at synapses \( a_1, b_1 \) and \( c_1 \) is a key factor for the potentiation of \( a_1 \) and \( b_1 \). This means that such potentiation may happen even if the spikes invading \( a_1 \) and \( b_1 \) do not come from neuron 1, but from a different neuron with correlated firing with neuron 1. This subsection will investigate the EA process when single post-synaptic neurons receive from correlated inputs.

Pre-synaptic neurons with correlated activities target close dendritic locations [72, 58] through synapses with cooperative plasticity: when a synapse is potentiated, the neighboring synapses can also be potentiated at the same time [37] or become easy to be potentiated by only weak stimulation [35]; when a synapse is depressed, the neighboring synapses can also be depressed at the same time [36]. Here we suppose that the synapses from correlated-firing neurons are mutated downward or upward simultaneously (group mutation, GM) instead of independently (individual mutation, IM), modeling the cooperative plasticity between neighboring synapses due to cellular mechanisms in the post-synaptic dendrite [35, 36]. We found that GM is essential for the success of EA learning.

To illustrate the mechanisms, we consider three pre-synaptic neurons (1, 2, 3) with correlated activities targeting a post-synaptic neuron through two dendrites (\( a, b \)), see Figure 5a, b. At a certain time, dendrite \( a \) is gated on while dendrite \( b \) is gated off, and the synapses from the three pre-synaptic neurons on dendrite \( a \) have large efficacy (Figure 5a). Under IM, each synapse is mutated independently with low probability. In this case, suppose synapse \( a_3 \) on dendrite \( a \) is mutated downward (Figure 5a, middle column, upper row). Due to the correlated activity of neuron 3 with neurons 1 and 2, the NO emitted from synapses \( a_1 \) and \( a_2 \) is likely to arrive at synapses \( a_3 \) and \( b_3 \) simultaneously with the spike from neuron 3, thereby inducing high potentiation eligibility traces in synapses \( a_3 \) and \( b_3 \) (Figure 5a, middle column, upper row). If dendrite \( a \) is parental so that high dopamine is induced, all the
synapses in both dendrites will become large (Figure 5a, right column, upper row): this means that the parental synaptic configuration on dendrite a (synapses $a_1$, $a_2$, and $a_3$ have large, large and small efficacies respectively) cannot be reproduced to dendrite b, which contradicts with the EA requirement. Under GM, however, the synapses from the three pre-synaptic neurons on dendrite a are mutated downward simultaneously. In this case, the NO received by the synapses on both dendrites is low, inducing low potentiation eligibility traces. If dendrite a is parental so that high dopamine is induced, all the synapses in both dendrites will become small (Figure 5a, right column, lower row): this means that the parental synaptic configuration on dendrite a gets reproduced to dendrite b. A similar situation happens when the three synapses on dendrite a have small efficacies (Figure 5b). Under IM (or GM), the synapses can receive NO from only synapse $a_3$ (or from all the synapses) on dendrite a, so that the potentiation eligibility traces are low (or high). If dendrite a is parental, its configuration will not (or will) be reproduced to dendrite b (Figure 5b, right column).

We demonstrate the above mechanism by training a spiking neural network on the XOR task. In this network (Figure 5c), a population of pyramidal neurons $P$ receives Poisson spike trains emitted from two neuronal groups $G_1$ and $G_2$, and then give output to a single neuron. The synapses from $G_1$ and $G_2$ to $P$ should be trained so that the output neuron has a high firing rate when one input group has a high firing rate while the other is silent, and the output neuron has a low firing rate when both input groups have high or low firing rates (Figure 5d).

We examined IM, when individual synapses are mutated independently, and GM, when the synapses from the same input group are mutated simultaneously. GM is a better mutation strategy than IM, because GM enables the network to quickly achieve high classification accuracy, whereas IM saturates the network at a low performance (Figure 5e). Therefore, cooperative plasticity between synapses from correlated input neurons is necessary for EA to train the brain’s neuronal network successfully. See more details in Supplementary Figure 1.

**EA-trained neural networks mimic brain dynamics**

To further illustrate the biological plausibility of EA, we trained recurrent neural networks to perform a context-dependent decision-making task [38] by EA, and compared the dynamics of EA-trained neural networks with that of the brain and backpropagation (BP) -trained neural networks [38]. In this task, neural networks are to make a binary choice according to one of its two input channels (representing the motion or color coherence of the random dots in the experiment of [38]) depending on whether the context signal is motion or color (Figure 6a). In the motion (color) context, the network should choose 1 or 2 depending on whether the motion (color) coherence is positive
Figure 5: The computational function of the cooperative plasticity. (a, b) Schematic to illustrate the advantage of group mutation. The green circles in dashed boxes represent pre-synaptic neurons with correlated activity. Large (or small) yellow triangles represent large (or small) synapses. Dashed triangles with question marks represent synapses with no matter large or small efficacy. Dendrite a or b is gated on or off by inactivated or activated SST interneurons (white or black circle). Upward (or downward) arrows indicate that the synapses are mutated upward (or downward). (c) The structure of the network to perform the XOR task. Pyramidal neurons receive from two input groups (whose activity levels can be high or low) through synapses (red arrows) on dendrites gated by SST interneurons. Red synapses are to be changed during training, and the other synapses are fixed. (d) Raster plot of the input neurons in one simulation trial (upper) and the single output neuron in multiple trials (lower) after training under group mutation. Vertical blue lines delimit four intervals in which the two input groups have different high or low activities. (e) Classification accuracy during the training process for individual (orange) or group (blue) mutation. Error belts indicate the standard error of the mean (s.e.m.) over 100 training trials.
or negative, neglecting the color (motion) signal. The neural networks contain firing-rate units, each representing a
group of spiking neurons with correlated activities in the brain. We trained the neural networks using the following
simple algorithm (EA-Simple) to model the group mutation of the synapses from the same neuronal group (Figure
5a, b):

(1) Each synaptic weight takes binary values, one positive $\Delta w$, one negative $-\Delta w$, modeling the effective synaptic
weight when the excitatory synapse has large or small efficacy under the global inhibition of PV interneurons (Figure
5c).

