

1 Simultaneous representation of multiple time 2 horizons by entorhinal grid cells and CA1 place cells

3 Prannoy Chaudhuri-Vayalambone^{1†}, Michael Rule^{2†}, Marius Bauza³, Marino
4 Krstulovic¹, Pauline Kerekes¹ Stephen Burton³, Timothy O’Leary² and Julija
5 Krupic^{1*}

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7 ¹Department of Physiology, Development and Neuroscience, University of Cambridge, UK

8 ²Department of Engineering, University of Cambridge, UK

9 ³Sainsbury Wellcome Centre, University College London, UK.

10 [†]These authors contributed equally

11 ^{*}Corresponding authors. Email: jk727@cam.ac.uk (J.K.)

12 Summary

13 Grid cells and place cells constitute the basic building blocks of the medial entorhinal-
14 hippocampal spatial cognitive map by representing the spatiotemporal continuum of an
15 animal’s past, present and future locations. However, the spatiotemporal relationship between
16 these different cell types is unclear. Here we co-recorded grid and place cells in freely foraging
17 rats. We show that average time shifts in grid cells tend to be prospective and are proportional
18 to their spatial scale, providing a nearly instantaneous readout of a spectrum of progressively
19 increasing time horizons ranging hundreds of milliseconds. Average time shifts of place cells
20 are generally larger compared to grid cells and also increase with place field sizes. Moreover,
21 time shifts displayed nonlinear modulation by the animal’s trajectories in relation to the local
22 boundaries and locomotion cues. Finally, long and short time shifts occurred at different parts
23 of the theta cycle, which may facilitate their readout. Together, these findings suggest that
24 progressively increasing time horizons of grid and place cells may provide a basis for
25 calculating animal trajectories essential for goal-directed navigation and planning.

26 **Main**

27 The firing patterns of spatially sensitive cells, such as grid and place cells, are normally viewed
28 as functions of an animal's current position. However, studies have shown that their firing is
29 often biased towards representing positions the animal has just visited or is about to visit¹⁻⁸. In
30 this way, activity in place cells and grid cells represents the spatiotemporal continuum of an
31 animal's past, current and future locations. In rats, on average, temporal biases in CA1 place
32 cells tend to be positive and span ~120 ms^{1,6,8}. Previous work suggests that inputs to CA1,
33 representing an animal's movement in a particular direction, may be used for shifting spatial
34 representations slightly ahead of the animal by defining a small future trajectory segment^{5,8-10}.
35 Grid cells, notably, share a constant temporal offset with co-recorded medial entorhinal speed
36 cells⁵. However, the spatiotemporal relationships between different grid modules and between
37 grid cells and place cells have not been established.

38 Here, we investigated temporal biases in how locations are represented by the medial entorhinal
39 grid cells and CA1 place cells. We found that both grid and place cells predominantly represent
40 space with a prospective bias in a two-dimensional foraging task. Similar to the spatial
41 organisation of the entorhinal grid and hippocampal place cells, which fundamentally depends
42 on the geometry of the enclosure¹¹⁻¹⁶, the time shifts of these cell types are modulated by the
43 animal's trajectory in relation to the enclosure boundaries. In addition, they are also non-
44 linearly modulated by the animal's locomotion properties, such as speed, angular velocity and
45 acceleration.

46 Importantly, the magnitudes of these time shifts correlate with the spatial scale of grid cell and
47 the size of the place cell field. Thus, simultaneously recorded grid cells of different grid
48 modules (a grid module is defined as a group of anatomically neighbouring grid cells with
49 similar scale and orientation¹⁷⁻¹⁹) together with different size place cells may, in principle,
50 provide a nearly simultaneous (occurring within 200 ms; see Methods) readout of multiple time
51 shifts available at any given location. Notably, time shifts in place cells are on average larger
52 than in grid cells. Finally, we found that both grid and place cells with different time shifts
53 tended to fire at different phases of the theta cycle, suggesting that the time horizon of a given
54 cell's coding window may be organised by the theta rhythm.

55 Results

56 Place cells have larger time shifts compared to grid cells

57 To establish how temporal biases are distributed along the dorsoventral axis of the medial
58 entorhinal cortex (mEC) and along the hippocampal-mEC network, we analysed data from a
59 study published by Krupic and colleagues¹¹, in which grid cells (n = 249, 6 rats) and place cells
60 (n = 95, 5 rats) were recorded while rats freely foraged in four geometrically distinct enclosures
61 (Fig. 1A). Grid cells and place cells were simultaneously recorded in three of these rats. To
62 investigate whether grid and place cell firing was predominantly prospective or retrospective
63 (or encoding an animal's current location), we shifted spike times by up to ± 2 seconds (in 20
64 ms steps) relative to the rats' trajectories. With each time shift, we measured the 'sharpness' of
65 the resulting rate map's firing fields by calculating the spatial variance of its rate map ("zero-
66 lag spatial autocorrelation" ZLAC; see Methods; "lag" here refers to spatial lag). The position
67 of the ZLAC peak nearest 0 ms was taken as the recording's optimal time shift (Fig. 1B). In
68 this study, positive time shifts indicate prospective firing, while negative shifts indicate
69 retrospective firing.

70 Overall, both grid cells and place cells were biased towards prospective firing (Fig. 1C, grid
71 cells (n = 249, blue): median 133 ms (interquartile range (IQR) 116), $Z = 13.4$, $P = 4.5 \times 10^{-41}$;
72 place cells (n = 95, red): 180 ms (IQR 133), Wilcoxon signed-rank test (WSRT): $Z = 8.2$,
73 $P = 2.8 \times 10^{-16}$), with place cells on average having significantly larger prospective time shifts
74 compared to grid cells (Mann-Whitney U test (MWUT): $Z = -2.8$, $P = 4.5 \times 10^{-3}$); N.B. there
75 was no significant difference between the median field sizes of the grid (median 0.22 m², IQR
76 0.25) and place cells (median 0.22 m², IQR 0.12; MWUT: $P = 0.67$, Fig. S1). Three rats with
77 co-recorded place cells and grid cells similarly showed significant differences between the
78 average time shifts of the two cell types, with place cells exhibiting larger time horizons (Fig.
79 1D, MWUTs: 3/3 significant after Bonferroni correction for multiple comparisons, False
80 Discovery Rate (FDR) $\alpha = 0.05$). In addition to using ZLAC, we calculated the optimal time
81 shifts by maximising spatial information²⁰ and peak firing rate, which are commonly used
82 properties for finding optimal time shifts in place cells^{4,5,7}. We found that the resultant optimal
83 time shifts were highly correlated (Fig. S2), ruling out the possibility that the results are due to
84 a hidden bias of our novel ZLAC method.

85 Enclosure boundaries modulate the optimal time shifts

86 If time shifts provide a horizon in which the animal's past, present and future state is being
87 represented, we might expect time shifts to be influenced by boundaries and obstacles that
88 constrain behaviour. Notably, it has been shown that spatial symmetry in grid cells
89 fundamentally depends on the geometry of the enclosure^{11,12,15,16}, and grid cell firing resets at
90 the walls^{21–23}, which may correct path integration errors and increase animals' certainty about
91 their current locations. It has been suggested that the amount of information available to the
92 animals may influence their predictive coding horizon: when local cues are abundant, rats show
93 consistent predictive coding, which degrades in cue-deprived environments¹. We, therefore,
94 hypothesised that the time bias may depend on enclosure shape and the rat's proximity to the
95 walls.

96 To address this, we compared the time shifts in four geometrically distinct enclosures, which
97 have been shown to induce local distortions in grid cell firing patterns and local shifts in place
98 fields close to the walls¹¹. We reasoned that if time shifts are used to represent immediate future
99 trajectories, which on average, extend further in a larger rectangular (“S4”) enclosure compared
100 to a smaller trapezoidal (“S1”) enclosure, prospective time bias should be larger in the former
101 enclosure compared to the latter.

102 To address this question, we regressed time shifts against enclosure shape (Fig. 2A; linear
103 regression $\Delta t = m n + b$, where n is the enclosure number ranging from 1: most trapezoidal to
104 4). Consistent with our predictions, grid cell time shifts increased in the larger arenas (inner
105 95% confidence interval for slope m : $P_{2.5-97.5} = 3.1-14$ ms/arena; cells from all subjects
106 combined). This effect was consistent in direction and magnitude across all subjects (Fig. S3).
107 For place cells, the inner 95% confidence interval for m ($P_{2.5-97.5} = -16-9.0$ ms/arena) included
108 zero, with no consistent effect across subjects (Figs. 2A, S3). Notably, in grid cells, the arena
109 shape remained a minor contributor to the overall time shifts, with the median time shift
110 remaining largely prospective even in the smallest arena (Δt : median 140 ms, IQR 60–200 ms).
111 This implies that arena deformation effects are superimposed on a larger, fixed prospective
112 shift.

