

# Whole-brain multivariate hemodynamic deconvolution for multi-echo fMRI with stability selection

Eneko Uruñuela<sup>a,b,\*</sup>, Javier Gonzalez-Castillo<sup>c</sup>, Charles Zheng<sup>d</sup>, Peter Bandettini<sup>c</sup>, César Caballero-Gaudes<sup>a,\*</sup>

<sup>a</sup>Basque Center on Cognition, Brain and Language, Donostia - San Sebastián, Spain

<sup>b</sup>University of the Basque Country (EHU/UPV), Donostia-San Sebastián, Spain

<sup>c</sup>Section on Functional Imaging Methods, National Institute of Mental Health, Bethesda, MD 20892, United States

<sup>d</sup>Machine Learning Team, Functional Magnetic Resonance Imaging Facility, National Institute of Mental Health, Bethesda, MD 20892, United States

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## Abstract

Conventionally, analysis of functional MRI (fMRI) data relies on available information about the experimental paradigm to establish hypothesized models of brain activity. However, this information can be inaccurate, incomplete or unavailable in multiple scenarios such as resting-state, naturalistic paradigms or clinical conditions. In these cases, blind estimates of neuronal-related activity can be obtained with paradigm-free analysis methods such as hemodynamic deconvolution. Yet, current formulations of the hemodynamic deconvolution problem have three important limitations: 1) their efficacy strongly depends on the appropriate selection of regularization parameters, 2) being univariate, they do not take advantage of the information present across the brain, and 3) they do not provide any measure of statistical certainty associated with each detected event. Here we propose a novel approach that addresses all these limitations. Specifically, we introduce MvME-SPFM (multivariate multi-echo sparse paradigm free mapping), a novel hemodynamic deconvolution algorithm that operates at the whole brain level and adds spatial information via a mixed-norm regularization term over all voxels. Additionally, MvME-SPFM employs a stability selection procedure that removes the need to select regularization parameters and also lets us obtain an estimate of the true probability of having a neuronal-related BOLD event at each voxel and time-point based on the area under the curve (AUC) of the stability paths. Besides, the formulation is tailored for multi-echo fMRI acquisitions, which allows us to better isolate fluctuations of BOLD origin on the basis of their linear dependence with Echo Time (TE) and to assign physiologically interpretable units (i.e., changes in the apparent transverse relaxation  $\Delta R_2^*$ ) to the resulting deconvolved events. We demonstrate that this algorithm outperforms existing state-of-the-art deconvolution approaches, and shows higher spatial and temporal agreement with the activation maps and BOLD signals obtained with a standard model-based linear regression approach, even at the level of individual neuronal events. Consequently, the proposed algorithm provides more reliable estimates of neuronal-related activity, here in terms of  $\Delta R_2^*$ , for the study of the dynamics of brain activity when no information about the timings of the BOLD events is available. This algorithm will be made publicly available as part of the *splora* Python package.

**Keywords:** multi-echo fMRI, hemodynamic deconvolution, inverse problems, stability selection

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## 1. Introduction

Functional magnetic resonance imaging (fMRI) data analysis relies on the blood oxygenation level-dependent (BOLD) contrast as a proxy to localize neuronal activity and to study the functional organization of the human brain in vivo when performing a task or at rest. Due to the nature of the paradigms, task and resting state fMRI data are analyzed with different techniques. Often, the analysis of task fMRI data is performed using general linear models (GLM) that calculate statistical parametric maps of brain activity by building hypothetical timecourses of the BOLD responses to the experimental paradigm, thus exploiting the knowledge of the timings of the stimuli. However, the analysis of other types of fMRI paradigms, such as resting-state, naturalistic paradigms, or clinically-relevant assessments, cannot be

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\*Corresponding authors

Email addresses: [e.uruñuela@bcbl.eu](mailto:e.uruñuela@bcbl.eu) (Eneko Uruñuela), [c.caballero@bcbl.eu](mailto:c.caballero@bcbl.eu) (César Caballero-Gaudes)

10 performed with a GLM given that the timings of the stimuli are unknown, inaccurate or insufficient, and  
11 hence requires a paradigm free approach. Such data are typically analyzed with correlation-based meth-  
12 ods; for example, static and dynamic functional connectivity (Prete et al., 2017), edge-centric measures  
13 (Faskowitz et al., 2020a), and inter-subject correlations (Hasson et al., 2004) Another method often used  
14 to analyze resting-state fMRI data are co-activation patterns (CAPs) (Liu et al., 2013, 2018).

15 However, as all these techniques operate on the BOLD signal, they are affected by the blurring that the  
16 hemodynamic response introduces to the signal, which makes the interpretation of the analyses uncertain.  
17 In order to undo this blurring effect and obtain more reliable estimates of the neuronal activity, various  
18 deconvolution techniques can be used (Glover, 1999; Gitelman et al., 2003; Gaudes et al., 2010, 2012, 2013;  
19 Caballero-Gaudes et al., 2019; Hernandez-Garcia & Ulfarsson, 2011; Karahanoglu et al., 2013; Cherkaoui  
20 et al., 2019; Costantini et al., 2022; Hütel et al., 2021). These techniques are able to blindly (i.e., with  
21 no information about the timings of neuronal events) estimate the neuronal activity that induces the  
22 BOLD response by assuming a hemodynamic response function (HRF) and solving an inverse problem  
23 with additional constraints to overcome the ill-posed nature of hemodynamic deconvolution (Uruñuela  
24 et al., 2021a).

25 The ability of deconvolution algorithms to estimate neuronal activity in a paradigm-free manner has  
26 been exploited in a number of applications. For instance, deconvolution techniques have been used on  
27 resting-state fMRI data to explore time-varying activity (Petridou et al., 2013; Karahanoglu & Ville,  
28 2015; Prete et al., 2017; Keilholz et al., 2017; Lurie et al., 2020; Bolton et al., 2020), to decode the  
29 flow of spontaneous thoughts and actions across different cognitive and sensory domains (Tan et al.,  
30 2017; Gonzalez-Castillo et al., 2019), and to investigate modulatory interactions within and between  
31 resting-state functional networks (Di & Biswal, 2013). These methods have also been applied in clinical  
32 conditions to detect the foci of interictal events in epilepsy patients without the use of EEG recordings  
33 (Lopes et al., 2012; Karahanoglu et al., 2013; Tobias et al., 2022), to investigate functional dissociations  
34 found during non-rapid eye movement sleep associated with reduced consolidation of information and  
35 impaired consciousness (Tarun et al., 2021), and to detect functional signatures of prodromal psychotic  
36 symptoms and anxiety at rest in patients with schizophrenia (Zöller et al., 2019).

37 Despite the range of deconvolution methods that have been developed, few capitalize on the various  
38 properties of fMRI data, such as the advantages of multi-echo fMRI for denoising fMRI data (Bright &  
39 Murphy, 2013; Kundu et al., 2017), or the use of tissue-based or parcellation-based information to improve  
40 the accuracy of the estimates of neuronal activity. Recent exceptions include deconvolution algorithms  
41 that incorporate a multivariate formulation to perform spatio-temporal deconvolution (Bolton et al.,  
42 2019a; Uruñuela et al., 2021b; Costantini et al., 2022). In addition, one deconvolution algorithm has  
43 been presented that exploits the mono-exponential decay model of the multi-echo fMRI signal: multi-echo  
44 sparse paradigm free mapping (ME-SPFM) (Caballero-Gaudes et al., 2019). Furthermore, approaches  
45 have been developed to estimate the likelihood of having a neuronal event at each time-point and for  
46 each voxel by means of logistic regression (Bush & Cisler, 2013; Bush et al., 2015) or Gaussian mixture  
47 models (Pidnebesna et al., 2019). Wouldn't it be nice to obtain a measure of the probability of each  
48 voxel containing a neuronal event at each time-point for regularized estimators while exploiting the  
49 mono-exponential decay and spatio-temporal properties of the multi-echo fMRI signal?

50 In this work, we propose a novel approach for the hemodynamic deconvolution of multi-echo fMRI  
51 data that operates at the whole-brain level (i.e., multivariate formulation) to incorporate spatial informa-  
52 tion through a mixed-norm regularization term. Furthermore, we propose a stability selection procedure  
53 (Meinshausen & Bühlmann, 2010) that makes the estimation of the neuronal activity more robust to the  
54 selection of the regularization parameters, while providing the likelihood of having a neuronal-related  
55 event at each time-point and for each voxel. Using multi-echo fMRI data acquired from 10 healthy  
56 subjects (16 datasets) we demonstrate that the proposed multivariate multi-echo paradigm free mapping  
57 (MvME-SPFM) algorithm not only provides more robust estimates of the neuronal activity, but also  
58 yields a measure of the probability of each voxel containing a neuronal event at each time-point. More-  
59 over, MvME-SPFM returns quantitative estimates of  $\Delta R_2^*$  in interpretable units ( $s^{-1}$ ), which is relevant  
60 for functional analysis across different acquisition methods and field strengths.

## 61 2. Theory

### 62 2.1. Voxel-wise signal model for multi-echo paradigm free mapping

63 The analysis of BOLD fMRI data usually assumes that the signal  $y(t)$  acquired for a voxel  $v$  is  
64 described by the convolution between the activity-inducing signal  $s(t)$  driving the BOLD response and  
65 the hemodynamic response  $h(t)$  itself (Boynton et al., 1996; Glover, 1999), plus an additional term

66  $e(t)$  representing noise. Considering that the signal measured by the scanner is sampled at every TR  
 67 seconds, the acquired signal can be written in discrete form as:  $y(n) = \sum_{i=0}^{L-1} h(i)s(n-i) + e(n)$ , for  
 68  $n = 1, \dots, N$ , where  $N$  is the number of observations in the time-series, and  $L$  is the discrete-time length  
 69 of the hemodynamic response function (HRF).