(2) Descendants are mutated from parental agents with a small probability of synaptic flip: $\epsilon$ (Figure 1f) is usually
zero, but with a small probability $2\Delta w$ (or $-2\Delta w$) if $w_{ij}^{\text{parental}}$ is $-\Delta w$ (or $\Delta w$).

EA-trained networks mimic the brain and BP-trained networks in the following four aspects [38]. First, EA-trained
networks successfully learned to perform the task, which is reflected in the behavioral psychometric functions (Figure
6b). Second, when making different choices, the trajectories of the network states move parallelly to a line attractor
toward different directions at a distance proportional to the coherence strength (Figure 6c, d). Third, after defining
three axes that respectively capture the most across-trial variance in the state space due to the choice, the motion
and color coherence, we found that context usually has no substantial effect on the directions of the axes of choice,
motion and color (Supplementary Figure 2c). Fourth, the selection vectors (i.e., the left eigenvector of the largest
eigenvalue) of a line of attractors are aligned with the motion (or color) input and orthogonal to the color (or motion)
input in the motion (or color) context (Figure 6e). This indicates that the relevant input pushes the network state
along the direction of the line attractor, whereas the irrelevant input has no effect. Our results imply that EA-trained
networks mimic the dynamics of the brain in cognitive tasks, which further supports the biological plausibility of
EA. See more details in Supplementary Figure 2.

**EA is a general-purpose and powerful learning paradigm**

To demonstrate the universality and capability of EA, we trained neural networks with biologically plausible binary
weights using the EA-Simple algorithm on a variety of tasks.

We first trained a sparse recurrent spiking neural network (SNN) to output arbitrary trajectory with time [73, 74]
(Figure 7a, see Methods). Despite the binary weights of the recurrent and output synapses, the network’s output can
well approximate the target trajectory (Figure 7b, Supplementary Figure 3a).
Figure 6: EA-trained neural networks mimic brain dynamics. (a) The architecture of the neural network model. In motion (or color) context, the neural network is to choose 1 or 2 depending on whether input signal indicating the motion (or color) coherence is larger than zero. (b) Psychometric curves show the percentage of choice 1 as a function of the motion and color coherence in the motion (black) and color (blue) contexts. Error bars represent s.e.m. over 8 training trials. (c) Dynamics of model population responses in the motion context in the subspace spanned by the axes of choice and motion, when the motion coherence takes different values. Each fixed point (red cross) has an eigenvalue close to zero, these fixed points together approximate a line attractor. After the inputs are turned off (orange dots and lines), the responses relax back towards the line attractor. Analogous to Figures 2 and 5 of [38]. (d) Similar to panel c, but in the color context and in the subspace spanned by the axes of choice and color. (e) The line attractor (black or blue crosses) and selection vector (green) at each fixed point, in the subspace spanned by the motion and color input weights. Analogous to Figure 6c of [38].
Figure 7: Training spiking and analog neural networks using EA. (a) Schematic of the SNN to generate trajectories. Recurrent units (yellow circles) are sparsely connected. The output unit (blue circle) is trained to generate an arbitrary trajectory. Both the recurrent and output connections (red arrows) are binary and plastic. (b) An example of the actual output (blue) and target (orange) trajectory. The input signal is on during the initial 50 ms (red shading) and off afterward. (c) The architecture of the ANN to classify MNIST images. When studying the Hebbian algorithm in panel d, the lower layers were trained using the Hebbian algorithm (blue arrows), and the top layer was trained by gradient-descent algorithm supervisedly (red arrows). (d) Classification accuracy on the test dataset when training the deep network using BP, EA and Hebbian rule. Error bars represent s.e.m. over 8 training trials.

We then trained a feedforward analog neural network (ANN) with two hidden layers (Figure 7c) to classify the MNIST images. The final classification performance is comparable with continuous-weight networks with the same structure trained by BP (Figure 7d, Supplementary Figure 3b). We also trained continuous-weight networks layer-by-layer using a competitive Hebbian algorithm [13], except that the last layer was trained by gradient-descent algorithm supervisedly (Figure 7c), and found worse final performance (Figure 7d, third bar). Interestingly, if we supervisedly trained the last layer on top of the first hidden layer, the performance got better than the two-hidden-layer case (Supplementary Figure 3c, d), similar to the finding in [14]. This phenomenon implies that Hebbian algorithms cannot coordinate weights in different layers to fulfill better performance.

We further demonstrate the capability of the EA by training deep neural networks with binary weights to play Atari games, whose performance is comparable to continuous-weight networks trained by gradient-based methods such as DQN and A3C [75, 76] and EA-based method [77] (Table 1). We also tried a Hebbian-Q algorithm, where low-level layers were trained by a competitive Hebbian algorithm [13], and the top layer was trained by the Q learning [75], and found that the resulted performance was significantly worse than DQN, A3C and EA (Table 1).
Table 1: Binary-weight networks trained by EA are competitive with continuous-weight networks trained by DQN and A3C in Atari games. In the fifth column, bold black numbers indicate the scores of EA-trained binary-weight networks that are higher than both the scores of DQN and A3C. Red (or blue) numbers indicate the scores that are higher than those of DQN (or A3C), but lower than A3C (or DQN).