113 Next, we asked whether there was a bias in time shifts depending on the animal's running
114 direction. We used the animal's movement direction ('heading') as a correlate of the animal's
115 past trajectory²² (i.e. the animal's going eastwards approximates a recent encounter with the

116 west wall, whereas the animal going southwards suggests its recent encounter with the north
117 wall etc.; N.B. heading and head direction are highly correlated and show consistent results;
118 Fig. S4). We used a polar regression $\Delta t = d \cos(\theta - \theta_{\max}) + c$, where d captures the depth of
119 directional modulation, and θ_{\max} captures the direction in which time shifts are most
120 prospective. For both grid cells and place cells, time shifts showed significant prospective
121 heading modulation (modulation depth spanning $d = 46\text{--}87$ ms in all subjects for grid cells and
122 $d = 62\text{--}280$ ms in 3/5 subjects for place cells, FDR TSBH $\alpha = 0.05$), with the largest time
123 horizons consistently reflecting eastward directions (Figs. 2B, S5 θ_{\max} ranges over all subjects:
124 heading GC: S82°E–S30°E, PC: N56°E–N77°E). Further investigation revealed that
125 directional modulation depended on the arena shape, showing a preference for eastward
126 directions in the rectangular arena and south-eastward directions in the trapezoidal arena. Such
127 directional bias suggests that the animals may be using the frame of reference associated with
128 the external room instead of the internal walls of the enclosure to estimate the field locations.
129 Namely, the rat reaches fields sooner than expected when it travels from west to east in the
130 rectangular enclosure compared to the trapezoidal enclosures. On the other hand, in the
131 trapezoidal enclosure, the fields, on average, may be ‘squeezed upwards’ as a result of
132 deformations and the animals will tend to show more south-east directed bias in time shifts
133 (Fig. 2C).

134 Finally, we asked whether the time shifts depended on the animal’s trajectories in relation to
135 the enclosure walls. We hypothesised that time shifts in grid cells and place cells may be
136 modulated by proximity to the walls, which act as reset points for updating the grid cell firing^{21–}
137 ²³ to reduce the accumulation of path integration errors (hence decreasing ambiguity about the
138 location, which is known to affect time shifts in place cells¹). Furthermore, such modulation
139 should change depending on whether the animal approaches or moves away from the wall. We
140 reasoned that if time shifts indeed reflect the look-ahead predictive time horizon of the medial
141 entorhinal-hippocampal network, they should be shorter when the animal approaches the wall
142 compared to when it is leaving the wall. To investigate this, we first divided each grid and place
143 cell rate map into individual firing fields and used these to segment the animal’s trajectory into
144 individual runs through single fields²⁴. For each spike train associated with a single run, we
145 calculated a ‘single-run’ time shift “ δ ” as the time difference between the “median spike” (Fig.
146 2D, green dots) and the point on the run closest to the field centre (positive shifts = prospective
147 coding; N.B. often the median spikes can be the only spikes). We confirmed that using this
148 alternative method, each cell’s grand median single-run time shift (i.e. the median of each

149 recording's median single-run time shift) was significantly correlated with the cell's ZLAC-
150 optimal time shift (Fig. 2E, grid cells ($n=242$ excluding outliers): Pearson's $r=0.59$,
151 $P=2.3 \times 10^{-23}$; place cells ($n=92$, excluding outliers): Pearson's $r=0.61$, $P=2.0 \times 10^{-8}$).

152 We then compared these single-run time shifts on runs towards and away from the walls
153 ("outward" and "inward" respectively) in fields near the walls (Fig. 2F-G; see Methods). On
154 inward runs (away from the walls), grid cells showed prospective time shifts (Fig. 2G, $n=73$
155 cells with sufficient runs: median 132 ms (IQR 167); WSRT: $Z=6.3$, $P=2.8 \times 10^{-10}$), as
156 expected based on grid cells' overall average time shifts. However, outward runs (i.e. when the
157 animal approached the wall) had significantly shorter time horizons (difference between time
158 shifts of inward and outward runs in each map: median 130 ms (IQR 214), WSRT: $Z=5.7$,
159 $P=1.4 \times 10^{-8}$); and were neither prospective nor retrospective (median 25 ms (IQR 156);
160 WSRT: $Z=-1.6$, $P=0.10$). Place cells showed similar results: inward runs tended to have
161 larger prospective shifts than outward runs (difference between time shifts of inward and
162 outward runs in each map: median 260 ms (IQR 281), $n=29$ place cells with sufficient runs,
163 WSRT: $Z=3.7$, $P=2.3 \times 10^{-5}$). However, for place cells, both inward and outward runs showed
164 significant prospective time shifts (inward runs: median 300 ms (IQR 310), WSRT: $Z=4.6$,
165 $P=4.8 \times 10^{-6}$; outward runs: median 70 ms (IQR 184), WSRT: $Z=3.0$, $P=2.6 \times 10^{-3}$).

166 Nonlinear modulation of time shifts by internal cues

167 To dissociate the influence of the wall from behaviour correlated with turning or stopping, we
168 checked for systematic differences in behaviour between inward and outward runs near walls.
169 Consistent with rats slowing as they approach the wall, acceleration was more positive in
170 inward runs for both cell types (median difference: GC: $0.035 \text{ m}\cdot\text{s}^{-2}$, IQR 0.054, $n=73$ cells
171 with >15 runs; PC: median difference $0.044 \text{ m}\cdot\text{s}^{-2}$ IQR 0.044, $n=29$ cells with >5 runs),
172 significant in both cases (GC: $p=3.9 \times 10^{-13}$, PC: $p=2.6 \times 10^{-6}$; WSRT). The median angular
173 velocity was similar between inward and outward runs in the data for grid cells (WSRT:
174 $p=0.91$), but significantly different for place cells (median difference $4.3 \text{ }^\circ/\text{s}$ (IQR 18.2°),
175 WSRT: $p=7.7 \times 10^{-4}$). Given these behavioural biases, we tested whether time shifts depended
176 on behaviour more generally. We measured the rank correlation between per-run time shifts
177 and acceleration, forward speed, and angular speed (Methods). We converted data to ranks on
178 a per-recording basis before combining to remove per-cell and per-rat effects.

179 Acceleration was significantly (after $\alpha = 0.05$ TSBH FDR control) positively correlated with
180 single-run time shifts for both cell types in all subjects save one (n.s for place cells in R2377),
181 with correlations spanning $\rho = 0.08 - 0.12$ (Figs. 3A, S6); the plots show the result of the rank
182 regression after converting back to physical units by inverting the rank transform (Methods).
183 Positive accelerations correspond with more prospective shifts and negative accelerations to
184 smaller (and sometimes negative) time shifts. This effect saturated for the most extreme
185 positive or negative accelerations and showed an inflection point around zero. This suggests
186 an almost binary switch between more prospective coding when accelerating and less
187 prospective coding when decelerating.

188 Consistent with previous works³⁻⁵, correlations between time shift and either forward or
189 angular speed were small (forward speed, GC: $\rho = -0.06-0.01$, PC: $\rho = -0.05--0.00$; angular
190 speed, GC: $\rho = -0.07-0.02$, PC: $\rho = -0.07--0.02$) and not significant in individual rats.
191 However, these correlations were negative in a majority of subjects and reached significance
192 when aggregating all runs across rats (forward speed: GC: $\rho = -0.006$, PC: $\rho = -0.03$; angular
193 speed: GC: $\rho = -0.02$, PC: $\rho = -0.04$). A closer inspection revealed a nonlinear relationship
194 (Figs. 3B-C, S7), whereby the average forward speed (and angular speed) was consistently
195 higher on runs with time shifts closer to zero and runs with either very large positive or negative
196 time shifts were associated with slower average forward and angular speeds. This finding is
197 consistent with the idea that time shifts may reflect the information about the future and past
198 actions usually occurring when the animal is relatively immobile^{25,26}.

199 To check whether these behavioural correlations with time shifts might be related to boundary
200 effects or arena shape, we compared rank correlations between three pairs of topographic
201 subsets of the data: arenas "S1" (trapezoid) vs "S4" (rectangle), near walls vs interior regions,
202 and the right vs left halves of the enclosure. For acceleration and speed, correlation coefficients
203 never differed more than expected by chance for more than one subject for any comparison (z-
204 test; TSBH FDR $\alpha = 0.05$). A more detailed comparison found no consistent topographic
205 dependence for time-shift–acceleration correlations (Fig. S8), and differences in directional
206 modulation d were never significant for any comparison (both heading and head direction; two-
207 tailed test on the bootstrapped difference in d ; TSBH FDR $\alpha = 0.05$). No significant regional
208 differences in the correlations with acceleration or speed emerged when aggregating cells from
209 all rats.

