ImeEEG: Mass linear mixed-effects modeling of EEG data with crossed random effects

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ABSTRACT

Given their ability to simultaneously model crossed random effects for subjects and items, mixed-effects models are the current standard for the analysis of behavioral studies in psycholinguistics and related fields. However, they are hardly applied in neuroimaging and psychophysiology, where the use of mass univariate analyses in combination with permutation testing would be too computationally demanding to be practicable with mixed models. Here we propose and validate an analytical strategy that enables the use of linear mixed models with crossed random effects in mass univariate analyses of EEG data (lmeEEG) overcoming the computational costs of standard available approaches (our method was indeed ≈250 times faster). Data and codes are available at osf.io/kw87a. Codes and a tutorial are also available at github.com/antovis86/lmeEEG.

Keywords: EEG; linear mixed-effects models; TFCE; mass-univariate testing; crossed random effects; Psycholinguistics
1. Introduction

Mixed-effects models are crucial to appropriately analyze data from experimental designs including both subjects and items as crossed random effects (Baayen et al., 2008; DeBruine & Barr, 2021). However, their use is limited in neuroimaging analyses also due to computational time constraints. Here, we introduce an analytical strategy for performing mass univariate linear mixed-effects model analyses of EEG data (lmeEEG), such as event-related brain potentials.

Consider, as an example of a design with crossed random effects, a psycholinguistic study in which participants are asked to judge a linguistic property of a set of words. When analyzing psychological experiments, researchers take into account the inter-individual variability (or random error) to draw general conclusions that are valid beyond the selected sample (this aspect is implicit in the majority of repeated-measures statistical models). Of note, also linguistic stimuli are sampled from the population of all words. As participants are treated as random variables to generalize results to their population, the same logic applies to items (Barr, 2017; Clark, 1973). Indeed, researchers are usually not interested in experimental effects that are valid only for the selected set of words. Hence, the inter-item variability has to be considered in order to generalize results to the population of words from which the experimental items are drawn. Although the modeling of by-item variability has mainly attracted the attention of psycholinguistics, it is needed whenever a researcher analyzes data from an experimental study (e.g., memory or emotion studies) in which stimuli are drawn from a larger population (Barr, 2017).

Given their ability to simultaneously model crossed random effects, mixed models are the current standard for behavioral studies in psycholinguistics and related fields (DeBruine & Barr, 2021), but they are less common in neuroimaging and psychophysiology. Focusing on electroencephalography (EEG) or magnetoencephalography (MEG), in recent years we are moving from traditional approaches that analyze selected channels and timepoints to mass univariate approaches, in which the whole Channel(Sensor) \times\ Timepoint data space is tested (Groppe et al., 2011; Woolrich et al., 2009). However, the use of mass univariate analyses in combination with resampling methods (i.e., permutation testing and bootstrapping) to control for the Family-Wise Error Rate (FWER) (Pernet et al., 2015) is too computationally demanding to be practicable with mixed models (Fields & Kuperberg, 2020; Nielson & Sederberg, 2017). Indeed, linear mixed models (LMM) are estimated using (restricted) maximum likelihood ((RE)ML) methods, which require too much time to be performed millions of times. This is probably one of the reasons why the available toolboxes for mass univariate analysis of M-EEG data (e.g., LIMO EEG; Pernet et al., 2011; SPM;
Kiebel & Friston, 2004; Unfold; Ehinger & Dimigen, 2019) include only statistical tests that can be reconducted to linear models (LM), which rely on ordinary least squares (OLS) solutions (considerably faster than (RE)ML estimations). These toolboxes perform random coefficient analysis (also called two-step linear regression, multilevel linear model, or hierarchical general linear model; Lorch & Myers, 1990), in which fixed effects coefficients are first estimated within each participant and then tested at the group level. Although these models deal with inter-individual variability, they cannot simultaneously model crossed random effects. It follows that they are not appropriate for the analysis of experimental designs in which stimuli are a sample from a population of items (Bürki et al., 2018).