<table>
<thead>
<tr>
<th>Game name</th>
<th>DQN, continuous weight, [77]</th>
<th>A3C, continuous weight, [77]</th>
<th>EA, continuous weight, [77]</th>
<th>EA, binary weight, ours</th>
<th>Hebbian-Q, continuous weight, ours</th>
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These results demonstrate that the biologically supported EA is a general-purpose algorithm capable of training deep neural networks on complicated tasks.

Discussion

Overall, by unifying a broad spectrum of biologically experimental evidence into a coherent picture, we propose that the brain uses an EA (Figures 1-5) to learn. Our EA provides new insights into the origin of brain dynamics in cognitive tasks (Figure 6), and manifests its general-purpose and powerful capability in training spiking or analog neural networks (Figures 7, Table 1). Compared to the homosynaptic mechanisms reviewed in [11], our mechanism also has the following computational advantages:

(1) EA is model-free. It does not depend on the task objective or neural network structure, thereby broadly applicable without the need for much gene-encoded knowledge. As we mentioned in the Introduction, previous proposals suffer from a variety of difficulties, such as contradicting with existing experimental evidence, restricting to specific network architecture, lacking the capability of training deep networks on hard problems, requiring an infeasibly huge amount of gene-encoded information, not considering biologically plausible binary-weight synapses, etc. Our EA,
however, is simple, powerful, biologically supported, and broadly applicable to spiking or analog neural networks with recurrent or feedforward architecture. The advantage of EA lies in its nature of target orientation, without the need to design sophisticated weight-updating rules.

(2) EA is particularly good at creatively dealing with multi-objective optimization problems [78]. Real-world problems usually require addressing several conflicting objectives. For example, because a cheap house in downtown is usually unavailable, a house buyer may consider both an expensive house in the downtown and a cheap house in the suburb, due to their own merits to be either convenient or cheap. Gradient-based methods can solve this problem using a single objective by considering it as a linear combination of these multiple objectives with pre-assigned preferential weights: e.g., a house buyer may emphasize more on price or convenience before making the decision. In contrast, a population of agents trained by EA can simultaneously enumerate a number of possible solutions, each with its unique merit (more precisely, the Pareto front [78]), thereby significantly improving the creativity in dealing with conflicting objectives. In neuro-imaging studies, it has been found that the functional connectivity of the brain is dynamically reconfigured, and this reconfiguration is closely related to the creativity of human subjects [79, 80]. In terms of our model, this dynamic reconfiguration is caused by the alternation of activated agents (Figure 1e), reflecting the solution enumeration process mentioned above.

(3) Natural selection creates our brain, endowing us with amazing learning capabilities, such as meta-learning [81], zero-shot (or few-shot) learning [82], transfer learning [83], continuous learning [84], etc. Therefore, EA is a universal philosophy to automatically create high intelligence in dealing with a complicated environment. By taking EA as its learning mechanism, the brain can potentially maximize its adaptation to the environment, facilitating the survival of the animal.

Compared with gradient-based methods, the drawback of EA is slow learning speed. Classification of the MNIST images requires over 10,000 learning epochs using EA (Supplementary Figure 3b), in contrast to dozens or hundreds of epochs using gradient-based methods. However, learning speed may not be a critical aspect, because EA mainly mediates the off-line learning during resting or sleeping (Figure 4), whose neuronal firing sequences are temporally compressed by about 20-fold relative to those in real experience [70]. Besides, the brain may use several strategies to speed up learning:

(1) First is the combination of homosynaptic mechanisms compatible with gradient-based methods [11]. To manifest the sole power of heterosynaptic plasticity, we do not add any Hebbian-type homosynaptic rules in our model. Adding Hebbian-type homosynaptic rules can guide the mutation direction using hints of gradients, potentially speeding up
(2) Curriculum learning (i.e., learning the easier aspects of the task first and then gradually increasing the task difficulty) can significantly improve the speed and success rate of learning complicated tasks [85, 86]. To achieve optimal learning effects, the brain may adapt the contents of replay (so that the replayed contents are neither too simple nor too complex) to its current capability in solving problems: a process called autocurriculum [86]. It has been found that the brain prioritizes high-reward memories to replay [87], which may be the mechanism of autocurriculum: the replayed contents prioritize the highest-reward and most complicated situations within the current ability of the brain, training the brain to be more powerful.

(3) A recent theoretical study [88] suggests that compared with EA with only mutation, EA with crossover operation is faster by polynomial order of the number of agents in a generation. Our model only contains the mutation operation, but can easily include crossover. Suppose the network $N$ (Figure 1d) has two modules $N_A$ and $N_B$, whose dendritic routes are separately controlled by two networks $A$ and $B$, such that when a neuron $a_i \in A$ (or $b_j \in B$) is activated, the dendritic routes labeled by $i$ (or $j$) in $N_A$ (or $N_B$) are gated on, with other routes gated off. If at the early stage of training, $a_1$ and $b_1$ (or $a_2$ and $b_2$) are activated simultaneously, such that dendritic routes with the same label 1 (or 2) in both $N_A$ and $N_B$ are gated on simultaneously; but at a later stage, $a_1$ and $b_2$ (or $a_2$ and $b_1$) are activated simultaneously, such that dendritic routes with different labels in $N_A$ and $N_B$ are gated on simultaneously: then this later stage fulfills crossover operation.

In our model, only excitatory synapses between pyramidal neurons are plastic, whereas the synapses to and from inhibitory interneurons are kept fixed. Physiologically, inhibitory plasticity surely plays an important role in the normal functioning of the brain [89, 90]. Inhibitory plasticity has more diverse forms than excitatory plasticity, so understanding its functional roles is challenging [89]. An emerging principle is that inhibitory plasticity restores the balance between excitation and inhibition up to dendrite-specific level, in response to the imbalance caused by excitatory plasticity or environmental change [91, 89]. For example, the activation of NMDA receptors potentiates the dendrite-targeting synapses from the SST interneurons [92], which may help the SST interneurons better gate off the dendrites frequently or strongly activated by excitatory inputs.