210 Finally, we checked whether the tendency toward larger time shifts in the larger arenas could
211 be attributed wholly to behavioural correlations. We measured the partial correlation between
212 arena shape (S1–S4) and time shift (ranks), conditioned on acceleration (ranks) and heading
213 angle ($\cos(\theta)$, $\sin(\theta)$). This summarises the linear variation in (ranked) time shifts that can be
214 uniquely explained by, e.g. arena shape and no other variables (N.B. for simplicity, we omitted
215 forward and angular speed analysis due to their substantially lower nonlinear influence on time
216 shifts, see above). The positive trend in grid cells persisted ($\rho_{\text{partial}}=0.029$, $p<10^{-11}$; c.f. raw
217 correlation $\rho_0=0.030$; single-field runs from all subjects aggregated). Likewise, significant
218 partial correlations between run direction (near walls) and time-shifts persisted (GC:
219 $\rho_{\text{partial}}=0.105$, $p<10^{-27}$, c.f. $\rho_0=0.145$; PC: $\rho_{\text{partial}}=0.148$, $p<10^{-6}$, c.f. $\rho_0=.221$; single-field runs
220 from all subjects aggregated). Finally, significant (albeit smaller) partial rank-correlations
221 persisted for (acceleration and time shift) when conditioning on run-direction near walls (GC:
222 $\rho_{\text{partial}}=0.068$, $p<10^{-11}$, c.f. $\rho_0=0.109$; PC: $\rho_{\text{partial}}=0.095$, $p<10^{-3}$, c.f. $\rho_0=0.175$; single-field runs
223 from all subjects aggregated).

224 In summary, time shifts are generally more prospective in the larger arenas when running
225 inward from the wall, when accelerating (vs decelerating), and when moving eastwards (vs
226 westwards). Acceleration and inward/outward run direction are correlated but not wholly
227 redundant. Heading and arena-shape correlations potentially reflect the effects of room
228 distortion, as previously explored in^{11,22}.

229 Time shifts are correlated with the spatial scale of firing patterns

230 Grid cells are topographically arranged into distinct grid modules along the dorsoventral axis
231 of the mEC, with smaller scales located dorsally and larger scales at ventral parts of the mEC^{17–}
232 ^{19,27} and were shown to exhibit independent neural processing^{18,28}. Hence, we asked whether
233 place and grid cells' time shifts also varied with their spatial scale (Fig. 4). We reasoned that if
234 larger grids represent larger distances, they may also exhibit longer time horizons. For this
235 analysis alone, we added a separate dataset recorded in larger enclosures ($\sim 2.8 \text{ m} \times 3 \text{ m}$, see
236 Methods), which included grid cells with much larger scales. We found a significant positive
237 correlation between grid scales and time shifts, with larger-scale grid cells having more
238 prospective time shifts (Fig. 4A-B, $n = 249 + 21$ cells from small + large enclosures
239 respectively: Spearman's $\rho = 0.44$, $P = 5.3 \times 10^{-14}$). Similarly, place cell optimum time shifts

240 were positively correlated with their field sizes (Fig. 4C-D, $n = 74 + 24$ cells from small + large
241 enclosures, respectively: Spearman's $\rho = 0.54$, $P = 1.3 \times 10^{-8}$).

242 To exclude the trivial explanation that these correlations resulted from a methodological bias
243 towards estimating larger time shifts in larger fields, we repeatedly simulated grid cells and
244 place cells of different scale and field sizes, respectively, with zero underlying time shifts,
245 matching the recorded data sets (Fig. S9, see Methods). In simulated maps with larger grid
246 scales/place field sizes, we found that the optimum time shift estimates had a higher variance,
247 but the means were not significantly positive or negative. This suggests that our methods do
248 not introduce bias towards identifying more positive time shifts in larger-scale firing rate maps.

249 Grid modules provide multiple simultaneous time shifts

250 We next asked whether on a moment-by-moment basis overlapping fields of co-recorded grid
251 cells from the same grid module had similar time shifts, whereas cells from different co-
252 recorded grid modules consistently showed larger time shifts. This would suggest that the time
253 shift is signalled at the grid module level and that a spectrum of multiple time shifts may be
254 read out nearly simultaneously (within < 200 ms time window; see Methods) from the outputs
255 of different scale grid cells. To address this, we again segmented the animal's trajectory into
256 individual runs through single grid fields²² and calculated the difference in “ δ ” time shifts
257 between overlapping individual runs for all simultaneously recorded grid cell pairs. Indeed, we
258 found that the difference between the optimal time shifts was not significantly different from
259 zero in co-recorded grid cells of the same grid module (Fig. 5A, mean \pm s.e.m.: -2.3 ± 5.0 ms,
260 $t_{6249} = -0.46$, $P = 0.64$, Student's t-test). However, such difference in time shifts was
261 consistently positive in larger grid modules (Fig. 5B, mean \pm s.e.m.: 78.8 ± 6.9 ms, $t_{10101} = 9.70$,
262 $P = 3.64 \times 10^{-22}$, two-sample t-test) indicating that multiple progressively increasing time
263 horizons within a few hundreds of milliseconds range can be readout nearly simultaneously at
264 any given point in the environment (N.B. the co-recorded grid fields of both larger and smaller
265 scales as well as place fields were distributed across the entire enclosure).

266 Time shifts show the differential distribution across theta phase

267 Cell activity across the hippocampal formation is organised in relation to the theta rhythm²⁹⁻³³.
268 Place cells in CA1 tend to fire near the trough (phase 0°) of the LFP theta oscillation in *stratum*
269 *pyramidale*^{29,30,32} and continue spiking at progressively earlier phases of each theta cycle as the

270 animal traverses a cell's firing field (a phenomenon known as phase precession^{20,31}). Both grid
271 and place cells can show theta precession^{20,31,34}. In grid cells, theta modulation is layer
272 dependent³⁴ with mEC layer II grid cells often phase precessing, whereas layer III grid cells
273 showing both theta precession as well as theta locking activity (i.e. the tendency to fire at a
274 specific theta phase). Furthermore, experimental evidence shows that several variables
275 correlate with the LFP theta phase, including the relative inputs that CA1 receives from the
276 medial entorhinal cortex and CA3^{29,32,35}; the ease of inducing long-term
277 potentiation/depression at CA3 synapses to CA1^{36,37}; and the timing of fast vs. slow gamma
278 oscillations in CA1²⁹. Such observations support models in which the encoding and retrieval
279 of spatial memories (e.g., associations between specific locations and food reward) occur on
280 separate phases of LFP theta³⁸. In such a context, it is possible that each cell's prospective bias
281 during a specific trial is correlated with the mean theta phase at which it tended to fire,
282 reflecting the balance between that cell's involvement in encoding and retrieval during the trial.

283 To explore this, we looked at place and grid cell firing with respect to the theta rhythm in CA1.
284 Previous studies have shown that the trough (phase 0°) of the theta rhythm in stratum
285 pyramidale of CA1 aligns with the phase associated with the highest probability of CA1 place
286 cell firing^{29,30,32}. Hence, for each rat, we shifted the measured theta phase such that 0°
287 corresponded to this peak in CA1 activity (Fig. S10). This accounted for differences in
288 recording location within each rat and enabled closer comparison with previous studies. To
289 allow a comparison of place cells with grid cells, we also analysed grid-cell firing in terms of
290 its relationship to the stratum pyramidale theta rhythm rather than the entorhinal theta rhythm.
291 After alignment, we found that CA1 place cells spiked maximally at $0.0 \pm 72.0^\circ$ (circular mean
292 \pm circular standard deviation), while grid cells spiked maximally at $268.4 \pm 78.1^\circ$, with place-
293 cell theta phases lagging grid-cell theta phases (Fig. 6A-B), consistent with cell firing
294 probabilities of principal cells in CA1 and mEC layer II³¹.

295 We next asked whether each cell's preferred firing phase relative to the theta rhythm was
296 correlated with the size of its prospective time shift. Several place cells (28/56) and grid cells
297 (29/39) showed significant theta precession (Methods). We could not detect a significant
298 difference between the time shifts of precessing and non-precessing cells (Table S1; MWUT:
299 place cells: $P = 0.23$; grid cells: $P = 0.31$). Hence, to investigate the relationship between
300 average time shift and theta phase at a single cell level, we limited this analysis to cells with
301 statistically significant theta phase locking, regardless of whether they showed phase
302 precession (Methods). We found that both place and grid cells showed a significant positive

303 correlation between their average preferred theta phases and their optimum time shifts (Fig.
304 6C; place cells, $n=239$ recordings: Kempter circular-linear correlation coefficient
305 (CLCC) = 0.36, $P=3.6 \times 10^{-6}$; Fig. 6D: grid cells, $n=117$ recordings: CLCC = 0.65,
306 $P=3.3 \times 10^{-11}$). This was also true at the level of cells from individual rats (Table S2).
307 Curiously, grid-cell time shifts showed a nonlinear relation with the theta phase, saturating at
308 small values for cells with earlier preferred theta phases (Fig. 6D). To summarise this trend,
309 we regressed a piecewise linear model $\Delta t = \max[\Delta_0, m \cdot (\theta - \theta_0)]$ to the combined grid-cell data
310 from all animals. Bootstrap confidence intervals suggest that grid-cell time shifts saturate
311 around $\Delta_0 = 19$ ms (median; IQR 8.8 ms), with the linear trend beginning around $\theta_0 = 220^\circ$
312 (median; IQR 11°). Overall, these findings suggest that the horizon of a cell's prospective
313 coding may depend on the phase of its coupling to the hippocampal theta rhythm in agreement
314 with previous studies^{2,29,39}. This could facilitate the readout of multiple increasing time
315 horizons from grid cells of different scales (Fig. S11) and place cells of different place field
316 sizes.