70 Hence, the signal model can be written in matrix notation as:

$$\mathbf{y} = \mathbf{H}\Delta\mathbf{s} + \mathbf{e} \quad (1)$$

71 where  $\mathbf{y}$ ,  $\Delta\mathbf{s}$ ,  $\mathbf{e} \in \mathbb{R}^N$  are the voxel's time-series, the activity-inducing signal changes and the noise term,  
 72 respectively, and  $\mathbf{H} \in \mathbb{R}^{N \times N}$  is the Toeplitz convolution matrix defined by the HRF (Gitelman et al.,  
 73 2003; Gaudes et al., 2013).

74 For gradient-echo fMRI acquisitions, the voxel's time-series in terms of the signal percentage change  
 75 has a linear relationship with the echo time (TE) as  $y(\text{TE}_k, n) \approx \Delta\rho(n) - \text{TE}_k \Delta R_2^*(n)$ , where  $\Delta R_2^*(n)$   
 76 denotes the BOLD-like signal changes and  $\Delta\rho(n)$  corresponds to changes in the net magnetization, for  
 77 instance due to head motion (Kundu et al., 2017). The signal changes associated to fluctuations in the  
 78 net magnetization can be effectively reduced in preprocessing, for example using multi-echo independent  
 79 component analysis (Kundu et al., 2012; Caballero-Gaudes et al., 2019), and are neglected hereinafter.  
 80 Hence, considering that neuronal-related signal changes  $\Delta\mathbf{s}$  produce a change in  $\Delta R_2^*$ , the signal model  
 81 in Eq.(1) can be adapted to contain the signal acquired at all  $K$  echo-times (TE) via concatenation:

$$\begin{bmatrix} \mathbf{y}_1 \\ \vdots \\ \mathbf{y}_K \end{bmatrix} = - \begin{bmatrix} \text{TE}_1 \mathbf{H} \\ \vdots \\ \text{TE}_K \mathbf{H} \end{bmatrix} \Delta\mathbf{s}, \quad (2)$$

82 which can be simplified into  $\bar{\mathbf{y}} = -\bar{\mathbf{H}}\Delta\mathbf{s}$ . An estimate of the activity that induces the BOLD response  $\hat{\mathbf{s}}$   
 83 can be obtained by solving an ordinary least-squares problem such as:

$$\Delta\hat{\mathbf{s}} = \arg \min_{\mathbf{s}} \frac{1}{2} \|\bar{\mathbf{y}} - \bar{\mathbf{H}}\Delta\mathbf{s}\|_2^2. \quad (3)$$

84 However, solving the equation above is an ill-posed problem given the high collinearity of the convolution  
 85 matrix  $\mathbf{H}$  due to the overlap between shifted HRFs, which introduces large variability in the estimates  
 86 of  $\mathbf{s}$ .

87 In practice, this excess of variability can be reduced by introducing additional assumptions about  
 88 the activity-inducing signal in the form of regularization terms. For instance, we could assume that  
 89 the activity-inducing signal is well represented by a reduced subset of non-zero coefficients at the fMRI  
 90 timescale that trigger the BOLD responses. This assumption can be mathematically represented with  
 91 a sparsity-promoting regularization term such as the  $\ell_1$ -norm that is added to the data fitting term in  
 92 Eq.(3) (Tibshirani, 1996; Gaudes et al., 2013).

93 Hence, the activity-inducing signal in a single voxel can be blindly detected from the multi-echo  
 94 signals by solving the following inverse problem (Caballero-Gaudes et al., 2019):

$$\Delta\hat{\mathbf{s}} = \arg \min_{\mathbf{s}} \frac{1}{2} \|\bar{\mathbf{y}} - \bar{\mathbf{H}}\Delta\mathbf{s}\|_2^2 + \lambda \|\Delta\mathbf{s}\|_1, \quad (4)$$

95 where  $\lambda$  is the regularization parameter that regulates the level of sparsity of the estimates given the  $\ell_1$ -  
 96 norm, which is defined as  $\|\Delta\mathbf{s}\|_1 = \sum_{n=1}^N |\Delta\mathbf{s}_n|$ . The tuning of the regularization parameter is challenging  
 97 and requires the careful selection of an adequate value in order to avoid overfitting (i.e., false detection  
 98 of the activity-inducing signal) or underfitting (i.e., no detection of the activity-inducing signal).

## 99 2.2. Whole-brain signal model for multi-echo paradigm free mapping

100 Assuming that the shape of the hemodynamic response can be similarly modeled across all brain  
 101 voxels, the previous voxel-wise (i.e., univariate) model in Eq.(2) can be extended straightforwardly to a  
 102 multivariate formulation that considers all the voxels  $V$  of the brain:

$$\begin{bmatrix} \mathbf{y}_{1,1} & \cdots & \mathbf{y}_{1,V} \\ \vdots & \ddots & \vdots \\ \mathbf{y}_{K,1} & \cdots & \mathbf{y}_{K,V} \end{bmatrix} = - \begin{bmatrix} \text{TE}_1 \mathbf{H} \\ \vdots \\ \text{TE}_K \mathbf{H} \end{bmatrix} [\Delta\mathbf{s}_1 \quad \cdots \quad \Delta\mathbf{s}_V], \quad (5)$$

103 which can be simplified into  $\bar{\mathbf{Y}} = -\bar{\mathbf{H}}\Delta\mathbf{S}$ , where  $\bar{\mathbf{Y}} \in \mathbb{R}^{KN \times V}$ ,  $\bar{\mathbf{H}} \in \mathbb{R}^{KN \times N}$  and  $\Delta\mathbf{S} \in \mathbb{R}^{N \times V}$ .

104 The inverse problem in Eq.(4) can be directly adapted to be solved at the whole-brain using the  
 105 multivariate formulation in Eq.(5). More interestingly though, solving the inverse problem at the whole-  
 106 brain level opens up many possibilities in the form of additional regularization terms to take advantage  
 107 of the spatial information for an informed estimation of the activity-inducing signal  $\Delta\hat{\mathbf{S}}$ . For instance,  
 108 mixed-norms in the form of  $\ell_{p,q}$  can be employed to separate coefficients into groups that are blind to each  
 109 other, while the coefficients within a group are treated together (Kowalski, 2009). Hence, regularization  
 110 terms based on mixed-norms can promote spatio-temporal structures that are observed in fMRI signals.

111 Here, we add an  $\ell_{2,1} + \ell_1$  mixed-norm regularization term (Gramfort et al., 2011) to the multi-  
 112 variate convex problem to promote the co-activation of the activity-inducing signal  $\Delta\hat{\mathbf{S}}$  considering the  
 113 coefficients of the voxels of the brain ( $V$ ) at time  $n$  as one group:

$$\Delta\hat{\mathbf{S}} = \arg \min_{\mathbf{S}} \frac{1}{2} \|\bar{\mathbf{Y}} - \bar{\mathbf{H}}\Delta\mathbf{S}\|_2^2 + \lambda\rho\|\Delta\mathbf{S}\|_1 + \lambda(1-\rho)\|\Delta\mathbf{S}\|_{2,1}, \quad (6)$$

114 where  $\ell_{2,1}$ -norm is defined as  $\|\Delta\mathbf{S}\|_{2,1} = \sum_{n=1}^N \sqrt{\sum_{v=1}^V \Delta\mathbf{S}_{n,v}^2}$ , and  $0 < \rho < 1$  is a parameter that controls  
 115 the tradeoff between the sparsity introduced by the  $\ell_1$ -norm and the grouping of voxels promoted by the  
 116  $\ell_{2,1}$ -norm so that the estimation of one voxel coefficient at time  $n$  is influenced by the estimates of the  
 117 rest of the brain voxels at the same time. Note that when  $\rho = 1$  Eq. (6) is the whole-brain equivalent of  
 118 Eq. (4) On the other hand, the regularization parameter  $\lambda$  can be adapted voxel-wise in order to account  
 119 for differences in the signal-to-noise ratio across voxels. Consequently, the multivariate deconvolution  
 120 problem can be written as:

$$\Delta\hat{\mathbf{S}} = \arg \min_{\mathbf{S}} \frac{1}{2} \|\bar{\mathbf{Y}} - \bar{\mathbf{H}}\Delta\mathbf{S}\|_2^2 + \rho\|\mathbf{D}\Delta\mathbf{S}\|_1 + (1-\rho)\|\mathbf{D}\Delta\mathbf{S}\|_{2,1}, \quad (7)$$

121 where  $\mathbf{D} = \text{diag}(\lambda_1, \dots, \lambda_V) \in \mathbb{R}^{V \times V}$  is a diagonal matrix with the voxel-specific values of  $\lambda$ . In practice,  
 122 a criterion must be used to select the voxel-specific  $\lambda$ s. Instead, we propose the use of stability selection  
 123 to avoid this critical choice (see Section 3.2).