Overall, our work provides new computational aspects to understand heterosynaptic plasticity in the brain, and suggests neuroscientists further investigate the biological backend of EA.
Methods

XOR task

In the network of Figure 5c, 20 pyramidal neurons receive from two groups of excitatory neurons (each group has 20 neurons) through 20 dendrites gated by different SST interneurons. These pyramidal neurons are inhibited by PV interneurons, which receive input from both groups of excitatory input neurons. Pyramidal neurons also give output to a single neuron. Pyramidal neurons, PV interneurons and the output neuron are modeled with leaky integrate-and-fire equations:

\[ C \frac{dV_i}{dt} = -g_L(V_i - V_L) - \sum_j g_{ij}(V_i - V_{ij}) + I_{\text{background}}, \] (1)

where \( V_L = -74 \text{mV}, g_L = 25 \text{nS}, C = 500 \text{pF}, \) and \( I_{\text{background}} = 425 \text{pA} \) (parameters are chosen according to [93]). When the membrane potential \( V_i \) reaches the firing threshold \( V_\theta = 54 \text{mV} \), a spike is recorded and \( V_i \) is reset to \( V_{\text{reset}} = -60 \text{mV} \). The reversal potential \( V_{ij} \) is 0mV if the synapse from \( j \) to \( i \) is excitatory, or -70mV if this synapse is inhibitory. The dynamics of the synaptic conductance \( g_{ij} \) is

\[ \tau_{ij} \frac{dg_{ij}}{dt} = -g_{ij} + w_{ij} \sum_l \delta(t - t_{ij,l}), \] (2)

with \( t_{ij,l} \) being the time of the \( l \)th spike of the \( j \)th neuron. The time constant \( \tau_{ij} \) is 10ms for the synapses from the PV interneurons to the pyramidal neurons and 5ms for the other synapses. The synaptic efficacy \( w_{ij} \) is 4.8nS from the excitatory input neurons to the PV interneurons, 2.4nS from the inhibitory input neurons to the PV interneurons, 2.4nS from the PV interneurons to the pyramidal neurons, and 4.8nS from the pyramidal neurons to the single output neuron. All the above synapses are connected with probability 0.4, and fixed during training. The synapses from the excitatory input neurons to the pyramidal neurons are all-to-all connected in every dendrite of the pyramidal neurons, and have either large efficacy of 2.4nS or small efficacy of 0nS. The excitatory input neurons produce Poisson spike trains. The firing rates of the two groups of excitatory input neurons are respectively (0Hz, 0Hz), (20Hz, 0Hz), (0Hz, 20Hz), and (20Hz, 20Hz) in the four successive intervals with duration 500ms in each training epoch (Figure 5d, upper panel). The pyramidal neurons also receive from a group of inhibitory neurons (not depicted in Figure 5c), which output Poisson spike trains of 40Hz. Our simulation was performed in the Brian simulator [94] using the exponential Euler method with time step of 0.1ms.

The potentiation eligibility trace of the synapse from a pre-synaptic excitatory input neuron to a post-synaptic pyra-
midal neuron through the dth dendrite is

\[ e_{pre,post,d}^{pot} = A(d) \sum_{l,m} \Theta(t_{m,post} - t_{l,pre}) e^{-\frac{(t_{m,post} - t_{l,pre})}{\tau_{pot}}}, \tag{3} \]

where \( \Theta(\cdot) \) is the step function, \( t_{m,post} \) (or \( t_{l,pre} \)) is the time of the mth (or lth) spike of the post- (or pre-) synaptic neuron, \( \tau_{pos} = 20 \text{ms} \) is the characteristic time window indicating the simultaneity of the pre- and post-synaptic spikes that induces potentiation eligibility trace, \( A(d) = 1 \) for a synapse on the gated-on dendrite, and \( A(d) = 0.5 \) for a synapse on a gated-off dendrite modeling the effect of diffusive NO emitted from the gated-on dendrite. Here we suppose that the potentiation effect of the NO emitted from a gated-on dendrite of a pyramidal neuron can only influence the dendrites of the same pyramidal neuron, and has no influence on the dendrites of other pyramidal neurons, based on the experimental observation that the spatial scope of this potentiation effect is comparable with the dendritic arbor of a pyramidal neuron [25, 24].

Synaptic efficacy was updated according to the eligibility trace and the dopamine level, satisfying the following rules or constraints, explained below:

\[
\begin{align*}
    w_{ij,d}(T) &= 0 \text{ or } w_{max} \\
    w_{ij,d}(T) &= w_{ij,d}(T - 1) + \Theta(e_{ij,d}^{pot}(T - 1) - e_\theta^{pot}) B(r(T), \{r(t)\}_{t<T}) \\
    \sum_j w_{ij,d}(T) &= \sum_j w_{ij,d}(T - 1) \\
    w_{ij,d}(T) &= w_{ij,d}(T - 1), \text{ if } h_{ij,d}(T - 1) > 0 \text{ and } w_{ij,d}(T - 1) = w_{max} \\
    h_{ij,d}(T) &= h_{ij,d}(T - 1) + \Theta(e_{ij,d}^{pot}(T - 1) - e_\theta^{pot}) C(r(T), \{r(t)\}_{t<T}), \text{ if } w_{ij,d}(T) = w_{max} \\
    h_{ij,d}(T) &= h_{ij,d}(T - 1) - 1 \\
    0 \leq h_{ij,d}(T) \leq h_{max}
\end{align*}
\]