Here, we propose a solution to the unfeasible computational costs derived from the use of resampling methods with LMM. Differently from other approaches that reduce the dimensionality of analyzed EEG datasets before performing LMM (Nielson & Sederberg, 2017), our approach (lmeEEG) enables the use of mixed models with mass univariate analyses. lmeEEG complements other mass univariate modeling techniques by providing a method to analyze experimental design with crossed random effects.

### 2. Description of lmeEEG

To introduce lmeEEG, we will describe its application to a simplified experiment. Three participants $s_1$, $s_2$, and $s_3$ perform a semantic decision task (i.e., judging whether a word is abstract or concrete) on four words belonging to the concrete ($w_1$ and $w_2$) and to the abstract ($w_3$ and $w_4$) experimental conditions. Analyses are conducted on EEG data collected from 19 channels at 50 timepoints locked with word onset.

lmeEEG consists of the following steps (Fig. 1):

1. **Conduct mixed models on each channel/timepoint combination.** For each EEG channel $ch$ and event-locked timepoint $t$, a linear mixed-effects model (LMM) is conducted on trial-wise EEG responses concatenated across participants ($EEG_{ch,t}$ in Eq.1). The LMM can be summarized as:

   $EEG_{ch,t} = X\beta + Zu + \varepsilon$  

   In (1), $X$ represents the fixed-effects design matrix, which includes in the present example a column of ones for the intercept and a column representing the experimental factor contrast. The $X$ matrix is multiplied by the population coefficients $\beta$, here consisting of $\beta_0$ for the intercept and $\beta_1$ for the contrast of abstract compared to concrete words.
Continuing with (1), Z represents the random-effects design matrix. In the proposed example, Z is composed of two grouping variables, a three-column variable for participants and a four-column variable for words, which are multiplied by the coefficients $u$, which indicate the value that must be added to the population intercept for each participant and each item. Finally, $\varepsilon$ represents the residual. A consideration needs to be done on the specification of the random-effects structure. In the present example we used a minimal structure, allowing only intercepts to vary across participants and words. Different approaches have been proposed for the random-effects specifications (Barr et al., 2013; Bates, Kliegl, et al., 2015; Matuschek et al., 2017). Here, we limit to suggest to be parsimonious for two main reasons: first, it is hard to assess and manage convergence and singularity issues with massive testing; second, random-effects parameters are more difficult to be estimated and their number rapidly increases as model complexity slightly increases, thus leading to important computational costs.

2. Perform mass univariate linear regressions on “marginal” EEG data.

Random-effects contributions are removed from EEG data:

$$\text{mEEG}_{ch,t} = \text{EEG} - Zu = X\beta + \varepsilon \quad (2)$$

As expressed in (2), what we call marginal EEG (mEEG) can be reconstructed by removing the fitted random values $Zu$ from the data (which is equivalent to adding conditional residuals to trial-wise marginal fitted values). It follows that mEEG can be explained using a (multiple) linear regression model (LM), since we can assume the independence of observations. In the next sections, we will show that the results from (2) are equivalent to fixed-effects results in (1). A single LM is conducted on each channel/timepoint combination. In this way, we obtain a channel-by-timepoint map of the observed $t$-values ($t$-map$_{OBS}$) for each fixed effect. In the proposed example, we obtain a 16×50 $t$-map for the abstract vs. concrete contrast.