317 It is important to note that this suggested correlation is not a necessary consequence of theta
318 precession. In theta precession, cells spike at progressively earlier phases of each theta cycle
319 as the animal traverses a cell's firing field. At the level of an individual cell, therefore,
320 precession does predict that spiking at later theta phases is technically more "prospective".
321 However, precession only describes this gradient in phases for an individual cell^{20,40}. It does
322 not constrain the mean phase at which each cell fires. Hence, different cells can display firing
323 locked to different mean phases (though these mean phases are usually clustered around 0°),
324 regardless of whether they precess.

325 Discussion

326 Here we show that time horizons (i.e. the average look-ahead time range) of the grid and place
327 cell positively correlate with their scale and field size, respectively, enabling nearly
328 simultaneous readout of a spectrum of progressively increasing time horizons over a few
329 hundred milliseconds. Distinct grid modules, whose scales increase along the dorsoventral
330 mEC axis^{17-19,27}, may provide a nearly instantaneous readout of future positions at multiple
331 time horizons to hippocampal place cells. In line with this hypothesis, our findings suggest that
332 place cells tend to be more prospective than their field-size-matched grid cells, potentially
333 reflecting the contribution of inputs from larger grid cell modules⁴¹⁻⁴³. Furthermore, in our
334 dataset, the firing of CA1 place cells appears to lag that of mEC grid cells in the theta cycle

335 (Fig. 6), which may reflect the flow of information from grid cells (likely mostly recorded in
336 mEC layer II/III based on post-hoc histology¹¹) to CA1 place cells.

337 Previously it has been suggested that observed prospective time shifts may reflect the animal's
338 'true' location, positioned under its nose¹. In contrast, our data show that place and grid cells
339 simultaneously encode a continuum of locations, which range from the centre of a rat's head to
340 slightly more forward from its nose (see Fig. S12 for median speed distribution) and are
341 determined by the scale and the field size of grid and place cells, respectively, as well as
342 modulated by multiple external and internal cues such as boundary conditions, heading
343 direction, acceleration, forward and angular speeds. The range of simultaneously encoded
344 locations hints that these time shifts may not simply represent the rat's 'true' single location,
345 but instead, they may reflect ongoing computations by the entorhinal-hippocampal network,
346 which take into account past, present and future states. We found that similar to spatial grid
347 structure, the time horizon fundamentally depends on the external boundaries of the
348 enclosure^{11,12,15}. In the most polarised trapezoid, the time horizon shrinks compared to a
349 rectangular enclosure suggesting that the time horizon expands when animals expect a longer
350 run path. This is further corroborated by time shifts showing a significant positive bias in a
351 direction towards the unchanging part of the environment (eastward). Furthermore, information
352 from the boundaries may reduce the overall uncertainty of the rat's whereabouts, resulting in
353 shorter time horizons as the rat approaches the boundaries. In addition to the effects of external
354 cues, internal cues also play a role in modulating the time horizons with a nonlinear
355 relationship. Namely, time shifts show sigmoidal dependence on acceleration. Previously,
356 positive acceleration was associated with an increase in theta frequency⁴⁴. Theta frequency is
357 associated with the pacemaker of entorhinal-hippocampal computations^{2,20,29,32,33,38,40,45,46}, thus
358 this increase would suggest that larger time horizons (i.e. further look-ahead mode) may be
359 used in more rapid entorhinal-hippocampal computations during accelerated movements and
360 may provide a more efficient way for error correction due to rapid acceleration. At the same
361 time, larger time shifts at lower angular and forward speeds are consistent with the idea that
362 time shifts may reflect the information about the future and past actions usually occurring when
363 the animal is relatively immobile^{25,26,47}. It is important to note that our data suggests that these
364 internal and external factors are not entirely redundant and may be shaping time shifts
365 independently of each other. Moreover, the combined influence of all these factors cannot fully
366 explain the observed positive time shift bias, suggesting that it may be an intrinsic property of
367 the entorhinal-hippocampal network.

368 Our second major finding provides compelling evidence that hippocampal theta oscillations
369 may coordinate the readout of the spectrum of locations encoded by multiple place and grid
370 cells. We found that spiking at later stratum pyramidale theta phases is associated with more
371 prospective time shifts in both grid cells and CA1 place cells. While in place cells, this
372 association appears to be linear, in grid cells, it is strongly nonlinear, hinting towards the
373 possibility that distinct grid modules may be providing inputs at different theta phases (Fig.
374 S11). Furthermore, it has been previously suggested that information processing by the
375 entorhinal-hippocampal network is organised with respect to theta and gamma
376 oscillations^{2,20,29,32,33,38,40,45,46}. Namely, the prospective coding in CA1 place cells tends to be
377 accompanied by slow gamma oscillations, whereas fast gamma oscillations are associated with
378 more retrospective coding². Fast and slow gamma were shown to occur at different theta phases
379 associated with distinct modes of communications between the CA1-mEC and CA1-CA3²⁹,
380 which may correspond to memory encoding and retrieval modes respectively³⁸.
381 Hasselmo&Eichenbaum^{48,49} explicitly predicted a nearly simultaneous representation of
382 sequential activity in the hippocampus and the superficial layers of the medial entorhinal
383 cortex, with the timing of such activity based on the hippocampal theta phase. Moreover,
384 subsequent computational models predicted that such sequences should include place cells with
385 multiple place field sizes in order to support ‘flexible’ navigation to the goals⁴⁷ (including distal
386 goals). Our findings provide some compelling evidence supporting such models and further
387 extend them. Namely, the multi-size-field sequences were primarily modelled for place cells.
388 We show that this applies to grid cells as well. We argue that grid cells may represent a more
389 robust system than place cells since they maintain their ‘canonical’ scales invariable across
390 different experimental conditions²⁷ and, therefore, may be read out using a consistent
391 relationship to theta phase. Moreover, we found that grid cells may encode the time shifts on a
392 module basis, which may help to decrease the accumulation of noise. Specifically, each module
393 has many colocalised ‘identical’ grid cells, whose sum input would be less noisy, while
394 multiple grid scales of each module would ensure that the distribution of these look-ahead
395 representations is sufficiently dense. Both of these features were identified as crucial for
396 reliable navigation to distal goals⁴⁷.

397 Finally, it is important to stress that in these computational models, ‘decision points’ during
398 navigation to a goal spanned much wider look-ahead ranges^{47,48,50,51}. Place cells’ replay
399 activity⁵²⁻⁵⁴, which occurs when the animal is relatively immobile, was suggested to correspond
400 to such look-ahead sequences^{47,51}. Here, we show another type of sequential firing in place

401 cells and grid cells, which is continuously present, including when the rat is moving and spans
402 much shorter ranges, which depend on the rat's acceleration, angular and forward speeds as
403 well as the rat's relations to the boundaries and the shape of the enclosure. It is possible that
404 introducing a goal would modulate time horizons in line with observations that goals may
405 significantly influence place^{55,56} and grid^{57,58} fields. The successor representation hypothesis⁵⁹
406 predicts that one role of grid cells may be to transform distal goals into immediate movement
407 commands. This requires information about both the position and the desired changes in
408 position (position derivative). By dynamically combining inputs from different grid modules
409 with progressively longer time horizons, hippocampal place cells may be able to generate
410 smooth future trajectories ahead of the animal, which may provide the basis for goal-directed
411 navigation and planning^{56,59}.

412 **Methods**

413 This work was conducted in accordance with the UK Animals (Scientific Procedures) Act
414 (1986). The following sections summarise the acquisition of the original data from¹¹ and
415 previously unreported data from 7 rats recorded in larger enclosures. Below we describe the
416 analytical methods applied to the data in this study.

417 **Animals**

418 10 adult male Lister Hooded rats were chronically implanted in the left and/or right hemisphere
419 with a microdrive (Axona) loaded with four tetrodes (9 rats), or a single Neuropixels probe⁶⁰
420 (1 rat; R2405). Tetrodes were aimed at the superficial layers of the medial entorhinal cortex
421 (mEC, 6 rats: 4.3–4.5 mm lateral to the midline; 0.2–0.5 mm anterior to the sinus; angled
422 forwards in the sagittal plane at 0–10° and 1.5 mm below the pia) and/or CA1 region (6 rats:
423 2.5 mm lateral to the midline; 4 mm posterior to bregma and 1.4–1.8 mm below the pia). See
424 Supplementary Information Fig. S1 in¹² for more details and histology results. In short, in most
425 of the cases the recordings were done from layer II or layer III of mEC.

426 Neural activity was recorded while the rats foraged for food in four familiar polygonal
427 enclosures (Fig. 1A), which varied in shape from a rectangle (poly180°, S4; 1.8 m × 1.0 m) to
428 a left trapezoid (poly129°, S1) and were presented in random order.

429 The rat's position was determined based on an array of two infrared light-emitting diodes
430 (LEDs) fixed to the rat's head and recorded with an overhead infrared camera (sampled at
431 50 Hz). The LEDs were fixed to the plastic screw post cemented to the head of the animal ~3
432 mm posterior from the bregma and ~3 mm lateral (always on the left-hand side) from the
433 midline. The position of the plastic screw post corresponds to the position of the rat at zero
434 time shift.