The first equation indicates that the synapse \( w_{ij,d}(T) \) from the jth excitatory input neuron to the ith pyramidal neuron through the dth dendrite at the Tth epoch has binary efficacies 0 or \( w_{max} = 2.4 \text{nS} \). The second equation indicates that \( w_{ij,d} \) is potentiated if \( e_{ij,d}^{pot} \) is larger than a threshold value \( e_\theta^{pot} = 0.45 \) and the factor \( B \) controlled by the reward \( r(T) \) at the Tth epoch is larger than zero. In our simulation, \( B = 1 \) if \( r(T) \) is not smaller than the second largest
reward obtained in the last 20 training epochs, and $B = 0$ otherwise. The reward at an epoch is defined as

$$r = -[o(\text{high}, \text{high})]_+ - [-o(\text{high}, \text{low}) + 10]_+ - [-o(\text{low}, \text{high}) + 10]_+, \quad (5)$$

where $o(\text{high}, \text{low})$ means the number of spikes of the output neuron during the interval of 500ms when Group 1 of the excitatory input neurons have high firing rate and Group 2 have low firing rate; $o(\text{high}, \text{high})$ and $o(\text{low}, \text{high})$ have similar definitions; $o(\text{low}, \text{low}) = 0$ all the time. The third equation in Eqs. 4 indicates dendrite-level synaptic homeostasis (Figure 3c, lower panel), which keeps constant the total synaptic efficacy on a dendrite (specifically, half of the synapses on a dendrite are large). The fourth equation indicates that if the hidden state $h_{ij,d}$ in a synapse with large efficacy is larger than zero, this synapse will remain unchanged in the current epoch. In other words, a large synapse can only be depressed when its hidden state is zero. Note that the hidden state remains at zero for synapses with low efficacy. The fifth equation indicates that if the eligibility trace $e_{ij,d}^{\text{pot}}$ is larger than $e_0^{\text{pot}}$ and the factor $C$ controlled by the reward is larger than zero, the hidden state in the large synapse will be increased. In our simulation, $C$ is 20 if $r(T)$ is not smaller than the largest reward obtained in the last 20 training epochs, $C$ is 10 if $r(T)$ is smaller than the largest but not smaller than the second largest reward obtained in the last 20 training epochs, and $C = 0$ otherwise. The sixth equation indicates that the hidden state in every large synapse is decreased by 1 in each training epoch, so that the synapse gradually becomes less stable. The seventh equation indicates that the hidden state is non-negative and no larger than a maximum value $h_{\text{max}} = 20$. The second and third equations in Eqs. 4 may conflict with each other. In our simulation, when too many synapses had large potentiation eligibility traces and were to be potentiated, we kept synaptic homeostasis (i.e., the third equation) non-violated, and set the synapses with largest potentiation eligibility traces at large efficacy. Specifically, at the $T$th epoch when $B > 0$, we collected the synapses either with $w_{ij,d}(T - 1) > 0$ and $h_{ij,d}(T - 1) = 0$ or with $w_{ij,d}(T - 1) = 0$ and $e_{ij,d}^{\text{pot}}(T - 1) > e_0^{\text{pot}}$ into a set $S$, sorted the potentiation eligibility traces in the synapses in $S$ in descending order, and let the first $N_{\text{input}}/2 - N_{\text{stable large}}$ synapses have large efficacy, and let the rest synapses have small efficacy. Here, $N_{\text{input}} = 40$ is the number of synapses on a single dendrite (which is the total number of excitatory neurons in the two input groups), and we constrained the number of synapses with large efficacy on a dendrite fixed at $N_{\text{input}}/2$, modeling dendrite-level synaptic homeostasis (the third equation of Eqs. 4). $N_{\text{stable large}}$ is the number of synapses with $w_{ij,d}(T - 1) > 0$ and $h_{ij,d}(T - 1) > 0$ on the $d$th dendrite.

In our model, each pyramidal neuron has 20 dendrites, labeled from 1 to 20. Every neuron in the two input populations gives output to every dendrite (Figure 5c). At each training epoch, the dendrites of all the pyramidal neurons
with the same randomly selected label were gated-on, with the other dendrites gated off (Figure 1e). Synaptic mutation was performed at the gated-on dendrites at the beginning of each epoch. In individual mutation, synapses with large efficacy and zero hidden state were randomly set to small efficacy with probability 0.25 at the gated-on dendrites; at the same time, the same number of randomly selected synapses with small efficacy were also set to large efficacy, to maintain the total number of large synapses on every gated-on dendrite. In group mutation, all the synapses (except for those synapses with non-zero hidden states) from a randomly selected input group to the gated-on dendrite of a randomly selected pyramidal neuron were set to small efficacy; at the same time, the same number of synapses on the gated-on dendrite from another input group to the selected pyramidal neuron were set to large efficacy to maintain the total number of large synapses on that gated-on dendrite. In both mutation strategies, a single large synapse with zero hidden state has almost equal probability 0.025 to be mutated (Supplementary Figure 1c), so that the performance difference under these two strategies (Figure 5e, Supplementary Figure 1b, c) is caused by these strategies themselves, instead of the difference of mutation rate.