124 Therefore, given the convex nature of the inverse problem in Eq. (7), estimates of  $\Delta\hat{\mathbf{S}}$  can be calculated  
 125 using the fast iterative shrinkage-thresholding algorithm (FISTA) (Beck & Teboulle, 2009) with the  
 126 following proximity operator for  $\ell_1 + \ell_{2,1}$ :

$$S_{n,v} = \frac{Z_{n,v}}{|Z_{n,v}|} (|Z_{n,v} - \lambda_v\rho|)^+ \left( 1 - \frac{\lambda_v(1-\rho)}{\sqrt{\sum_v (|Z_{n,v}| - \lambda_v\rho)^+{}^2}} \right)^+, \quad (8)$$

127 where  $\Delta\mathbf{S} = \text{prox}_{\lambda(\rho\|\cdot\|_1 + (1-\rho)\|\cdot\|_{2,1})}(\mathbf{Z}) \in \mathbb{R}^{N \times V}$ ,  $(x)^+ = \max(x, 0)$  for  $x \in \mathbb{R}$ , and  $\frac{0}{0} = 0$  by convention.

### 128 3. Methods

#### 129 3.1. fMRI data acquisition and preprocessing

130 The evaluation of the proposed MvME-SPFM was performed on ME-fMRI data acquired in 10 sub-  
 131 jects using a multi-task rapid event-related paradigm. Six subjects performed two functional runs, the  
 132 other 4 subjects only performed 1 run due to scanning time constraints (i.e., a total of 16 datasets). All  
 133 participants gave informed consent in compliance with the NIH Combined Neuroscience International  
 134 Review Board-approved protocol 93-M-1070 in Bethesda, MD. A thorough description of the MRI ac-  
 135 quisition protocols and experimental tasks in the experimental design can be found in (Gonzalez-Castillo  
 136 et al., 2016), only those details that are relevant to this analysis are given here.

137 MRI data was acquired on a General Electric 3T 750 MRI scanner with a 32-channel receive-only  
 138 head coil (General Electric, Waukesha, WI). Functional scans were acquired with a ME gradient-recalled  
 139 echoplanar imaging (GRE-EPI) sequence (flip angle =  $70^\circ$  for 9 subjects, flip angle =  $60^\circ$  for 1 subject,  
 140 TEs = 16.3/32.2/48.1 ms, TR = 2 s, 30 axial slices, slice thickness = 4 mm, in-plane resolution =  $3 \times 3$   
 141 mm<sup>2</sup>, FOV 192 mm, acceleration factor 2, number of acquisitions = 220). Functional data was acquired  
 142 with ascending sequential slice acquisitions, except in one subject where the acquisitions were interleaved.  
 143 In addition, high resolution T1-weighted MPRAGE and proton density images were acquired per subject  
 144 for anatomical alignment and visualization purposes (176 axial slices, voxel size =  $1 \times 1 \times 1$  mm<sup>3</sup>, image  
 145 matrix =  $256 \times 256$ ).

146 Each run of data acquisition consisted of 6 trials with 5 different tasks each: biological motion  
 147 observation (BMOT), finger tapping (FTAP), passive viewing of houses (HOUS), listening to music

148 (MUSI), and sentence reading (READ). We refer the reader to that paper for details on the preprocessing  
149 steps, and comparison with alternative single-echo models for deconvolution. This data had previously  
150 been employed, preprocessed and ME-ICA denoised for the evaluation of the ME-SPFM algorithm in  
151 (Caballero-Gaudes et al., 2019).

### 152 3.2. Stability selection and the regularization parameter $\lambda$

153 The choice of the regularization parameter  $\lambda$  is crucial to obtain accurate estimates of  $\Delta\hat{\mathbf{S}}$ . Although  
154 the value of  $\lambda$  of each voxel could be fixed ad-hoc, previous work has opted for the use of model selection  
155 criteria, such as the Bayesian Information Criterion (BIC), on the regularization path (Caballero-Gaudes  
156 et al., 2019), computed by means of the least angle regression (LARS) algorithm (Efron et al., 2004).  
157 Even though the use of BIC performed well for ME-SPFM (Caballero-Gaudes et al., 2019) and its single-  
158 echo counterpart (SPFM) (Gaudes et al., 2013), due to its high specificity, it can be problematic for  
159 certain voxels where the BIC curve might present multiple local minima or even fail to present a clear  
160 minima for the evaluated range of  $\lambda$ .

161 In this work, we propose a more robust procedure to address this shortcoming with the usage of the  
162 stability selection method (Meinshausen & Bühlmann, 2010). Moreover, the stability selection procedure  
163 presented here yields the probability to have a non-zero coefficient in the activity-inducing signal at  
164 each time-point. Specifically, our implementation of the stability selection procedure generates  $T = 30$   
165 surrogates by randomly subsampling 60% of the time-points (we also tested a more computationally  
166 expensive version with  $T = 100$  surrogates that yielded very similar results). The convolution matrix  $\mathbf{H}$   
167 is subsampled accordingly. The subsampled data is then employed to solve the inverse problem in Eq. (6)  
168 for a range of different values of  $\lambda$  using the fast iterative shrinkage thresholding algorithm (FISTA).  
169 For each voxel, we select a logarithmically spaced sequence of 30 values between 5% and 95% of the  
170 voxel-specific maximum  $\lambda$  possible to more accurately sample the lower range. Then, for each time-point  
171 and value of  $\lambda$ , stability selection calculates the ratio (probability) of surrogates where the estimated  
172 coefficient at each time-point is non-zero. As illustrated in Figure 1, these probabilities build the so-  
173 called stability paths, which resemble the well-known regularization paths of conventional regularized  
174 estimators (e.g., LASSO, Ridge Regression) that plot the amplitude of the coefficients for each  $\lambda$ .

175 Unlike the original stability selection procedure, which sets a given probability threshold to select  
176 the final set of non-zero coefficients (Meinshausen & Bühlmann, 2010), we calculate the area under the  
177 curve (AUC) of the stability path of each time-point as an index of confidence of having a non-zero  
178 coefficient across the evaluated range of  $\lambda$ . As a result, the AUC timecourse provides a measure of  
179 the probability of having neuronal-related activity at each time-point and voxel. Next, the AUC time-  
180 series are thresholded according to the histogram of AUC values in a region of non-interest (hereinafter,  
181 denoted as the null AUC histogram) to yield a sparse representation of the signal. Alternatively, a null  
182 distribution of AUC values could be generated from surrogate data (Liégeois et al., 2021). Accordingly,  
183 when employing stability selection, the individual voxels' estimates might not be equivalent to the voxels'  
184 estimates in any single one of the whole-brain models that can be formulated with a given value of  $\lambda$   
185 in Eq. (6) or  $\mathbf{D}$  in Eq. (7)), but are rather obtained by computing area-under-the-curve (AUC) values for  
186 neuronal-related events.

187 Finally, we apply a fitting step to each voxel by defining a reduced convolution model with the selected  
188 non-zero coefficients and fitting it by means of a conventional orthogonal least squares estimator. This  
189 step reduces the bias towards zero imposed by the sparsity-promoting regularization terms, and thus  
190 obtains more realistic estimates of the neuronal-related signal (here, in terms of  $\Delta R_2^*$ ) (Caballero-Gaudes  
191 et al., 2019).

### 192 3.3. Balancing the spatial regularization

193 The  $\ell_{2,1}$ -norm regularization term in Eq. (6) promotes structured spatio-temporal sparsity in the  
194 sense that the estimates of all brain voxels at a given time-point are treated as a group and this term  
195 forms a constraint on the number of groups with at least one non-zero estimate to model the data.  
196 Assuming that  $\rho = 0$ , either the value of all the voxel estimates at one time point can be non-zero or all  
197 of them are nulled. Hence, this regularization term considers spatial information from all brain voxels  
198 for the deconvolution since the value of a given voxel coefficient also depends on the rest of the voxels.

199 To illustrate the effect of the corresponding regularization parameter  $\rho$ , in this work we solve the  
200 multivariate regularization problem in Eq. (6) using stability selection for  $\rho = 1$ ,  $\rho = 0.5$  and  $\rho = 0$ ; i.e.,  
201 applying the sparsity-promoting  $\ell_1$ -norm only, equally weighting the sparsity and spatial regularizations,  
202 and employing the  $\ell_{2,1}$ -norm spatial regularization only, respectively.

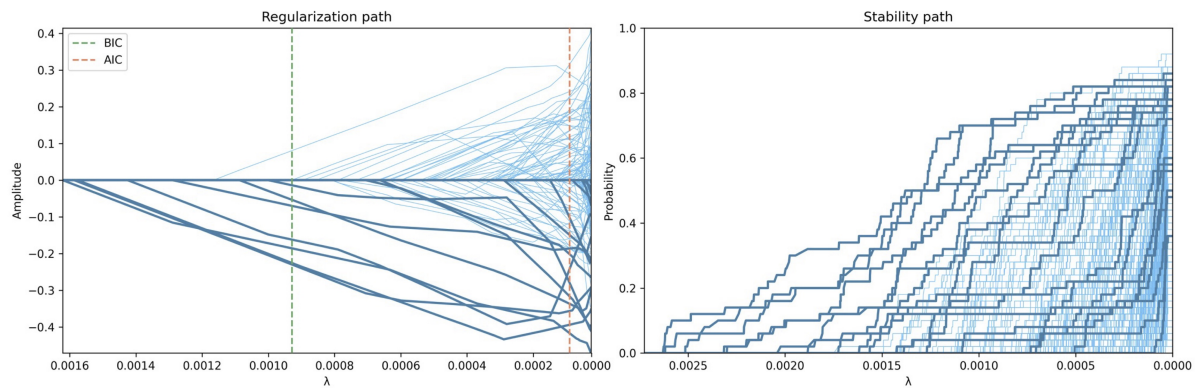


Figure 1: Example of the regularization path and the stability path for a voxel timeseries with  $\rho = 1$ . On the left, the regularization path shows the amplitude of each coefficient estimate  $\Delta\hat{\mathbf{S}}$  (one per TR). At first, all the coefficients are zero and successively they become non-zero as  $\lambda$  decreases towards zero, which corresponds to the orthogonal least squares solution (i.e., no regularization). On the right, the corresponding stability path plots the probability that each coefficient estimate is non-zero for each value of  $\lambda$  based on the stability selection procedure. Note that both paths can have a different maximum value of  $\lambda$  given the subsampling step in the stability selection. The darker lines denote the coefficient estimates corresponding to the TRs during the task-related events.