435 To analyse the relationship between firing scale and time shifts over a larger range of grid cell
436 scales (Fig. 4), we included electrophysiological data collected from 7 rats during a separate
437 series of experiments in larger enclosures. During these experiments, rats foraged in two
438 polygonal enclosures, which differed only in the configuration of their western wall: a 2.8 m
439 × 3 m (north-south × east-west) rectangular enclosure (Fig. 4C) and a right trapezium (Fig.
440 4A; the northwestern corner is displaced by 1.2 m horizontally between the two enclosures).
441 The rats were trained for 3–4 weeks to forage for sweetened rice randomly scattered throughout

442 these enclosures. Subjects were then implanted with a pair of 8 tetrodes (2 rats) or two single-
443 shank Neuropixels⁶⁰ probes (5 rats) in the left mEC and right CA1 region (the surgical
444 procedures described above). During a trial, each rat would forage in one of the two enclosures
445 for at least 60 minutes; each experimental session consisted of two trials in different enclosures.
446 The rat's position was determined based on an array of two infrared LEDs fixed to the rat's
447 head as described above (sampled at 50 Hz). Speed was smoothed using a moving average
448 (box) filter with a span of 20 samples (400 ms).

449 It should be noted that the large-arena datasets were included only in the analysis relating to
450 field scale and time shifts to include grid and place cells with as large scale/field sizes as
451 possible (Fig. 4). These large-arena recordings did not include simultaneously recorded grid
452 cells of multiple scales or the hippocampal LFP; hence it this new data could not be used for
453 the analysis in Figs. 5-6. Furthermore, we restricted our results to the data from¹¹ in Figures 1
454 and 2 to avoid substantial differences in field sizes between the grid and place-cell populations
455 and to enable direct comparison between the prospective temporal properties and the results on
456 spatial deformations reported in¹¹.

457 Rate maps and smoothing

458 We divided each enclosure area into 2.6×2.6 cm bins and then used the recordings to make
459 maps of (a) the number of spikes fired in each bin and (b) the time spent in each bin (“dwell
460 time”). We smoothed each of these maps by convolution with a Gaussian kernel and produced
461 each smoothed rate map by dividing binned spike counts by dwell times. We defined the kernel
462 width as $\sigma = l/2\pi$, where l was the position of the first trough in the radial average SAC. Place
463 cells were smoothed with a fixed kernel width of $\sigma = 3$ bins (7.9 cm). Smoothed rate maps were
464 used for all subsequent methods unless specified otherwise. Unvisited bins within the bounds
465 of the map are shown in white in Fig. 1A. For all ZLAC calculations, these bins were assigned
466 rates by interpolating between adjacent bins.

467 Cell classification

468 Grid cells were identified as cells with a gridness score of $> 0.27^{11,61}$. Only non-directional grid
469 cells were used in the current study to make sure that the results are not due to directional bias.
470 Only CA1 cells with clearly defined fields were classified as place cells and included in this
471 analysis as described in¹¹.

472 We identified conjunctive grid cells⁶² and similar direction-sensitive place cells by calculating
473 the head direction (HD) score of each recording⁶³. We produced a polar histogram showing
474 each cell's firing rate as a function of HD, using 1° bin and smoothing the resulting counts with
475 a rolling 25-bin box filter. To assess the directionality of this distribution, we calculated the
476 Rayleigh vector as follows:

$$477 \quad D = \frac{\sum_{\theta=1}^{360} \rho_{\theta} \cdot e^{i\theta}}{\sum_{\theta=1}^{360} \rho_{\theta}}$$

478 where ρ_{θ} represents the polar firing rate as a function of HD in the bin $[\theta-1, \theta]$.

479 This vector's magnitude, $|D| \in [0, 1]$ was used as the HD score. Cells with $|D| > 0.5$ during any
480 of the trials were excluded from subsequent analysis. This procedure and the thresholds used
481 are identical to those used by⁶³.

482 Spatial autocorrelation (SAC) and cross-correlation (SCC)

483 Let λ_1 and λ_2 denote two unsmoothed firing rate maps, where $\lambda(x, y)$ is the value of rate map λ
484 at the coordinates (x, y) . The normalised spatial cross-correlation (SCC) between these maps is
485 calculated for each possible discrete spatial lag (τ_x, τ_y) (measured in bins) as follows, as defined
486 by²⁵:

$$487 \quad R(\tau_x, \tau_y) = \frac{n \cdot \sum (\lambda_1(x, y) \cdot \lambda_2(x - \tau_x, y - \tau_y)) - \sum(\lambda_1(x, y)) \cdot \sum(\lambda_2(x - \tau_x, y - \tau_y))}{\sqrt{n \cdot \sum(\lambda_1(x, y)^2) - \sum(\lambda_1(x, y))^2} \cdot \sqrt{n \cdot \sum(\lambda_2(x, y)^2) - \sum(\lambda_2(x, y))^2}}$$

489 where each summation Σ is over all n bins for which rates were estimated in both λ_1 and λ_2 .
490 Setting λ_1 equal to λ_2 makes the above an equation for spatial autocorrelation (SAC). Spatial
491 cross-correlograms were smoothed with a Gaussian kernel ($\sigma = 2$ bins, or 5.2 cm), before
492 subsequent analysis.

493 Zero-lag autocorrelation (ZLAC)

494 The formula defined above is equivalent to calculating Pearson's correlation coefficient of a
495 rate map with a copy of itself at different spatial lags; hence, $R(0, 0) = 1$. To measure the
496 'sharpness' of each rate map, we used the following simplified version of the autocorrelation
497 equation:

498
$$R(\tau_x, \tau_y) = \frac{1}{n} \sum (\lambda(x, y) \cdot \lambda(x - \tau_x, y - \tau_y))$$

499 This version of autocorrelation removes the normalisation usually applied to a spatial
500 autocorrelogram. This means that the autocorrelation value at zero spatial lag, i.e. $R_{\text{sim}}(0, 0)$, is
501 no longer guaranteed to be equal to 1. Instead, when the time-shifted version of a rate map has
502 a higher $R_{\text{sim}}(0, 0)$ than the original, it can be inferred that the map's fields are sharper than
503 before. This is equivalent to the mean squared value of each rate map. Each map's optimum
504 time-shift was determined as the position of the ZLAC peak nearest 0 ms. These correlograms
505 were smoothed with the same Gaussian kernel used for normal SACs ($\sigma = 2$ bins, or 5.2 cm).

506 Radial average SAC

507 We determined each grid cell rate map's scale by calculating its radial average SAC. These
508 were calculated by averaging of all bins within shells of increasing radius l from the smoothed
509 autocorrelogram:

510
$$R_{\text{radial}}(l) = \frac{1}{n} \sum_{\tau_x, \tau_y \in \mathcal{R}_l} R(\tau_x, \tau_y) \forall l = 0, 1, 2 \dots$$

511 where each summation is over the set \mathcal{R}_l of all n bins where $(l - 1) < \sqrt{\tau_x^2 + \tau_y^2} \leq l$. To find
512 the grid cell's scale, $R_{\text{radial}}(l)$ was smoothed using a moving average (box) filter over a span of
513 9 bins; the grid cell's scale was taken as the position of the first peak at $l > 0$.

514 Spatial Information

515 We calculated the spatial information S using a method similar to²⁰:

516
$$S = \sum_{i=1}^n p_i \frac{\lambda_i}{\Lambda} \log_2 \frac{\lambda_i}{\Lambda}$$

517 where Λ is the mean firing rate of the cell; λ_i is the mean value of bin i in the smoothed firing
518 rate map; and p_i is the mean value of bin i in the smoothed map of dwell times. Each map is
519 divided into spatial bins $i = 1, \dots, n$. Note that we use bin values from the smoothed versions of
520 each map.

521 Time shifting

522 We used the following procedure to “time shift” recording data. During each trial, the rat’s
523 position and head direction were recorded at 50 Hz. Hence, we organised the spikes fired by
524 each cell into a vector of spike counts, also sampled at 50 Hz. This spike count vector was
525 shifted by a given number of samples to lead or lag the position and head direction vectors
526 (leading = “positive”, lagging = “negative” time shifts). Each sample corresponds to a shift of
527 20 ms; recordings were time-shifted by up to ± 2 seconds. Rate maps were then recalculated
528 from these vectors as normal, using the original smoothing parameters.

529 Validation of ZLAC analysis of time shifts

530 We observed a significant correlation between cells’ optimal time shifts and the size of their
531 firing fields. To identify any methodological bias, we generated grid- and place-cell activity
532 patterns with scales/field sizes matching the real data (Fig. S9A-B; see sections below). We
533 combined these with randomly selected trajectories from the actual dataset to simulate several
534 spike counts and the resulting rate maps (Fig. S9C-D; see sections below). We then used the
535 same ZLAC analysis applied to the experimental data to test whether identified ZLAC peaks
536 showed any consistent deviation from the correct time shift of 0 (Fig. S9E). We found that the
537 median identified time shift was more variable with increasing field size (Fig. S9F-G).
538 However, there was no evidence to suggest that the ZLAC analysis showed a bias towards non-
539 zero median shifts when firing fields were larger (Benjamini-Hochberg test, $\alpha = 0.05$).