Context-dependent decision-making task

Our model to study the context-dependent decision-making task in Figure 6 closely follows that of [38]. The recurrent neural network contains $N = 100$ neurons, and is all-to-all connected with the following dynamics:

$$\tau \dot{x} = -x + J r + b^c u_c + b^m u_m + b^{cc} u_{cc} + b^{cm} u_{cm} + c^x + \rho_x$$

$$r = \tanh(x)$$

$$z = w_x^T r + c^z$$

The variable $x(t)$ is a $N$-dimensional vector containing the total synaptic current of each neuron in the network, and $r(t)$ are the corresponding firing rates. Each neuron has a time constant $\tau = 10$ms. The matrix $J$ defines the recurrent connections in the network. The network receives 4-dimensional input, $u(t) = [u_c(t), u_m(t), u_{cc}(t), u_{cm}(t)]^T$, through synaptic weights, $B = [b^c, b^m, b^{cc}, b^{cm}]$. These four inputs represent, respectively, the sensory evidence for color and motion, and the contextual cues instructing the network to integrate either the color or the motion input. Finally, $c^x$ is a vector of offset currents and $\rho_x$ is a white noise drawn at each time step with standard deviation 0.1. The output $z$ of the network is a weighted sum of the firing rates, with weights $w_x^T$ and bias $c^z$. All the synaptic
connections in $J$, $B$ and $w_z$ are binary, taking values of $\pm 1/\sqrt{N}$, $\pm 1$ and $\pm 1/\sqrt{N}$, respectively. During training, the network dynamics were integrated for the duration $T = 750$ ms using Euler updates with $\Delta t = 1$ ms. After training, model dynamics were integrated for an additional 200 ms with the sensory inputs turned off, according to which we plotted the orange dots and lines in Figure 6c, d.

The contextual inputs $u_{cm}$ and $u_{cc}$ were constant for the duration of the trial. In the motion context $u_{cm}(t) = 1$ and $u_{cc}(t) = 0$, while in the color context $u_{cm}(t) = 0$ and $u_{cc}(t) = 1$. The motion and color inputs $u_m$ and $u_c$ are one-dimensional white-noise signals:

$$u_m(t) = d_m + \rho_m(t)$$
$$u_c(t) = d_c + \rho_c(t)$$

The white noise terms $\rho_m$ and $\rho_c$ have zero mean and standard deviation 1. During training, the offsets $d_m$ and $d_c$ were randomly chosen on each trial from the range $[-0.1875, 0.1875]$. During simulations after training, $d_m$ and $d_c$ took 6 values ($\pm 0.009, \pm 0.036, \pm 0.15$), corresponding to weak, intermediate, and strong evidence toward either choice. In the psychometric curves (Figure 6b), the coherence value is normalized so that $\pm 0.15$ of $d_m$ or $d_c$ corresponds to $\pm 1$ of the horizontal coordinate in the figure.

The target $p$ of network training is that the output $z$ approaches 1 (or -1) at time $T$ when $d_m > 0$ (or $< 0$) in the motion context, and approaches 1 (or -1) when $d_c > 0$ (or $< 0$) in the color context. In the EA, the number of agents was 100, the number of elite agents in each generation was 5. These agents were evaluated by averaging $(p - z(T))^2$ over 128 simulation trials. At each training epoch, every non-parental agent copied the synaptic weights $J$, $B$ and $w_z$ as well as the bias $c^x$ and $c^z$ from a randomly chosen elite agent, with the following mutation: every synaptic weight was flipped with probability 0.0005; to every bias was added a random Gaussian number with zero mean and standard deviation 0.002. The mutation of bias models the mutation of synaptic weights inputted from other brain areas.

The axes of choice, motion and color in Figure 6c, d were obtained by first linear regressing the trajectory $x(t)$ of all simulation trials and then orthogonalizing the regression coefficient using QR-decomposition [38]. Similar to [38], we plotted $x(t) - \langle x(t) \rangle_{\text{trial}}$ in Figure 6c, d, with the urgency signal $\langle x(t) \rangle_{\text{trial}}$ being the average trajectory over all simulation trials in the motion (Figure 6c) or color (Figure 6d) context.
The attractors in Figure 6c-e were obtained by minimizing the function

$$q(x) = \frac{1}{2} |F(x)|^2,$$ (7)

where $F(x)$ is the right-hand side of eq. 6 after setting $u_c = u_m = \rho_x = 0$, representing the speed of changing of neural state. To get the attractors in the motion context, $u_{cm}(t) = 1$ and $u_{ce}(t) = 0$; to get the attractors in the color context, $u_{cm}(t) = 0$ and $u_{ce}(t) = 1$. We used the L-BFGS-B algorithm of the ‘minimize’ routine of the Scipy package to minimize $q(x)$, initializing $x$ to be a random point in the trajectory of $x(t)$. To find slow-dynamic points on the line attractor instead of the two stable fixed points, we set the tolerance parameter ‘ftol=0.002’ to early stop the minimization algorithm. The found slow-dynamic points have a close-to-zero negative eigenvalue and many large negative eigenvalues. The small bars in Figure 6e are the left eigenvectors of the close-to-zero eigenvalues of these points (i.e., selection vectors), projected in the subspace spanned by the input weights $b^c$ and $b^m$.

**Trajectory generation task**

In the trajectory generation task (Figure 7a, b), we considered a network of $N = 1000$ randomly and sparsely connected (connection probability $p = 0.1$) quadratic integrate-and-fire neurons with dynamics [95]

$$\tau \frac{dv_i}{dt} = I_{in}(t) + \sum_k \mathbb{I}(k) \sum_j s_{ij,k}(t) + v_i^2,$$ (8)

where $\tau = 10\text{ms}$; $v_i$ is a dimensionless variable representing the membrane potential; $s_{ij,k}$ is the synaptic current from neuron $j$ to neuron $i$ through dendrite $k$; and $\mathbb{I}(k) = 1$ or 0, indicating whether or not dendrite $k$ is gated on. The dynamics of $s_{ij,k}$ is

$$\tau_s \frac{ds_{ij,k}}{dt} = -s_{ij,k} + w_{ij,k} \sum_l \delta(t - t_{l;j}),$$ (9)

where $\tau_s = 20\text{ms}$, $w_{ij,k}$ is the synaptic weight from neuron $j$ to neuron $i$ through dendrite $k$, and $t_{l;j}$ is the time of the $l$th spike of neuron $j$. To simulate the dynamics of quadratic integrate-and-fire neurons, we used theta neuron model derived by a simple change of variables $v_i = \tan(\theta_i/2)$, getting

$$\tau \frac{d\theta_i}{dt} = 1 - \cos \theta_i + (I_{in}(t) + \sum_j \mathbb{I}(k)s_{ij,k}(t))(1 + \cos \theta_i).$$ (10)