### 3.4. Comparison with conventional timing-based GLM analyses

To evaluate how the multivariate formulation combined with stability selection improves the accuracy of the estimates of  $\Delta\hat{\mathbf{S}}$  compared with its univariate counterpart ME-SPFM using the BIC for voxel-wise selection of  $\lambda$  (Caballero-Gaudes et al., 2019), we calculated the spatial sensitivity, specificity and overlap (using a Dice coefficient metric) of the MvME-SPFM activation maps using the trial-level GLM-based activation maps ( $p \leq 0.05$ ) as the ground truth. These GLM-based maps were obtained from the optimally combined and ME-ICA denoised data, and only negative  $\Delta R_2^*$  (i.e.,  $\Delta\hat{\mathbf{S}} < 0$  that generate a positive BOLD response) were considered for the computation of the Dice coefficients.

For the MvME-SPFM, we considered the following two strategies for thresholding the AUC timeseries in order to define the corresponding activation maps:

- **Static thresholding:** The estimates of  $\Delta\hat{\mathbf{S}}$  obtained with the novel MvME-SPFM technique that utilizes stability selection, where the AUC threshold was chosen as the 95th percentile of the histogram of AUC in deep white matter voxels (i.e., a fixed, static threshold), which were labeled after tissue segmentation of the T1-weighted anatomical MRI using *3dSeg* in AFNI, and eroding 4 voxels of the resulting white matter tissue mask at anatomical resolution.
- **Time-dependent thresholding:** The estimates of  $\Delta\hat{\mathbf{S}}$  obtained with the novel MvME-SPFM technique with stability selection, where the AUC threshold varies temporally according to the 95th percentile of the null histogram of AUC at each time-point. This implementation was based on the hypothesis that a time-dependent (TD) threshold would be able to better control for widespread spurious deconvolved changes in  $\Delta\hat{\mathbf{S}}$ , for instance due to head motion or deep breaths.

## 4. Results

The output of deconvolution algorithms such as ME-SPFM and the proposed MvME-SPFM is a 4D dataset that matches the dimensions (both spatial and temporal) of the input data, i.e., it is a movie of the estimated  $\Delta R_2^*$  maps. In addition, the use of stability selection generates the area under the curve (AUC) 4D output dataset, which indicates the probability of having a neuronal-related event at each time-point for every voxel in the brain.

Figure 2 depicts the area under the curve (AUC) time-series and maps obtained with stability selection for  $\rho = 0.5$  in representative voxels of each task in the paradigm (indicated with a cross in the maps), where the AUC maps correspond to single time-points signaled by the blue arrows. The AUC time-series of the ST and TD thresholding approaches are shown on top of the original AUC time-series. The AUC maps depict spatial patterns of  $\Delta R_2^*$  where regions that are typically involved in the tasks show higher probabilities of having neuronal-related activity compared with other brain regions.

Figure 3 displays the comparison of the  $\Delta R_2^*$  maps obtained by solving the inverse problem in Eq. (6) with a fixed selection of  $\lambda$  (1<sup>st</sup> row) and with the use of stability selection (2<sup>nd</sup>, 3<sup>rd</sup> and 4<sup>th</sup> rows) for

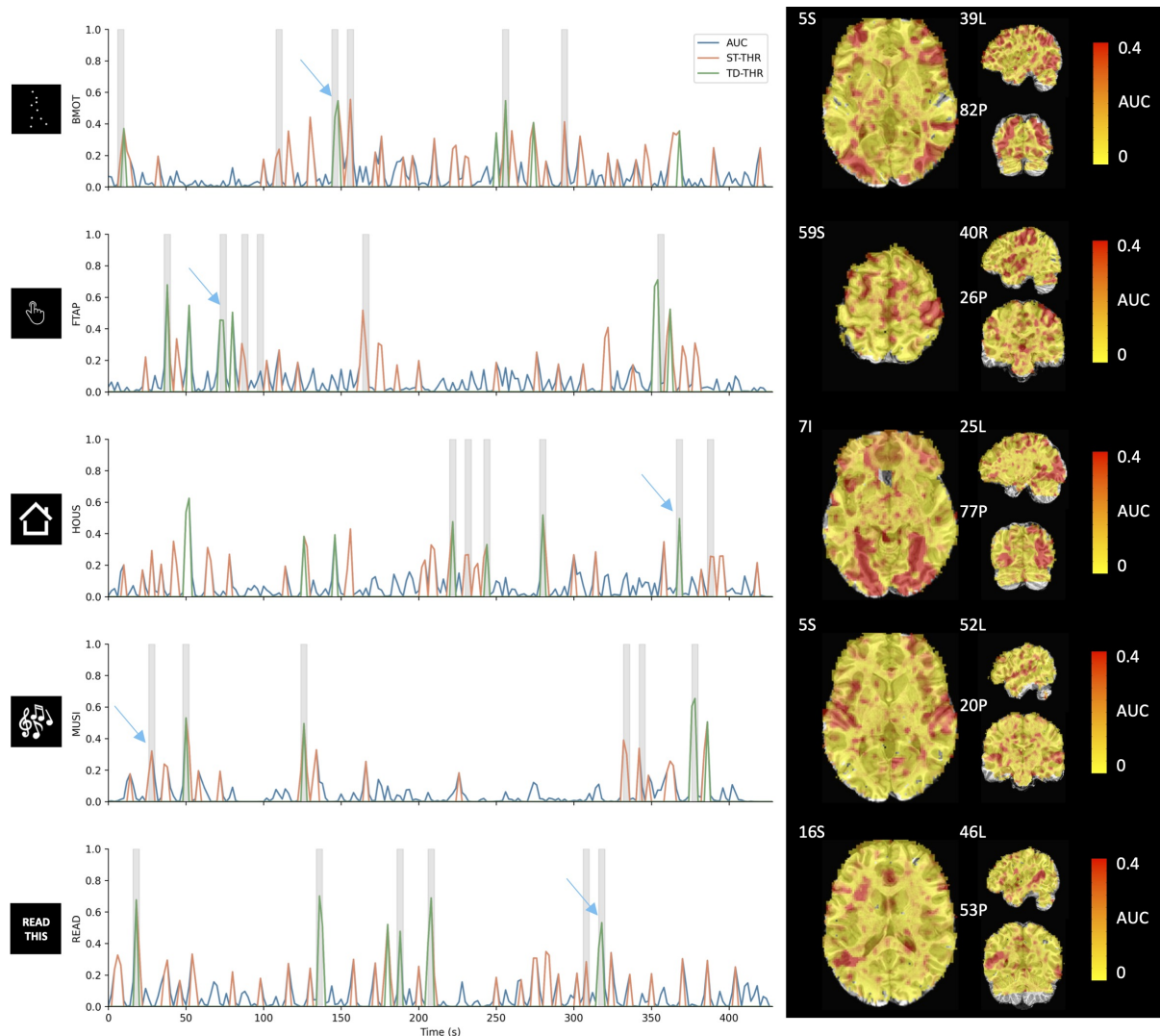


Figure 2: **Left:** Original (blue), ST thresholded (orange) and TD thresholded (green) AUC time-series for a representative voxel for each task in the paradigm ( $\rho = 0.5$ ). Note that the three time-series are overlaid; i.e., the static and time-dependent time-courses are thresholded versions of the original AUC. Gray blocks depict the onset and duration of each trial. **Right:** AUC maps at the time-points signaled by the blue arrows.

237  $\rho = \{0, 0.5, 1\}$ . The  $\Delta R_2^*$  maps obtained with a fixed selection of  $\lambda$  equal to the noise estimate of the  
 238 first echo volume (1<sup>st</sup> row) are very sensitive to the selection of  $\rho$ . Similar observations were obtained  
 239 with other values of  $\lambda$ . With a selection of  $\rho = 1$ , only the  $\ell_1$ -norm regularization term is applied, which  
 240 produces  $\Delta R_2^*$  maps with few non-zero coefficients. With  $\rho = 0$ , only the  $\ell_{2,1}$ -norm spatial regularization  
 241 is applied, which yields a  $\Delta R_2^*$  map that covers the entire brain and does not exhibit a spatial pattern  
 242 in concordance with the task. However, a selection of  $\rho = 0.5$  yields a  $\Delta R_2^*$  map that is more similar to  
 243 the activity maps often observed when participants are asked to look at the image of a house, depicting  
 244 negative  $\Delta R_2^*$  in bilateral fusiform regions. In contrast, the use of stability selection yields AUC maps  
 245 (row 2) and the corresponding  $\Delta R_2^*$  maps after each thresholding strategy (rows 3-4) reveal activation  
 246 patterns concordant with those often seen for viewing houses regardless of the selection of  $\rho$ . In other  
 247 words, the  $\Delta R_2^*$  maps obtained with stability selection are less sensitive to the selection of  $\rho$  while  
 248 obviating the need to choose  $\lambda$ . In fact, the spatial correlations between the AUC maps for each pair of  
 249  $\rho$ 's were nearly equal to 1 for all time points (average correlations are 0.97 between  $\rho = \{0, 0.5\}$ , 0.98  
 250 between  $\rho = \{0, 1\}$ , and 0.97 between  $\rho = \{0.5, 1\}$ ). In addition, it can be seen that using a TD threshold  
 251 yields BOLD signal changes that are more confined to the expected areas in bilateral fusiform cortices  
 252 than the ST threshold. Due to the high similarity of the AUC maps for any value of  $\rho$ , only the results  
 253 for  $\rho = 0.5$  are discussed hereinafter.