3. Perform permutation testing and apply the Threshold Free Cluster Enhancement (TFCE).

The TFCE (Mensen & Khatami, 2013; Smith & Nichols, 2009) is used to assess the significance of $t$-maps$_{OBS}$. First, the design matrix $X$ is permuted thousands of times (e.g., at least 2000 to properly estimate the critical statistics with an alpha level of .05). At each iteration, the permuted $X$ is used for the mass linear regressions as in step 2. Notably, LM are solved using OLS, which is much faster than the RE(ML) method used for LMM, and hence feasible for performing permutation testing in the whole data space (just this simplified example would require 1,600,000 tests). For each effect of interest, TFCE is applied on the corresponding $t$-map$_{OBS}$ and on the $t$-maps obtained from each
permutation (t-maps$\text{PERM}$). The maximum TFCE values from the t-maps$\text{PERM}$ (maxTFCE) are then extracted to build the empirical distribution of maxTFCE values under H$_0$, which is used to evaluate the statistical significance of t-maps$\text{OBS}$. Attention should be paid to two aspects. First, the use of permutation testing is important not only for multiple comparison corrections. It also allows us to overcome the difference between LMM and LM estimation. The fixed-effects $t$ values are computed as $\beta$ divided by their standard errors (SEs). As shown in Section 3.3, $\beta$ values are equal between LM and LMM testing. Conversely, since the covariance matrix of fixed-effects coefficients is derived differently in LM as compared to LMM (Bates, Mächler, et al., 2015), SEs differ between LM and LMM, although being correlated to $\sim$1. It follows that $t$ values are correlated to $\sim$1 between LMM and LM, but they differ in value. This aspect is not an issue since significance is assessed not on the basis of the absolute $t$ value, but on the basis of the empirical distribution of maxTFCE values under H$_0$. A further consideration is that we want to test fixed-effects, hence permutation of the random part of the model is not required. One last remark regards the fact that we opted for the TFCE approach, but in principle our method can be easily adapted to every resampling method used for multiple comparison correction (e.g., Frossard & Renaud, 2022).
Figure 1 | Illustration of the analytical steps in lmeEEG. In step 1 (top), a linear mixed model (LMM) is massively conducted at each electrode and timepoint combination on epoched EEG data vectors comprising all trials from all subjects. The LMM design matrix is composed of a fixed-effects part (X) and a random-effects part (Z). In step 2 (middle), marginal EEG data (mEEG) vectors for each channel and timepoint combination are obtained by removing random effects contributions estimated in step 1. Mass univariate linear regression models
(LM) - composed on only X - are conducted on mEEG and a map of t values (t-map\textsubscript{OBS}) is obtained for each predictor of interest (one predictor in the example). In step 3 (bottom), X is permuted and used for mass LM. Then, threshold-free cluster enhancement (TFCE) is applied on the t-maps obtained from each permutation (t-maps\textsubscript{PERM}) and maxTFCE values from each permutation used to build an empirical distribution of maxTFCE values under H\textsubscript{0}. The empirical distribution of maxTFCE values is used to assess the statistical significance of TFCE values of t-map\textsubscript{OBS}.

### 3. Validation

In order to validate our analytical strategy, we applied lmeEEG to a simulated EEG dataset. Validity was assessed in terms of equivalence between the lmeEEG results and the results obtained from the highly computationally expensive LMM permutation testing.

The analyses were carried out using MATLAB R2022b on a DELL Precision 7920 Tower, Processor Intel Xeon Gold 5220R (24 cores up to 2.20 GHz), 64 GB RAM, OS Windows 10.

The simulated dataset and MATLAB scripts to perform lmeEEG are available at osf.io/kw87a.

### 3.1. Simulation data

Event-related EEG datasets were simulated using the MATLAB-based toolbox SEREEGA (Krol et al., 2018). Simulated EEG data included a P1-N2-P3 complex with different intercepts for subjects (N = 30) and items (N = 10). Moreover, the P3 was differently modulated according to two experimental conditions (i.e., a two-level experimental factor: A vs. B). In detail, for each subject, item, and experimental condition we simulated 50 epochs of 1100 ms (100 ms of pre-stimulus) at 100 Hz. Each epoch consisted of the summed activity of 19 simulated EEG sources spread across the brain and projected onto a standard 19-channel montage. Of the 19 simulated components, 14 were a mixture of white and brown noise (amplitude of each type of noise = 2 µV) with random source locations. The remaining components were two bilateral P1, two bilateral N2, and a P3a components. Configurations of these ERP components were taken from the P100, N135 and the P3a templates in SEREEGA. Random intercepts for subjects and items were added to the amplitude of all ERP components (random intercepts were sampled from a normal distribution with mean = 0 µV and SD = 0.2 µV). Moreover, 0.5 µV were added to the P3 amplitude in the epochs belonging to the experimental condition B.
3.2. Validation analyses