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The output of the network is

\[ o(t) = \sum_k I(k) \sum_j w_{oj,k} s_{oj,k}(t) \tag{11} \]

where \( w_{oj,k} \) is the output weight of neuron \( j \) through dendrite \( k \), \( s_{oj,k} \) is the synaptic current with the dynamics

\[ \tau_s \frac{ds_{oj,k}}{dt} = -s_{oj,k} + w_{oj,k} \sum_l \delta(t - t_{l;ij}). \tag{12} \]

The recurrent synapse \( w_{ij,k} \) takes binary values: either \( 20/\sqrt{pN} \) or \( -20/\sqrt{pN} \). The output synapse \( w_{oj,k} \) also takes binary values: either either \( 1/\sqrt{N} \) or \( -1/\sqrt{N} \).

The target trajectory (orange curve in Figure 7b) was defined as \( f(t) = A \sin(2\pi(t - T_0)/T_1) \sin(2\pi(t - T_0)/T_2) \), where \( A, T_0, T_1 \) and \( T_2 \) were randomly sampled from intervals \([0.5, 1.5] \), \([0, 1000 \text{ ms}] \), \([500 \text{ ms}, 1000 \text{ ms}] \) and \([100 \text{ ms}, 500 \text{ ms}] \) respectively. At the beginning of each simulation session, every neuron was stimulated for 50 ms with constant external stimulus that had random amplitude sampled from \([-1, 1]\) (red shading in Figure 7b). The pattern of the stimulation was the same for the same target function. Simulations were performed in the Brian simulator [94] using the Euler method with time step of 0.1ms.

Our EA was performed with the aim to reduce the loss function

\[ \text{loss} = \sum_{t>50\text{ms}} (f(t) - o(t))^2. \tag{13} \]

In the EA, the number of agent was 100, the number of elite agents in each generation was 5. At each training step, every non-parental agent copied the synaptic weights of a randomly chosen elite agent, then every synaptic weight was flipped with probability 0.0005.

**MNIST classification task**

In Figure 7c, d, we train a multi-layer perceptron with 2 hidden layers with 256 and 128 neurons respectively to classify the MNIST images (size \( 28 \times 28 = 784 \) pixels) into 10 classes. Adjacent layers in the feedforward network are all-to-all connected. The non-linear activation function is ReLU.

The EA was performed on networks whose synaptic weights took binary values \( \{-\sqrt{1/N_{\text{in}}}, \sqrt{1/N_{\text{in}}} \} \), where \( N_{\text{in}} \) represents input size: 784 for weights between the input layer and the first hidden layer; 256 for weights between
the first and second hidden layers; and 128 between the second hidden layer and the output layer. We initialized 100 network configurations. At every training epoch, negative log-likelihood loss averaged over all the 60000 images in the training dataset was evaluated for every configuration. The elite configuration (the one with the smallest loss) remained unchanged, whereas each weight of the other configurations was updated to be the corresponding weight of the elite configuration with probability 0.99995, and to be different with probability 0.00005 (mutation). Therefore, there is only one parent in each generation of agents in this algorithm. We also studied the case of dendrite-level synaptic homeostasis (Figure 3c, lower panel), in which the number of synapses with large efficacy received by a neuron in a configuration was kept fixed after the random initialization of synaptic weights. Adding this homeostasis mechanism hardly influences the classification performance (Supplementary Figure 3b).

Back-propagation was performed on networks whose synaptic weights took continuous values. We used Adam optimizer of Pytorch for the training. Batch size was 100, learning rate was 0.0001, the number of training epochs was 400.

The Hebbian learning was also performed on continuous-weight networks. All the low-level layers (except the top layer) were trained by competitive Hebbian learning [13]. In every step, the weights were updated by

\[
\Delta w_{ij} \propto g_i [2u_j - (\sum_k w_{ik}u_k)w_{ij}], \tag{14}
\]

where \(w_{ij}\) is the synaptic weights from the \(j\)th pre-synaptic neuron to the \(i\)th post-synaptic neuron, \(u_j\) is the input from the \(j\)th pre-synaptic neuron, \(g_i\) is 1 for the highest activated post-synaptic neuron, \(-0.4\) for the second highest activated post-synaptic neuron, and 0 for others. Batch normalization was performed on the pre-synaptic input \(u_j\). Layers were trained one after another, such that when a higher-level layer was trained, all the lower-level layers were fixed. Adam optimizer was used to supervisedly train the top layer. Following [13], the learning rate of Hebbian learning linearly decreased from the maximal value 0.04 at the first epoch to 0 at the last epoch, and the learning rate of supervised learning was kept at 0.0001. Both Hebbian and supervised training was performed with minibatch size 100 for 1000 epochs.

**Atari game task**

The structure of the deep neural network to play Atari games closely followed those of [75] and [77]. Specifically, the neural network mapped 4 recent frames of size 84 \times 84 to actions through 3 convolutional layers and 2 fully-
connected layers. Of the three convolutional layers, the kernel sizes were $8 \times 8$, $4 \times 4$ and $3 \times 3$ respectively, the strides were 4, 2 and 1 respectively, and the numbers of features were 32, 64 and 64 respectively. The first fully-connected layer had 512 neurons. ReLU activation function was used.