540 Choosing place cells with single fields

541 When analysing the relationship between place-field size and time shifts, we restricted our
542 analysis to cells with single fields to simplify interpretation. We found the rates (λ) and
543 positions of all local maxima on the place cell’s rate map after smoothing, including the bin
544 with the highest overall rate (λ_{\max}). If any of the other local maxima had rates $\lambda > 0.4 \cdot \lambda_{\max}$, we
545 classified that map as containing multiple potential fields and excluded it from this particular
546 analysis. Each field area was defined as the area of pixels with rates $\lambda > 0.2 \cdot \lambda_{\max}$ surrounding the
547 highest-rate bin.

548 Simulating grid cell activity functions

549 We modelled grid-cell firing fields as three overlapping 2D plane waves oriented at 60° offsets
550 to each other (Fig. S9A). First, we calculate the total wave vector, k :

$$551 \quad k = [k_x k_y] = \frac{1}{L} \sum_{i=1}^3 [\cos\theta_i \sin\theta_i]$$

552 Each wave has wavelength $L = \frac{1}{2\pi} \frac{\sqrt{3}}{2} \cdot l$, where l is the grid scale. For a given point (x, y) , the
553 grid cell's activity $f_{x,y}$ is calculated as follows:

$$554 \quad f_{x,y} = \exp\left(\cos\left(\frac{x \cdot k_x + y \cdot k_y}{2}\right)^{2s}\right) - 1$$

555 The parameter s determines the 'sharpness' of the resulting grid; higher values decrease the
556 width of each firing field. For all simulated grid cell rate maps, $s=0.5$.

557 Simulating place cell activity functions

558 We modelled place cell firing fields as 2D Gaussian functions (Fig. S9B), i.e. for a given point
559 $x=[x,y]$, the place cell's activity is calculated as

$$560 \quad f_{x,y} = \exp\left(-\frac{1}{2} (x - \mu)^T \Sigma^{-1} (x - \mu)\right),$$

561 where the vector μ represents the coordinates of the place field centre, and $\Sigma = [\sigma 0 0 \sigma]$. The
562 parameter σ controls the width of the resulting place field, which is circular.

563 Drawing spike vectors from simulated rate maps

564 The activity functions described above specify the probability that a grid cell will spike in a
565 particular position (as the rate parameter of an inhomogeneous Poisson distribution). To
566 simulate a grid cell's firing, we specified its activity function, selected a trajectory taken by a
567 rat during a randomly selected trial, and used the function to assign a probability of spiking to
568 each point along this trajectory. We then assigned a fixed number of spikes to points along this
569 trajectory in line with said probabilities (grid cells: 2000 spikes, place cells: 500 spikes) (Fig.
570 S9C-D).

571 We simulated grid cell rate maps that varied in their scale (0.3 to 1.2 m, in 0.3 m steps) and
572 place cell rate maps that varied in the size of the (single) firing field (covering 0.1 to 0.4 m², in

573 0.1m² steps). Grid cell rate maps varied in their phase offset, so all possible horizontal and
574 vertical phase offsets of each specified map were uniformly sampled. Similarly, we randomised
575 the position of the field on each place cell rate map. Each combination of scale and time shift
576 was sampled 2000 times.

577 Defining individual firing fields and runs through fields

578 To study time shifts on single runs through grid cells' firing fields, we segmented each rate
579 map into fields by applying MATLAB's inbuilt watershed function to the negative rate-map.
580 This identifies each minimum's "drainage basin"; each identified basin corresponds to a field
581 surrounding a peak in the original rate map. To avoid over-identifying spurious peaks as
582 potential field centres, we applied MATLAB's extended minima transform to remove minima
583 whose depth was less than $\sigma/5$, where σ is the standard deviation of all of the image's pixels.
584 We excluded fields whose peaks were less than 15 cm from the edge of the enclosure. We
585 removed pixels on the edges of each field whose firing rates were <20% of its peak firing rate.
586 This 20% threshold was also used for defining each place cell's total firing field area.

587 The rat's trajectory was divided into segments, or "runs", passing through each field. We
588 assigned each spike on this run (Fig. 2D, red dots) an individual, 'single-run' time shift relative
589 to the point on the run passing closest to the field centre. Spikes fired before (after) this point
590 were assigned positive (negative) time shifts. The overall time shift for the run was defined as
591 the median of these time shifts (Fig. 2D, green dots). We only considered runs that (a) had an
592 associated spike train and (b) passed within half the field radius of the field's centre.

593 Investigating time shifts of spike trains when approaching or leaving walls

594 We divided each enclosure into inner and outer zones of equal area (Fig. 2F). We focused on
595 fields where $\geq 30\%$ of each field fell within the outer zone (called peripheral fields). We
596 measured the distance between each bin on the edge of the peripheral field and the closest bin
597 on the enclosure boundary and used these distances to divide these bins into "inner" and "outer"
598 groups of equal size, where the "outer" group represents the half of the field's circumference
599 closest to the wall (Fig. 2F-G). We then isolated valid runs (see criteria above) that started in
600 the inner group and ended in the outer group or vice versa and labelled these runs "outward"
601 or "inward" runs, respectively. For each map, we collated the time shifts of inward and outward
602 runs across all valid peripheral fields. We excluded grid-cell maps with fewer than 15 inward

603 or 15 outward runs in total. As place cell fields covered a smaller proportion of each arena, we
604 reduced the minimum numbers of inward and outward runs to 5 each.

605 In each recording with sufficient runs, we looked at the median time shift on inward and
606 outward runs and the difference between them. The final value assigned to each cell was the
607 grand median of these values for all trials in which said cell was recorded during a given
608 session.

609 Comparing time shifts of spike trains from pairs of co-recorded grid cells

610 To estimate the difference between time shifts in overlapping grid fields, we calculated all the
611 median time shifts of all individual laps through the fields. The runs from different cell pairs
612 were considered overlapping if they occurred within 200 ms from each other and the distance
613 between the average respective field centres was less than 5 bins (i.e. < 13 cm). We then
614 calculated the time-shift differences between all co-recorded runs of all co-recorded grid-cell
615 pairs. We split these differences in time shifts (larger field - smaller field) into intra-module
616 (grid scale ratio < 1.1) and inter-module (grid scale ratio ≥ 1.4), which were compared using a
617 two-sample t-test. We also used Student's t-test to test whether the time shift differences of
618 inner-grid modules differed significantly from zero.

619 Analysis of external and internal cues' effects

620 When regressing time-shift (Δt) against enclosure shape (Fig. 2A) and heading (Fig. 2B), we
621 removed per-cell (and per-subject) effects by standardising time-shifts across cells. We
622 replaced the per-cell median Δt with the population median Δt before aggregating. We
623 regressed time-shift (Δt) against direction θ by fitting $\Delta t = a \cos(\theta) + b \sin(\theta) + c$ using least-
624 squares. This is equivalent to the model $\Delta t = d \cos(\theta - \theta_{\max}) + c$, where the modulation depth
625 “ d ” is given by $d^2 = a^2 + b^2$, and $\theta_{\max} = \tan^{-1}(b/a)$ is the direction in which time shifts tended
626 to be the largest. We assessed significant tuning by re-fitting the model on shuffled θ , and
627 calculated confidence distributions for d and (differences therein) by bootstrap resampling
628 (1000 samples).

629 We calculated per-run acceleration (angular speed) as the change in forward speed (head
630 direction) during a run through a single field divided by the run's duration. We calculated per-
631 run heading (head directions, forward speeds) as the average heading (head direction, forward
632 speed) throughout every single run. We calculated Spearman's rank correlations between time

633 shifts and per-run acceleration, forward speed, and angular speed. To remove per-cell effects,
634 we converted time shifts to ranks before aggregating. To remove per-subject effects, we
635 converted acceleration and speed data to ranks per-subject before aggregating. To invert the
636 rank transform when plotting Fig. 3A, we took the average time-shift (over aggregated cells)
637 corresponding to each rank (in bin sizes of two percentage points), and smoothed the resulting
638 function with a Gaussian (σ : 2% percent) to remove further variance. To calculate partial
639 correlations, we used the Python package "Pingouin"^{64,65}

640 Theta analysis

641 The LFP signal was recorded in the hippocampus using one of the tetrode wires. It was
642 amplified 2000–8000 times, bandpass filtered at 0.34–125 Hz and sampled at 250 Hz. The LFP
643 signal was only available for analysis from multi-tetrode recordings. The signal phase was
644 calculated as the argument of the Hilbert transform $H(t)$ (MATLAB).