To validate lmeEEG, we first applied our method as described above. The LMM of Equation 1 was specified as the following Wilkinson-notation formula:

$$\text{EEG}_{\text{ch},t} \sim 1 + \text{Condition} + (1|\text{Subject}) + (1|\text{Item})$$  \hspace{1cm} (3)

where the fixed-effects portion of the model corresponds to 1 for the intercept and Condition is the two-level factor of interest. The random-effects portion specifies random intercepts for subjects and items. Next, mEEG data are reconstructed by adding conditional residuals to the trial-wise marginal fitted values. Finally, mEEG were used to perform the steps 2 and 3 described above. In step 3, we used the \texttt{ept_mex_TFCE2D} function from the 	exttt{ept_TFCE} toolbox (github.com/Mensen/ept_TFCE-matlab; Mensen & Khatami, 2013).

To validate lmeEEG, step 3 was also performed using LMM on the original EEG dataset. Specifically, the design matrix vector was permutated within each subject and the EEG data were explained in each permutation using Equation 3. The permutated Condition vector at each permutation was the same between analyses with EEG and mEEG datasets. Since performing permutation testing with LMM is too computational expensive (we propose lmeEEG to overcome this limitation) and here we are only interested in demonstrating that the use of our strategy is closely equivalent to performing LMM permutations, we limited the number of permutations to 500 (along with simulating a dataset with a small number of channels and time points per epoch). The results from both LMM and LM permutation tests were compared in terms of correlations between the \textit{t}-maps \textit{OBS} and between \textit{t}-maps \textit{PERMS}, as well as in terms of equivalence between the empirical TFCE distribution under \textit{H}_0.

3.3. Validation results

As anticipated above, the fitted $\beta$ coefficients were identical when estimated using LMM on EEG data and when using LM on mEEG data. This aspect was crucial to validate our method. The results showed that both $t$-maps\textit{OBS} and $t$-maps\textit{PERMS} had a correlation of $\sim$1 between the LMM and LM tests ($r > 0.99999997$) because, as explained above, the standard errors differed between the two analytical methods, although they were correlated to $\sim$1. Importantly, however, all the dichotomous decisions based on null hypothesis significance testing (i.e., significant vs. non-significant effects) were the same between the two methods, meaning that there were neither Type 1 nor Type 2 errors. Furthermore, the
equivalence of \( \beta \) coefficients (i.e., raw effect sizes) between the two methods prevents the possibility of either Type S or Type M errors (Gelman & Carlin, 2014), ensuring an accurate estimation of experimental effects.

However, the p-value maps obtained from the two procedures were almost identical. Indeed, 9% of the \( p \) values differed by one position in the obtained empirical TFCE distributions under \( H_0 \) (i.e., the mean difference was .002; SD < 0.00071). This negligible difference in \( p \) values, which represents the price of a substantial decrease in computation costs, can nevertheless be reduced by increasing the number of permutations and, thus, the granularity of the TFCE distributions under \( H_0 \).

The estimation of LM models for the permutation test (step 3) had a median duration of 0.75 ms (interquartile range IQR = 0.72 ms), while the estimations of LMM had a median duration of 184.92 ms (IQR = 31.25 ms). Consequently, our permutation strategy was almost 250 times faster than LMM permutations.