EA was used to train neural networks whose synaptic weights took binary values $\{-\sqrt{2/N_{in}}, \sqrt{2/N_{in}}\}$, with $N_{in}$ being the input size. The training protocol closely followed that of [77]. Specifically, each generation had 1000 agents, each was evaluated by one episode (i.e., from the start to the end of a game, when the player succeeded or was killed in the game). The top 20 agents were selected to be parents and reproduced the next generation by flipping their binary weights with probability 0.002 (mutations). Then the top 10 agents were further evaluated by 30 episodes, based on which the elite agent was selected to be the agent that achieved the highest score. The elite agent became a member of the next generation without any mutation. The performance of EA at any training epoch was the average score of the elite agent in further 200 episodes. We trained the network in $2.5 \times 10^8$ epochs. Each number of EA in Table 1 represents the median value of the scores in the final epochs of 5 training trials.

To perform Hebbian-Q learning, batch normalization was inserted between each layer of the network. The low-level layers were trained using Eq. 14, and the last layer was trained supervisedly using Adam optimizer. When training a higher-level layer, all lower-level layers were kept fixed.

When unsupervisedly training a low-level layer, at each time step, a random action was taken of a game [96], then minibatches of size 32 were randomly extracted from a buffer of $10^6$ recent frames to train the network using Eq. 14. To train convolutional layers, we averaged over the weight updating at different spatial locations to update the sharing weights. Similar to [13], learning rate linearly decreased from the maximal value 0.02 at the first epoch to 0 at the last epoch. The three convolutional layers were trained in $1.6 \times 10^5$ epochs, and the first fully-connected layer was trained in $4.8 \times 10^5$ epochs.

When supervisedly training the top layer, minibatches of size 32 were randomly extracted from a buffer of $10^6$ recent frames at each epoch in a similar protocol to [75]. The action policy during training was $\epsilon$-greedy, with $\epsilon$ linearly annealed from 1 to 0.1 in the first $10^6$ epochs. The target network was cloned from the Q-network every 10000 epochs. Every $2 \times 10^5$ epochs, the performance of the agent was evaluated by the average score over 200 episodes of games. We trained the top layer for $2 \times 10^6$ epochs, and found no sign of continuing increase of performance. Each number of Hebbian-Q in Table 1 represents the maximal value of the evaluation scores in 2 training trials.
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References


Evolutionary learning in the brain by heterosynaptic plasticity:

Supplementary Information

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Supplementary Figure 1: Cooperative plasticity of the synaptic weights in the XOR task. (a) Probability density function (P. D. F.) of the standard deviation (S. D.) of \( \{ w_{d,i} \}_{i \in G_k} \), with \( \{ w_{d,i} \}_{i \in G_k} (k = 1, 2) \) being the efficacies of all the synapses the neurons in group \( G_k \) through a given dendrite \( d \), under individual mutation (orange) and group mutation (blue). The dashed vertical line indicates the S. D. when the synapses in \( \{ w_{d,i} \}_{i \in G_k} \) have either large or small efficacies with equal probability. This panel means that the synapses from the same group either mostly have large efficacy or mostly have small efficacy. (b) The probability that the majority value of \( \{ w_{d,i} \}_{i \in G_k} \) changes at a given epoch during the training process. This panel means that the majority value of \( \{ w_{d,i} \}_{i \in G_k} \) frequently changes under group mutation, but hardly changes under individual mutation, consistent with Figure 5a, b. (c) The mutation rate of large synapses with zero hidden states under individual mutation (orange) or group mutation (blue) as a function of training epoch. Notice that they are almost the same at our parameter values, so the performance difference under these two mutation strategies (Figure 5e) is due to these strategies themselves rather than the difference of mutation rate. The results in the three panels are average over 100 training trials.
Supplementary Figure 2: The dynamics of EA-trained neural networks in the context-dependent decision-making task.

(a) Left: Dynamics of model population responses in the motion context in the subspace spanned by the axes of choice and motion, when the motion coherence takes different values. Middle: Same data as the left panel, but in the subspace spanned by the axes of choice and color. Right: Same data as the middle panel, but re-sorted according to the direction and strength of the irrelevant color input. (b) Responses in the color context, analogous to panel a. (c) $\beta_{\text{v,motion}}$ (v =choice, motion, color) indicates the unit vector along the choice, motion or color axis in the motion context; $\beta_{\text{v,color}}$ indicates the unit vector in the color context. This panel means that the choice, motion or color axes in different contexts are almost colinear. Error bars represent quantiles over 8 simulation trials. (d) Motion context. $w_{\text{input}}^v$ (v =motion, color) is the input weight of the motion or color signal; $I_{\text{selection}}$ is the selection vector of the fixed points along the line attractor. This panel means that the motion input has a larger projection on the selection vector than the color input in the motion context. Error bars represent s.e.m. over 1024 fixed points found in 8 simulation trials. Blue asterisk: $p < 0.05$, Wilcoxon rank-sum test. (e) Similar to panel d, but in the color context. This panel means that the color input has a larger projection on the selection vector than the motion input in the color context. (f) The loss function during the training process. Error bars represent s.e.m. over 8 simulation trials.
Supplementary Figure 3: The trajectory generation task and the MNIST classification task. (a) Trajectory generation task. The loss function during the training process. (b-d) MNIST classification task. (b) Left: Classification accuracy during the training process of EA. The horizontal dashed line with the number indicates the final accuracy on the test dataset. Right: The same as the left panel, except under dendrite-level homeostasis. Notice that dendrite-level homeostasis hardly influences performance. (c) Classification accuracy on the test dataset when training deep networks with 2 hidden layers or 1 hidden layer using Hebbian rule. (d) Schematic of the structures of the networks examined in panel c, one with two hidden layers and the other with one hidden layer. Lower layers were trained using the Hebbian algorithm (blue arrows), and the top layer was trained by gradient-descent algorithm supervisely (red arrows). In panels a and c, error bars represent s.e.m. over 8 training trials. Error bars in panel b (not shown) are comparable with the width of the lines.