254 Figure 4 provides an in-depth view of how the time-dependent thresholding operates when motion-  
 255 and respiration-related artifacts are present in the data. The grayplot (Power, 2017) in Figure 4A clearly

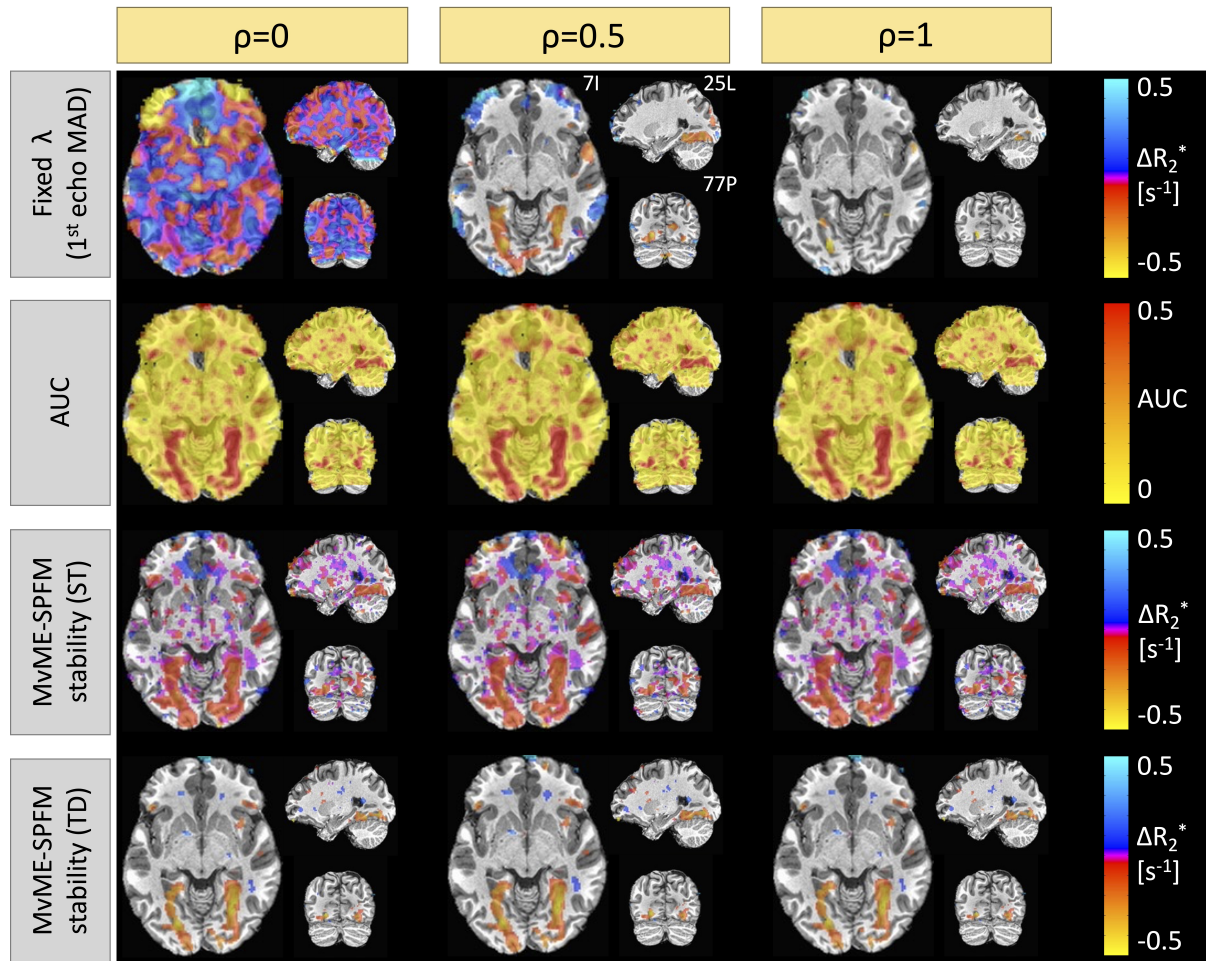


Figure 3: Comparison of the  $\Delta R_2^*$  maps obtained with a fixed selection of  $\lambda$  (row 1) and the use of stability selection (rows 2-4: AUC, stability selection with static thresholding (ST), and stability selection with time-dependent thresholding (TD)) for  $\rho = 0$  (column 1),  $\rho = 0.5$  (column 2), and  $\rho = 1$  (column 3). These maps correspond to a single-trial event of the house-viewing task (HOUS).

256 shows bands spanning throughout the entire brain that illustrate significant changes in the amplitude of  
 257 the signal. The source of these signal changes can be attributed to head motion events (see Euclidean  
 258 norm in Figure 4C) and deep breaths (see arrows for respiration volume signal (Chang et al., 2009) in  
 259 Figure 4D). The respiration-related events cause a drop in the global signal (see Figure 4B) seconds after  
 260 the peak in the respiration volume signal. Interestingly, our results show a decrease in the equivalent  
 261 ST percentile that corresponds to the 95th TD threshold (Figure 4E) at the instances of these large  
 262 respiratory-related events. This decrease can also be observed in the corresponding AUC value of the  
 263 TD thresholding strategy as shown in Figure 4F. The distributions of AUC values at the time-points with  
 264 respiratory- and motion-related artifacts have a shorter tail than the distribution of the AUC values at  
 265 the time-points where subjects performed the task. Hence, in these events the TD thresholding strategy  
 266 is able to adjust the threshold so that the final estimates of  $\Delta R_2^*$  specifically capture task-activated  
 267 voxels while excluding voxels that are affected by artifacts. The higher specificity of the TD thresholding  
 268 strategy can be clearly seen in the ROC curves shown in Figure 4H-L. The use of stability selection  
 269 with the TD threshold yields more specific estimates of  $\Delta R_2^*$  than with ST thresholding or the original  
 270 ME-SPFM method, while the sensitivity is slightly reduced. On the other hand, the use of stability  
 271 selection with a ST threshold improves the sensitivity of the  $\Delta R_2^*$  estimates compared to the original  
 272 ME-SPFM technique while preserving its specificity.

273 Figure 5 illustrates the activation maps of representative single-trial events of each task for the  
 274 same subject depicted in Figure 4. We compared the activation maps of the proposed MvME-SPFM  
 275 formulation using the two thresholding approaches with the activation maps obtained with a single-trial  
 276 GLM and the previous ME-SPFM approach. While all PFM methods exhibit activation maps that  
 277 highly resemble those obtained with the single-trial GLM analysis, differences between the methods can