4. Application to a real EEG dataset

In this section, we will apply lmeEEG to a real EEG dataset collected during a psycholinguistic experiment. Fifty-eight students performed a semantic decision task. In particular, participants were asked to decide if a word presented at the center of the screen denoted an abstract or a concrete concept. The experimental stimuli consisted of 176 Italian words derived from the Italian affective norms (Montefinese et al., 2014). During the task, the EEG signal was recorded at a sampling rate of 500 Hz from 58 scalp electrodes mounted on an elastic cap according to the 10–10 International System. Participants’ EEG datasets were preprocessed using an ICA-based pipeline described in Visalli et al. (2021). Clean epochs (from -100 to 1000 ms at 100 Hz) time-locked to word onset were merged across participants. The dimensions of the final EEG dataset were 58 channels \( \times \) 111 timepoints \( \times \) 10072 epochs. The LMM was specified as:

\[
\text{EEG}_{ch,t} \sim 1 + \text{Concreteness} + (1|\text{Subject}) + (1|\text{Word})
\] (4)

The lmeEEG results are presented in Figure 2. The main finding was a less pronounced N400 event-related potential (ERP) for the abstract compared to concrete words at fronto-central scalp electrodes (Huang & Federmeier, 2015).
Figure 2 | ImeEEG results on the real EEG dataset. (a) Raster diagram showing significant effects elicited by the concreteness predictor. Rectangles in warm and cold colors indicate significantly modulated electrodes/timepoints. The color bar indicates the $t$ values. Gray rectangles indicate electrodes/timepoints for which no significant modulations were observed. (b) Trace-plot depicting the beta values estimated in ImeEEG step 2. Specifically, the intercept (blue line) represents the estimated EEG responses in the “concrete” condition. The $\beta_1$ (red line) represents the value to add to the intercept to obtain the estimated EEG responses to abstract words (yellow line). The displayed beta values are averaged across FCz and the eight surrounding electrodes. (c) Topoplot showing the $t$ values (same color scale of the raster diagram) averaged in the indicated time window.

5. Conclusion

In the present work, we proposed and validated an analytical strategy (ImeEEG) that allows researchers to use mixed models with mass univariate analyses. Essentially, it avoids the
unfeasible computational costs that would arise from massive permutation testing with LMM using a simple solution: removing random-effects contributions from EEG data and performing mass univariate LM analysis and permutations on the obtained marginal EEG. Analyses on simulated data showed that the estimated experimental effects and the relative statistical inferences yielded by ImeEEG were equivalent to those obtained by mass univariate analyses with LMM permutations, but almost 250 times faster.

Until now, the use of LMM for EEG analyses required either the (a priori) selection of the data to be tested (thus avoiding permutations for multiple comparison correction) or some dimensionality reduction procedure (Nielsen & Sederberg, 2017). In the present study we showed that ImeEEG is a valid, easy, and feasible method for performing LMM on the entire spatiotemporal with them using the entire Channel $\times$ Timepoint data space.

In presenting ImeEEG, we focussed on simple ERP studies with one experimental factor and crossed random intercepts for subjects and items. However, our method can be easily applied to a wide variety of experimental studies with more complex fixed structures. Concerning the type of dataset, it can be used for the analysis of even larger EEG data, such as, time-frequency data or source-reconstructed ERP, MEG data, or even pupillometry (Montefinese et al., 2018) and eye movement data (Lao et al., 2017). It can also account for designs with “nested” random effects, such as, in multi-site neuroimaging studies.

Mixed models are the gold standard for the data analysis in psycholinguistics, where experimental designs always include crossed random variables. Mixed models have several advantages besides modeling crossed random effects, such as: increased power; managing unbalanced datasets or incomplete designs; considering trial-, subject- and item-related covariates and nested dependencies between data (Baayen et al., 2008). Despite these advantages, mixed models are not yet a common practice in neuroimaging and psychophysiology. Thanks to the possibility given by our method, we anticipate that LMM will become increasingly important in neuroscience.

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**Ethics approval:** Real-data collection was performed in accordance with the ethical standards of the 2013 Declaration of Helsinki for human studies of the World Medical Association. The data collection procedures were approved by the Ethical Committee for the Psychological Research of the University of Padova (protocol number: 2945).

**Data and code availability:** Simulation data and codes are available at the Open Science Framework: [osf.io/kw87a](http://osf.io/kw87a). Codes and a tutorial for lmeEEG are also available at [github.com/antovis86/lmeEEG](http://github.com/antovis86/lmeEEG).

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