645 Pairwise phase consistency

646 We restricted our analyses of theta phase-precession to cells that showed significant coupling
647 to the theta rhythm, as assessed by phase-locking statistics. The theta phase-locking value for
648 a given recording was calculated as the magnitude of the average complex-valued phase-vector
649 (“phasor”):

$$650 \quad |z| = \left| \frac{1}{N} \sum_{j=1}^N e^{i\varphi(t_j)} \right|$$

651 where t_j is the time at which spike j was fired, N is the number of spikes fired by the cell during
652 the trial, and $\varphi(t)$ is the theta phase in radians. To decrease the bias induced by finite sampling,
653 we calculated the pairwise phase consistency (PPC, or \hat{Y} ; ^{66,67}), which corrects for this bias:

$$654 \quad \hat{Y} = \frac{N|z|^2 - 1}{N - 1}$$

655 For large N , $\hat{Y} \approx |z|^2$. We assessed each recording’s degree of theta phase locking by
656 comparing \hat{Y} to a shuffled distribution, produced by circularly shifting the recorded LFP signal
657 2000 times before recalculating \hat{Y} (minimum offset 10 seconds in either direction). Any
658 recordings with a \hat{Y} value that exceeded the 99th percentile of their respective shuffled

659 distributions were deemed to display significant phase locking, and subsequent in analyses
660 correlating preferred theta phase with time shifts.

661 Assessment of correlations

662 Unless otherwise stated, we evaluated correlations using Spearman's rank correlation
663 coefficient (ρ), which is appropriate when the underlying correlation is not necessarily linear.
664 There were three major exceptions to this. We used Pearson's correlation coefficient to
665 evaluate the correlation between optimum shifts and the inverse of the running speed and the
666 correlation between single-run time shifts and 'fixed' time shifts estimated by ZLAC (Fig. 2C),
667 since any correlations were expected to be linear in both cases. We calculated correlations
668 between preferred theta phases and time shifts using Kempter's circular-linear correlation
669 coefficient (CLCC)⁶⁸, since the phase is a circular variable.

670 Data availability

671 The full raw dataset is available from the corresponding author on reasonable request.

672 Code availability

673 All custom code written for reported analyses are publicly available at [insert Github link] or
674 via request to the corresponding author.

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872 **Author contributions**

873 J.K., M.B. and T.O'L. conceived the study. S.B., M.B., M.K., P.K. and J.K. collected the data.
874 P.C.V. and M.R. analysed the data with contributions from M.B., T.O'L. and J.K. J.K, M.R.
875 and P.C.V. wrote the manuscript with contributions from M.B. and T.O'L.

876 **Competing interests**

877 The authors declare no competing interests.

878 **Corresponding author**

879 Correspondence and requests for materials should be addressed to J.K.

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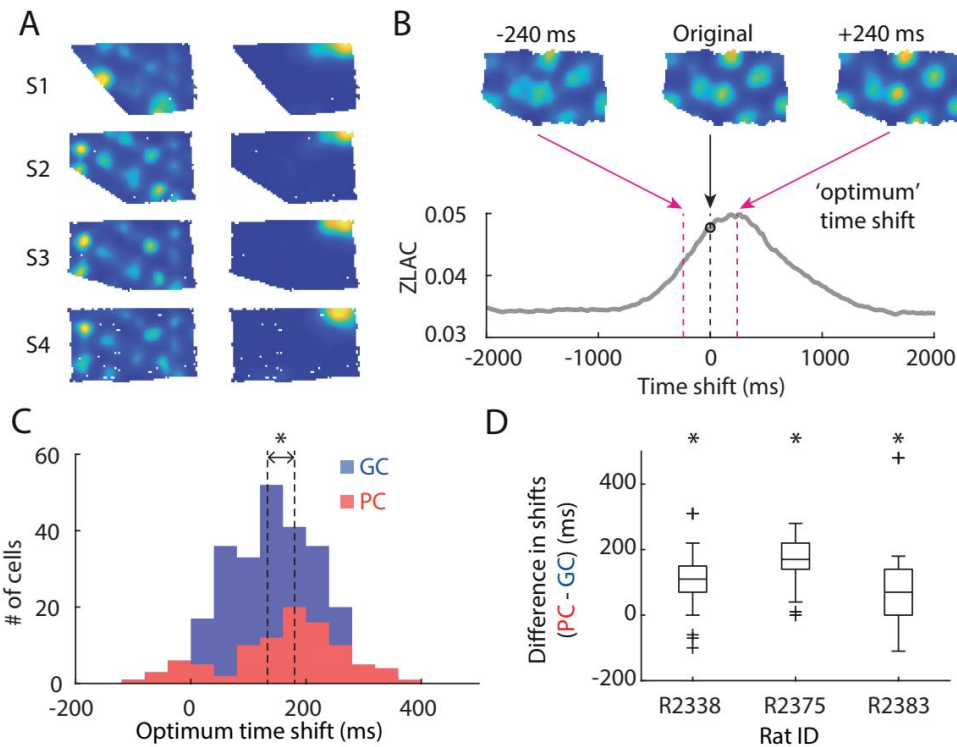
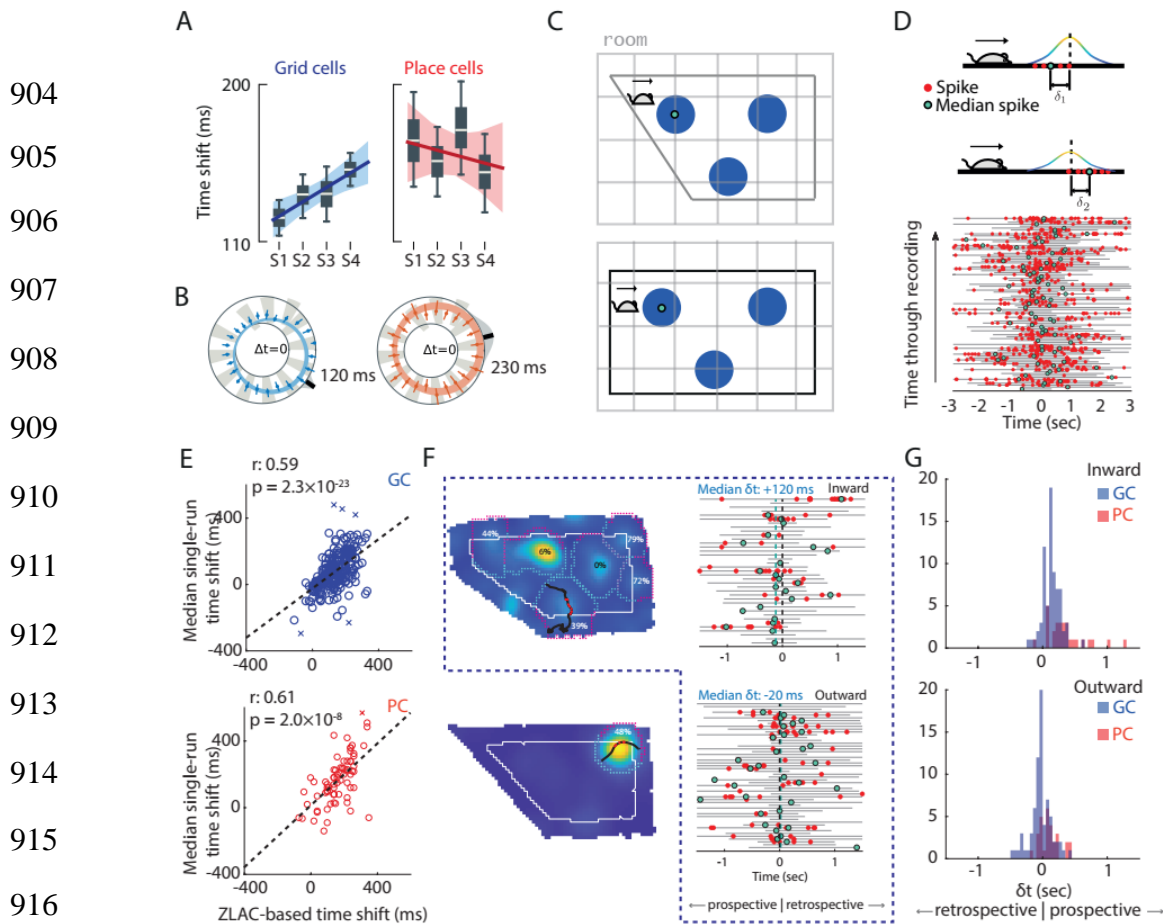


Figure 1. Grid cells and place cells encode space prospectively. (A) Rate maps of a typical grid cell (left column) and a place cell (right column) recorded in each of four differently shaped enclosures, “S1”-“S4”. (B) Shifting a cell’s spiking location along the trajectory of the rat sharpens firing fields, as measured by the zero-lag autocorrelation. ZLAC: zero-lag autocorrelation. (C) Time shift distribution from all 8 rats. This includes each cell’s median optimum time shift across all enclosures. On average, place cells had significantly larger prospective time shifts than grid cells GC: grid cell; PC: place cell. (D) The differences between trial-averaged grid and place cell time shifts in each of the three animals with co-recorded grid and place cells (outliers indicated by crosses). On average, recorded place cells showed more prospective time-shift values than grid cells.