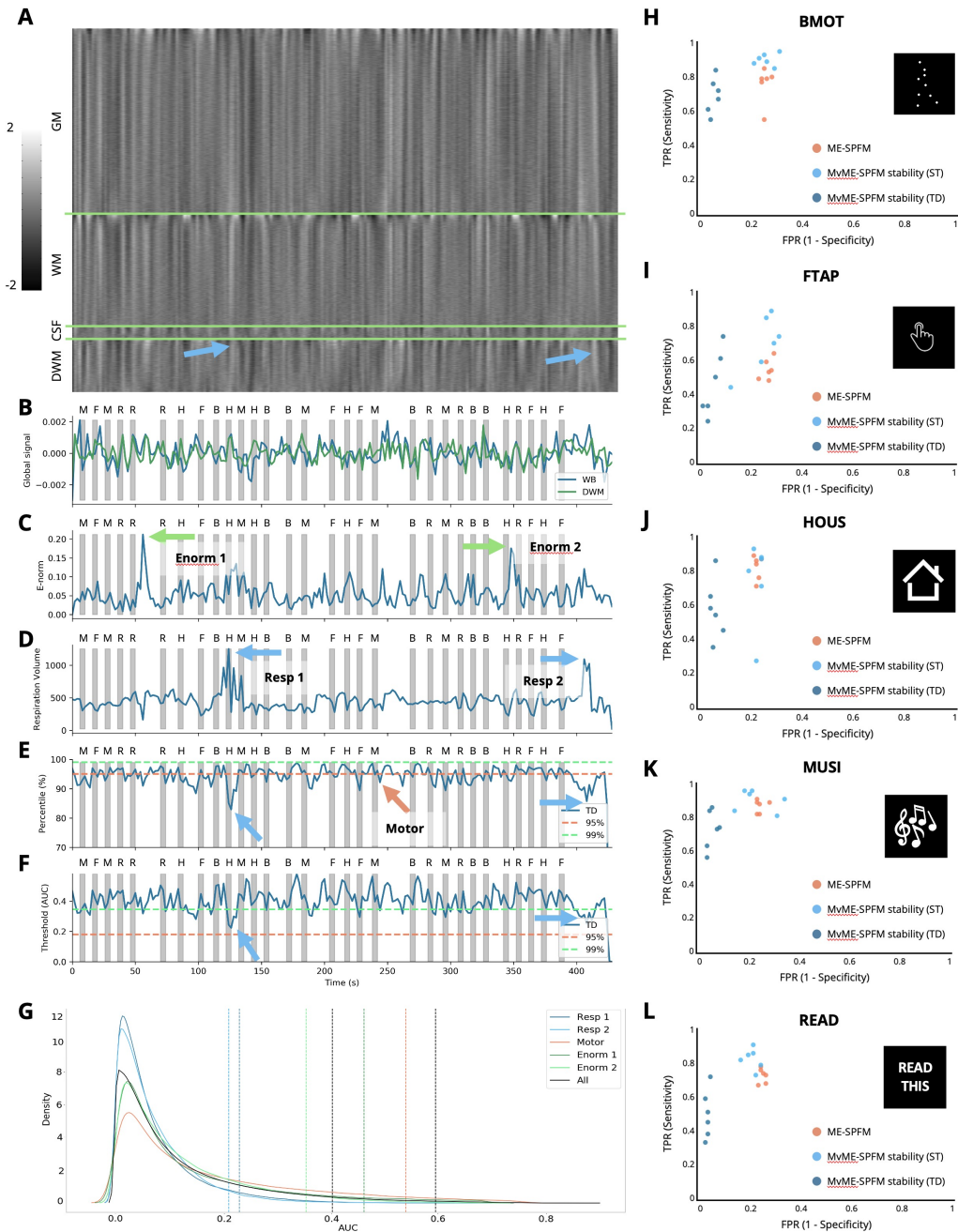


Figure 4: A look at the data of a representative subject with motion and respiration artifacts. **A**: Grayplot of the second echo volume. The grayplot is divided into 4 sections: gray matter (GM), white matter (WM), cerebrospinal fluid (CSF), and deep white matter (DWM). **B**: Time-series of the global signal calculated in the whole brain (WB, blue) and the deep white matter (DWM, green). **C**: Euclidean norm (e-norm) of the temporal derivative of the realignment parameters. **D**: respiration volume signal. **E**: AUC percentile corresponding to the time-dependent threshold (lines at 95th and 99th percentiles are shown for reference). **F**: AUC values corresponding to the time-dependent threshold are shown in blue. The horizontal dashed lines indicate the 95th (orange) and 99th (green) percentiles corresponding to ST thresholding. Gray bars in **B-F** indicate the onset and duration of each trial in the paradigm, with their respective initials on top. Blue arrows point out two respiration-related events, green arrows point out two motion-related events, and the orange arrow points out a finger-tapping event. **G**: Probability density functions (estimated by kernel density estimate) of the AUC values corresponding to the instances of the two respiratory-related events (blue lines), a representative time-point of one finger-tapping trial (orange line), the two largest peaks in the e-norm trace (green lines), and the overall AUC distribution (black). The corresponding coloured vertical dashed lines indicate the AUC value for the 95th percentile of the TD thresholding approach, along with the 95th and 99th AUC values of ST thresholding. **H-L**: Receiver operating characteristic (ROC) curves for the original ME-SPFM (orange), and proposed MvME-SPFM technique with the use of stability selection with the ST (light blue) and TD (dark blue) thresholding approaches for this dataset. The ROC plots depict the sensitivity and specificity of the methods at correctly estimating the activity maps that correspond to the 6 trials of each task in the paradigm.

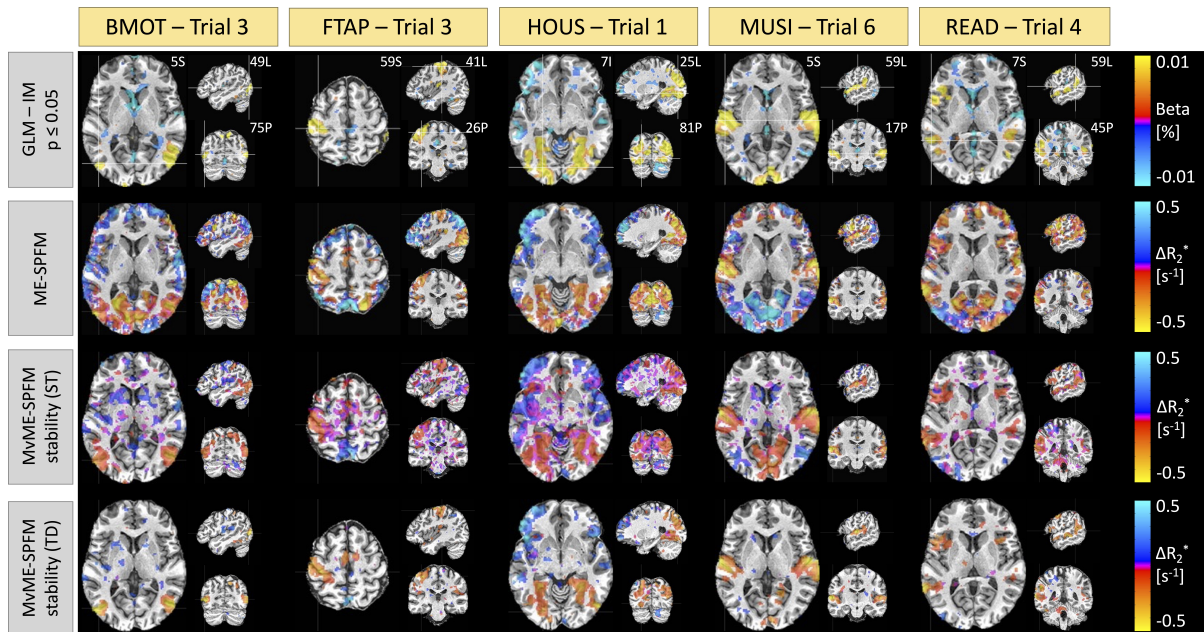


Figure 5: Comparison of single-trial activation maps obtained with a GLM (row 1) thresholded at  $p \leq 0.05$ , the original ME-SPFM formulation with a fixed selection of  $\lambda$  (row 2), the novel MvME-SPFM technique with stability selection,  $\rho = 0.5$  and a static threshold (ST, row 3), and using a time-dependent threshold (TD, row 4). A representative trial is shown for each task. All the maps correspond to the same subject and run shown in Figure 4.

278 be observed. For instance, although the use of stability selection with a ST thresholding approach yields  
 279 maps with clusters of activation of comparable size and location to those found with ME-SPFM, in  
 280 certain noisy trials (e.g., see HOUS-Trial 1), the ST-thresholding MvME-SPFM maps can yield reduced  
 281 spatial specificity, probably related to spurious, scattered changes in  $R_2^*$ . Across all tasks, the maps  
 282 obtained with TD thresholding exhibit a notably larger resemblance to the single-trial GLM, showing  
 283 higher spatial specificity and lower sensitivity compared to the other two PFM methods.

284 Figure 6 depicts the time-series of the estimated  $\Delta R_2^*$  and denoised BOLD, i.e.,  $\Delta R_2^*$  convolved with  
 285 the HRF, for a representative voxel of each task for the subject depicted in Figures 4 and 5 and compared  
 286 to a reference voxel in the lateral ventricles. The location of the voxels is shown in the corresponding  
 287 maps in Figure 5. The ST thresholding approach detects  $\Delta R_2^*$  events of the activity-inducing signal  
 288 that correctly match the timings of the stimuli (i.e., high temporal sensitivity), but also shows events  
 289 that occur in the resting state and do not coincide with any activity-evoking trial. Based on comparison  
 290 with the events detected in the time series extracted from the lateral ventricles, it can be conjectured  
 291 that some of these events might be due to artifactual and physiological fluctuations that remain in the  
 292 signal after preprocessing. On the other hand,  $\Delta R_2^*$  values estimated with the TD thresholding approach  
 293 match the timings of the stimuli almost perfectly with few missed trials (high temporal specificity). This  
 294 is supported by the few  $\Delta R_2^*$  events obtained for the reference voxel in the ventricles. Likewise, the  
 295 denoised BOLD time-series obtained with the TD thresholding approach clearly describes signal changes  
 296 associated with the trials, whereas the denoised BOLD time-series estimated with the ST thresholding  
 297 strategy fits the original data very closely, which could be interpreted as a signature of overfitting.

298 As illustrated in Figure 7, the Dice coefficient between the estimated single-trial  $\Delta R_2^*$  activity maps  
 299 and the reference GLM activity maps ( $p \leq 0.05$ ) demonstrates only a slight improvement over the  
 300 original ME-SPFM formulation when employing an ST thresholding approach with the novel MvME-  
 301 SPFM technique. In contrast, the dice coefficients obtained with TD thresholding show a very notable  
 302 increase of nearly 50% in the median of the distribution of dice coefficients compared with the original ME-  
 303 SPFM approach. Similarly, the sensitivity and specificity distributions of ST thresholding demonstrate  
 304 a slight improvement with respect to the original ME-SPFM formulation. On the other hand, the use  
 305 of TD thresholding offers nearly perfect specificity ( $\geq 95\%$ ) at the cost of reduced sensitivity across  
 306 all experimental conditions. Hence, increasing the specificity of the  $\Delta R_2^*$  maps is more beneficial for  
 307 increasing the concordance with the GLM maps than increasing the sensitivity. The receiver operating  
 308 characteristic (ROC) curves in Figure 8 corroborate these observations regardless of the value of  $\rho$  used  
 309 in the MvME-SPFM method. The estimates obtained with the ST threshold reveal an overall higher

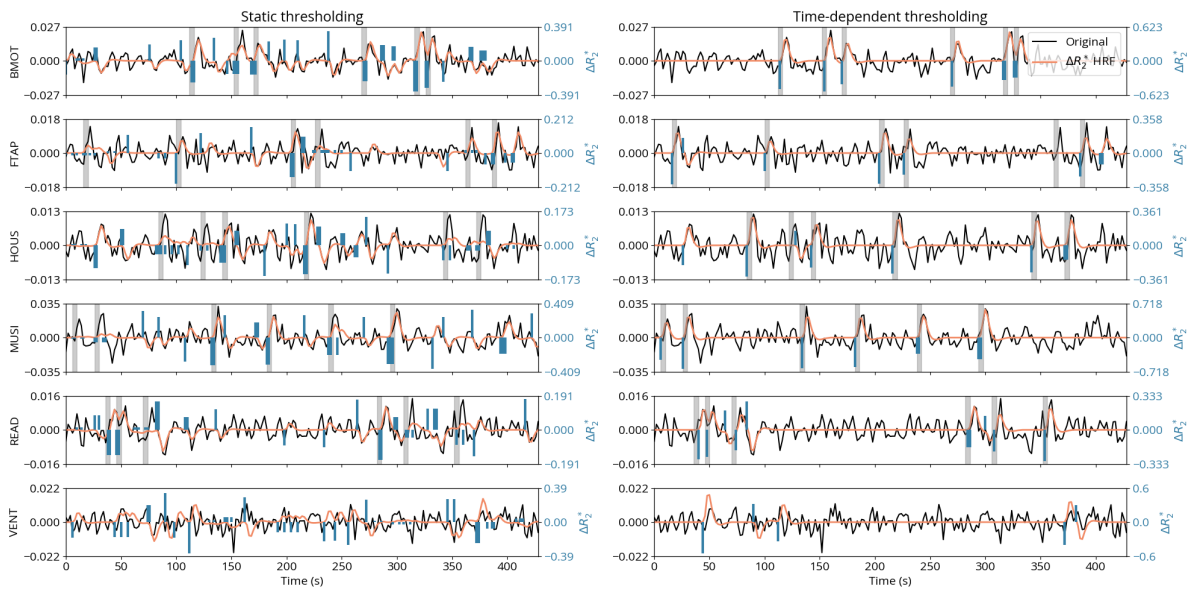


Figure 6: Comparison of the estimated  $\Delta R_2^*$  (blue) and denoised BOLD (orange), i.e.,  $\Delta R_2^*$  convolved with the HRF, time-series when employing the ST (left) and TD (right) thresholding approaches, for representative voxels of each task (rows) as well as one voxel from the lateral ventricle for reference. The estimates shown here were obtained with  $\rho = 0.5$ . The preprocessed time series is shown in black. The gray bars indicate the onset and duration of each trial for each task of the experimental paradigm.

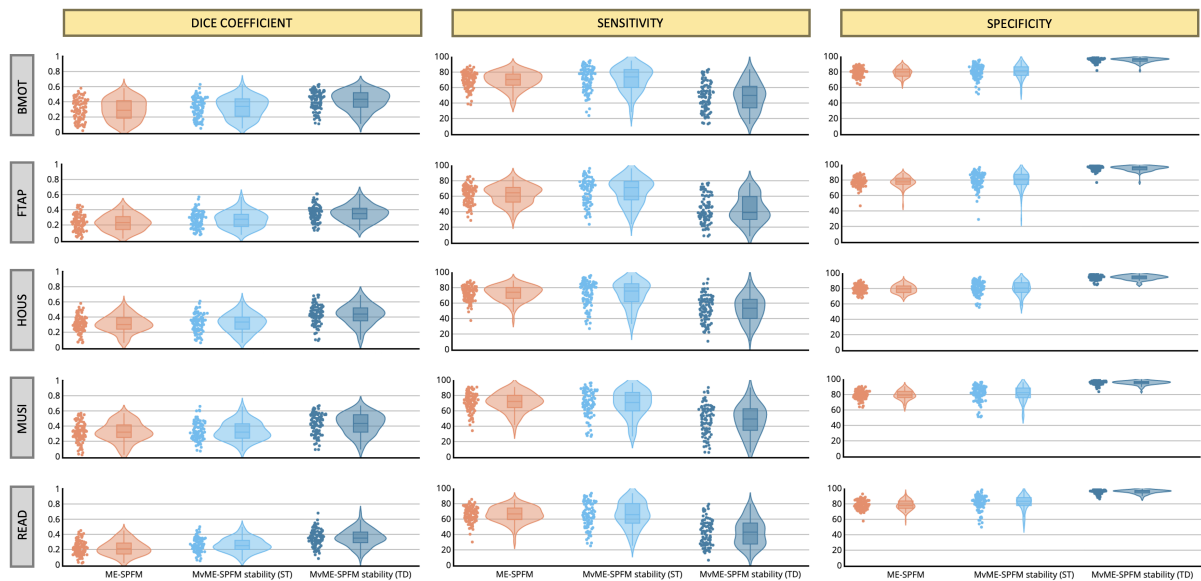


Figure 7: Dice coefficient (i.e., spatial overlap), sensitivity and specificity coefficients of the single-trial activation maps for each of the experimental conditions obtained with ME-SPFM, MvME-SPFM with stability selection and a static thresholding approach (ST), and MvME-SPFM with stability selection and a time-dependent thresholding approach (TD). These metrics were obtained with a selection of  $\rho = 0.5$ . Reference activation maps were obtained with a single trial GLM analysis and thresholded at uncorrected  $p \leq 0.05$ . The density plot shows the shape of the distribution of the dice coefficients, and the box plot depicts the median with a solid line, with each box spanning from quartile 1 to quartile 3. The whiskers extend to 1.5 times the interquartile range.

310 sensitivity and a slightly higher specificity compared to the original ME-SPFM technique. In contrast,  
 311 the ROC curves for the TD thresholding approach show a clear improvement in specificity but lower  
 312 sensitivity. These findings are in line with the results shown in Figures 3, 5 and 6, as the dice and ROC  
 313 curves certify that the use of stability selection yields robust activation maps regardless of the selection of  
 314 the spatial regularization term  $\rho$  and obviating the need to choose the temporal regularization parameter  
 315  $\lambda$ . An interactive version of Figures 7 and 8 is available on the GitHub repository provided in Section 7.

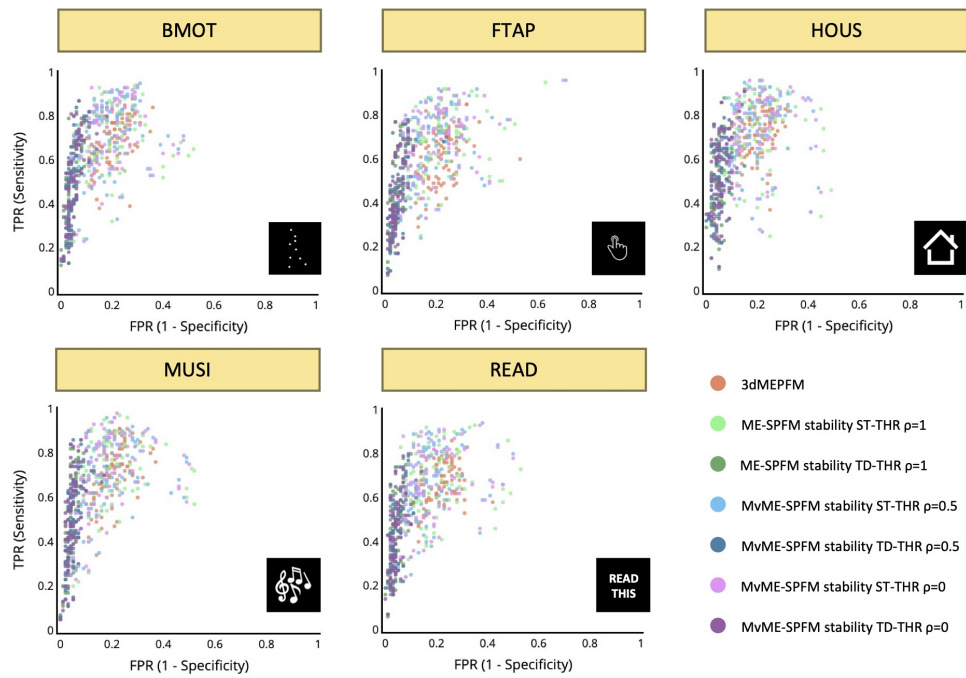


Figure 8: Receiver operating characteristic (ROC) curves with the sensitivity and specificity of each single trial’s activation map for all conditions and the reference map obtained with a single-trial GLM. Different colors are used for the different analyses: the original ME-SPFM, and the novel MvME-SPFM approach using stability selection with three different selections of the spatial regularization parameter  $\rho$  and the two different thresholding methods: static (ST) and time-dependent (TD). In each analysis each dot represents a single trial, depicting all trials across all datasets.

## 316 5. Discussion

317 The proposed whole-brain (i.e., multivariate) formulation for hemodynamic deconvolution of multi-  
 318 echo fMRI data with the use of stability selection achieved a closer agreement with the activation maps  
 319 obtained with a single-trial GLM analysis than the original ME-SPFM method (Caballero-Gaudes et al.,  
 320 2019), while obviating the need to select the temporal regularization parameter  $\lambda$  (see Figure 5). In  
 321 addition, our results illustrated that the stability selection procedure also offers robustness against the  
 322 choice of the spatial regularization parameter  $\rho$ , as the AUC maps for different selections of  $\rho$  were  
 323 practically identical, as shown in Figure 3. Hence, although stability selection could be employed with a  
 324 double selection of the regularization parameters  $\lambda$  and  $\rho$ , this can be avoided for computational reasons  
 325 with little influence in the results. In any case, extending the proposed stability selection technique to  
 326 other formulations of the hemodynamic deconvolution problem, such as the voxel-wise (i.e., univariate)  
 327 single-echo (Gaudes et al., 2013; Uruñuela et al., 2020), univariate multi-echo (Caballero-Gaudes et al.,  
 328 2019), or low-rank and sparse formulations (Uruñuela et al., 2021b; Cherkaoui et al., 2021), is relatively  
 329 straightforward. Moreover, considering that synthesis-based models, such as Paradigm Free Mapping  
 330 (Gaudes et al., 2013), and analysis-based models, such as Total Activation (Karahanoğlu et al., 2013),  
 331 for temporal hemodynamic deconvolution yield identical results (Uruñuela et al., 2021a), and the fact  
 332 that a multi-echo formulation provides higher accuracy for deconvolution (Caballero-Gaudes et al., 2019),  
 333 we argue that the proposed MvME-SPFM method with stability selection should result in more reliable  
 334 estimates of the activity-inducing signal.

335 One of the most interesting features of the proposed stability selection procedure is the estimation of  
 336 the area under the curve (AUC) measure, which provides a new perspective for exploring fMRI data: a 4D  
 337 movie with the probability of each voxel and time point containing a neuronal-related event. Therefore,  
 338 the AUC time-series and maps provide complementary information to the estimates of  $\Delta R_2^*$ , and serve as  
 339 a reliability measure. Even though the AUC measures were employed here to produce the final estimates  
 340 of the activity-inducing signal, they could also be exploited on their own. For instance, they could be  
 341 exploited to constrain functional connectivity analysis (Tagliazucchi et al., 2016; Faskowitz et al., 2020b)  
 342 to voxels and instants with a high probability of containing a neuronal-related event. Furthermore,  
 343 the stability selection and the AUC metric can also be interpreted from a machine learning perspective,  
 344 where the outputs from a collection of lasso learners are combined with an ensemble regression approach.  
 345 Alternatively, the stability selection procedure could also be linked to Bayesian approaches where the

346 prior is given by the range of values of the regularization term  $\lambda$  and the total posterior probability of  
347 the neuronal event is calculated as the integration of the stability paths, i.e., the AUC (see discussion in  
348 Meinshausen & Bühlmann, 2010).

349 Although the estimation of the AUC eliminates the need to select the spatial and temporal regular-  
350 ization parameters  $\lambda$  and  $\rho$ , it requires the use of a thresholding approach given the nature of the AUC  
351 measure, which cannot be equal to zero by definition. Here, we adopted two data-driven thresholding  
352 strategies, static (ST) and time-dependent (TD), based on the AUC values of a region where no BOLD  
353 signal changes related to neuronal activity are assumed to occur (e.g., deep white matter voxels). The  
354 use of a static AUC thresholding approach yielded higher sensitivity than the original ME-SPFM method  
355 (Caballero-Gaudes et al., 2019) while maintaining the specificity as demonstrated in Figure 8. Notably,  
356 this improvement was seen in all trials with the exception of one outlier run, regardless of the choice of  
357 the spatial regularization parameter  $\rho$ . Nevertheless, the use of a time-dependent thresholding approach  
358 may be even justified by the increased specificity and nearly perfect retrieval of the activity-inducing  
359 signal (row 3 in Figure 6) when motion- and respiration-related artifacts are visible in the data (see  
360 arrows in Figure 4). However, the application of the time-dependent threshold may reduce sensitivity at  
361 the single-trial level in some cases. Hence, the results shown in Figure 8 encourage the use of the static  
362 thresholding approach as an exploratory step before employing the time-dependent threshold. Other  
363 thresholding criteria could involve the comparison of AUC values obtained from surrogate (null) data  
364 (Liégeois et al., 2021) with the AUC values obtained with the original data.

365 Furthermore, the extension of the original ME-SPFM algorithm from a voxel-wise to a whole-brain  
366 (i.e., multivariate) regularized problem paves the way for more refined formulations that exploit the  
367 spatial characteristics and information available in fMRI and complementary imaging data into the spatial  
368 regularization term in order to improve the estimation of  $\Delta R_2^*$ . For instance, the spatial regularization  
369 could be constrained within brain regions delineated by commonly used parcellations (e.g., the Schaefer-  
370 Yeo atlas) (Karahanoğlu et al., 2013) or within neighbouring gray matter voxels (Farouj et al., 2017).  
371 Moreover, the multivariate formulation could exploit complimentary multimodal information such as  
372 structural connectivity from diffusion-based MRI data (Bolton et al., 2019b). In addition, the proposed  
373 formulation can be easily adapted to model the changes in neuronal activity in terms of its innovations,  
374 which can be more appropriate to capture sustained BOLD events (Uruñuela et al., 2021a).

375 Similar to the results obtained with ME-SPFM (Caballero-Gaudes et al., 2019), we observed that  
376 MvME-SPFM also detects hemodynamic events with physiologically plausible  $\Delta R_2^*$  and relatively high  
377 AUC values in periods between trials when the subjects are not engaged in any activity-evoking task,  
378 whereas analysis approaches that model events with known timings (e.g., GLM) cannot find these spon-  
379 taneous events. Consequently, MvME-SPFM can provide robust estimates of the activity that drives  
380 BOLD responses occurring in spontaneous brain fluctuations (Finn et al., 2015; Tanner et al., 2022), to  
381 study individual differences in naturalistic paradigms (Finn et al., 2020), to blindly decode the sub-  
382 ject’s engagement in a particular cognitive process from the activation maps (Poldrack, 2011; Poldrack  
383 & Yarkoni, 2016; Gonzalez-Castillo et al., 2019; Tan et al., 2017), or in clinical conditions such as the  
384 study of the urge-to-tic in patients with Tourette’s syndrome (Jackson et al., 2020).

385 One limitation of the proposed MvME-SPFM technique is the assumption of a particular shape of the  
386 hemodynamic response to construct the HRF matrix for deconvolution in Eq. (5). The proposed model  
387 does not account for the variability in the temporal characteristics of the HRF across the brain, which  
388 originates from differences in stimulus intensity and patterns, short inter-event intervals, or differences in  
389 the HRF shape between resting-state and task-based paradigms (Yeşilyurt et al., 2008; Sadaghiani et al.,  
390 2009; Chen et al., 2021; Polimeni & Lewis, 2021). To resolve this issue, given that the performance of  
391 MvME-SPFM is not time-locked to the trials, the current formulation could be extended to account for  
392 variability in the onset of the activity-inducing signal, as well as to introduce flexibility in the model, by  
393 employing multiple basis functions (Gaudes et al., 2012). Finally, the computational demands involved  
394 in the stability selection procedure, which solves the regularization problem in Eq. (6) for a range of  $\lambda$   
395 values on a number of subsampled surrogate datasets, are higher than solving the regularization path  
396 and finding an adequate solution via model selection criteria as in ME-SPFM (Caballero-Gaudes et al.,  
397 2019).

## 398 6. Conclusion

399 In summary, this work proposes a new approach (MvME-SPFM) for the deconvolution of multi-echo  
400 fMRI data that exploits spatial information of the fMRI data with a whole-brain (i.e., multivariate)  
401 formulation and an  $\ell_{2,1}$ -norm, and yields more robust estimates of changes in  $\Delta R_2^*$  through the use of

402 the stability selection procedure. Moreover, this work introduced a novel metric based on the area under  
403 the curve (AUC) of the stability paths that depicts the probability of having neuronal-related events  
404 at each voxel and time-point. We demonstrated that the proposed approach yields more robust and  
405 superior estimates of  $\Delta R_2^*$  compared with the original ME-SPFM approach, and shows high spatial and  
406 temporal agreement with activation maps obtained with a GLM, while having no information about the  
407 timings of the BOLD events.

## 408 7. Code and data availability

409 The code and materials used to generate the figures in this work can be found in the following GitHub  
410 repository: [https://github.com/eurunuela/MvMEPFM\\_figures](https://github.com/eurunuela/MvMEPFM_figures).

411 The Python package is available as part of *splora* in the following GitHub repository: <https://github.com/eurunuela/splora>.

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## 419 9. CRediT

420 Eneko Uruñuela: Conceptualization, Methodology, Software, Formal Analysis, Investigation, Writing  
421 (OD), Writing (RE), Visualization, Funding Acquisition. Javier Gonzalez-Castillo: Data Curation,  
422 Writing (RE). Charles Zheng: Writing (RE). Peter Bandettini: Funding Acquisition, Writing (RE).  
423 César Caballero-Gaudes: Conceptualization, Methodology, Software, Formal Analysis, Investigation,  
424 Data Curation, Writing (OD), Writing (RE), Visualization, Funding Acquisition.

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