917 **Figure 2. Time shifts depend on external boundaries.** (A) Changes in time shifts with enclosure
 918 shape. Time shifts significantly increased with enclosure size for grid, but not place cells. (B) Time
 919 shifts are consistently larger for eastward headings. Log-polar plots show heading on the angular axis
 920 and Δt on the radial axis (Inner axis ring: $\Delta t = 0$; Coloured rings: 95% confidence intervals for polar
 921 least-squares regression; Points: median Δt in 15° bins with (bars) 95% confidence). The exterior black bar
 922 indicates the regressed direction of largest Δt (“ θ_{\max} ”). Grey-shaded regions on the exterior reflect
 923 a von Mises model of uncertainty in θ_{\max} . All plots reflect single-run time shifts aggregated over cells
 924 from all subjects after removing per-cell variability. (C) Schematics of a rat heading eastwards. Grid
 925 fields (blue) close to the west wall shift westwards in the rectangle compared to the trapezoid. These
 926 shifts are consistent with time shifts becoming more prospective when the rat is moving eastwards.
 927 Grey: room frame of reference. The shifts are shown not scale to facilitate visualisation. (D) Schematics
 928 (left) and a plot (right) showing the timing of individual spikes (red dots) and median spikes (green
 929 dots) on all ‘valid’ runs for a particular grid cell recording. Note that several median spikes represent
 930 the only spike fired on a given run. (E) Scatter plot comparing ZLAC-estimated fixed time shifts with
 931 median single-field-run time shifts. Each point represents the mean measure for each cell (249 grid
 932 cells, 95 place cells). There is a significant positive correlation between the two measures. Crosses (\times)
 933 mark outliers calculated from cells without enough valid runs. (F) Left: Each rate map was split into
 934 inner and outer zones of equal area (white line, solid) based on the relative occupation in each zone
 935 (“peripheral” if $\geq 30\%$ of its area fell in the outer zone). Runs were deemed “inward” if they started on
 936 the outer half and ended on the inner half of each field and “outward” if vice versa. Black trajectory: an
 937 inward run; red dots indicate spikes fired along this run. Right: inward and outward runs from the grid
 938 cell rate map shown, and the spikes fired on each run. Dashed green and black lines indicate grand
 939 median single-run time shift and zero time shift: positive on the top and slightly negative in the bottom.
 940 (G) Median spike times on inward and outward runs across all 73 grid cells with sufficient runs.

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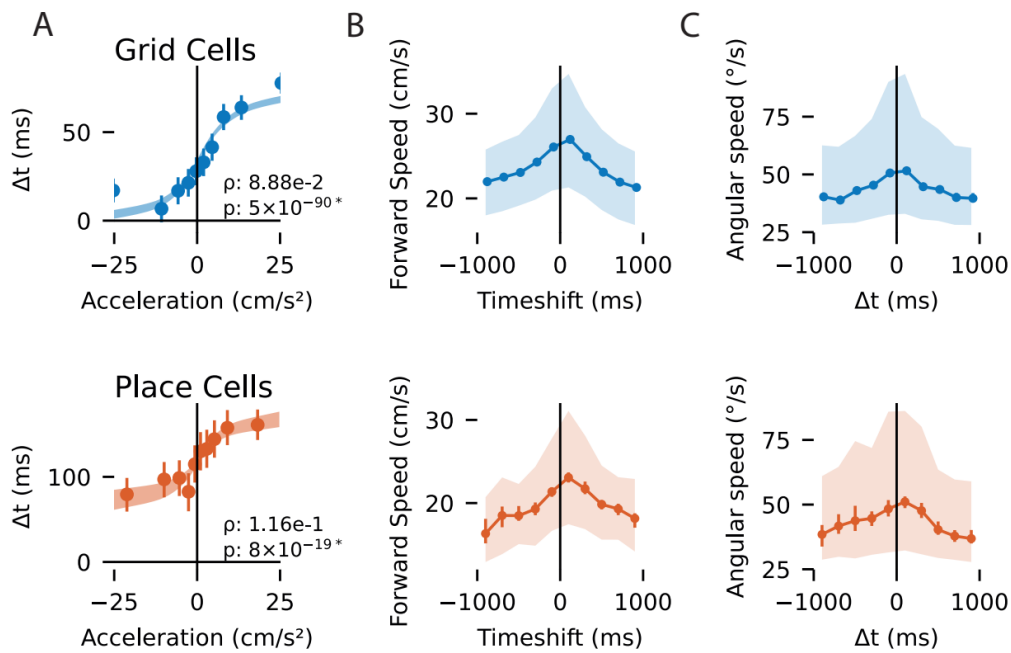
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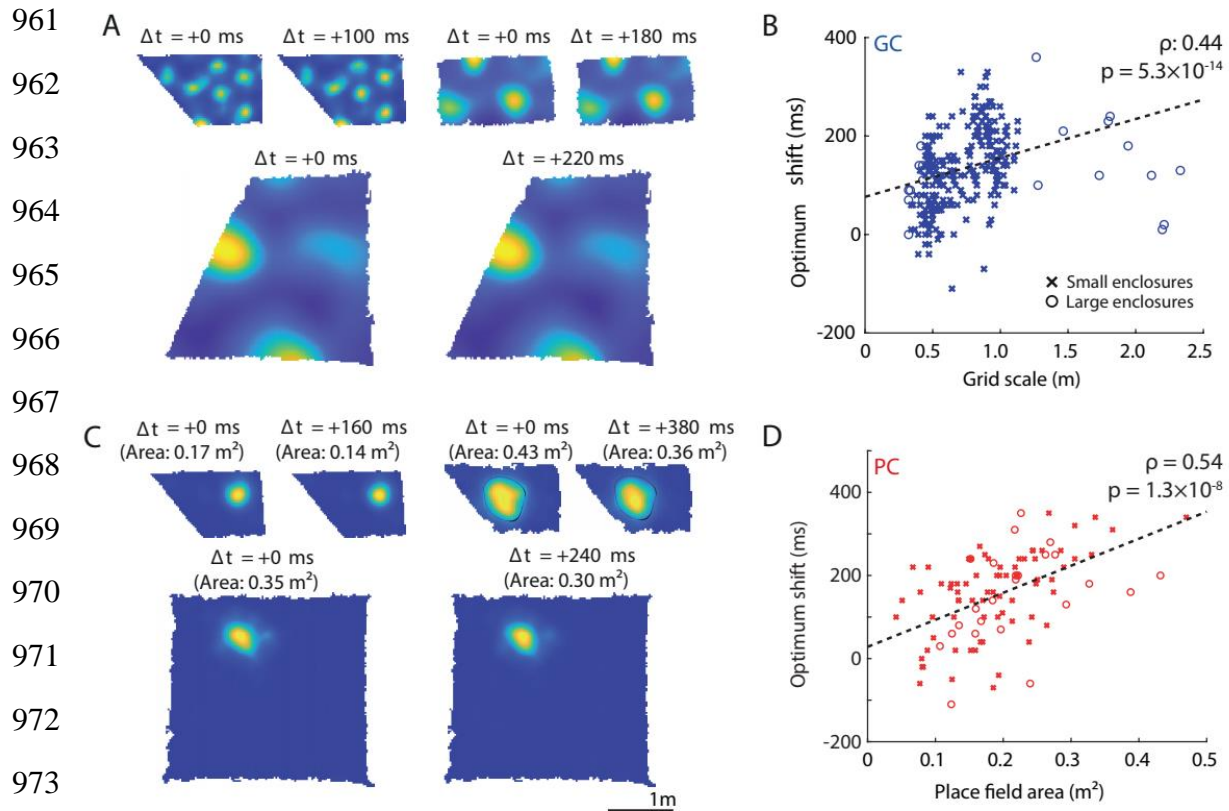
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952 **Figure 3. Nonlinear relationships between time shifts and behaviour** for place (red, top) and grid
953 cells (blue, bottom). (A) Positive accelerations correlate with more prospective time shifts (Δt). Plots
954 show the estimated linear rank regression of time shifts given acceleration after inverting the rank
955 transform back to physical units (shaded: 95% confidence of regressed fit; points: mean Δt within each
956 decile; lines: 95% confidence). (B) The largest positive and negative time-shifts are associated with
957 slower running speeds (consistent across subjects, see Fig. S7; Points: median within histogram bins;
958 Bars: 95% confidence in the median; Shaded: inter-quartile range within each bin). Faster running
959 speeds are associated with more typical time shifts (c.f. Fig 1C). (C) The relationship between time
960 shifts and the angular speed of head directions was similar to that of forward speed, albeit noisier.



974 **Figure 4. Time shifts in grid and place cells are correlated with firing field size.** (A) Three example
975 grid cells with different scales (upper left: 0.45m, upper right: 0.92 m, lower: 2.10m, in the large
976 trapezoidal enclosure described in Methods), before and after time shifting (Δt : applied time shift). Note
977 that these examples, while representative of optimum time shifts at those scales, result in very small
978 visual changes to each firing rate map. (B) There is a significant positive correlation between grid scale
979 and optimum time shift. (C) Three example place cells with different firing field sizes, before (left) and
980 after time shifting (right). Top right field is outlined with a blank line to facilitate visualisation of the
981 decrease in the area with time shifts. (D) There is a significant positive correlation between place cell
982 firing area and optimum time shift. Linear regression was used to draw a line of best fit (dashed black).
983 The same scale bar is used for all the rate maps.

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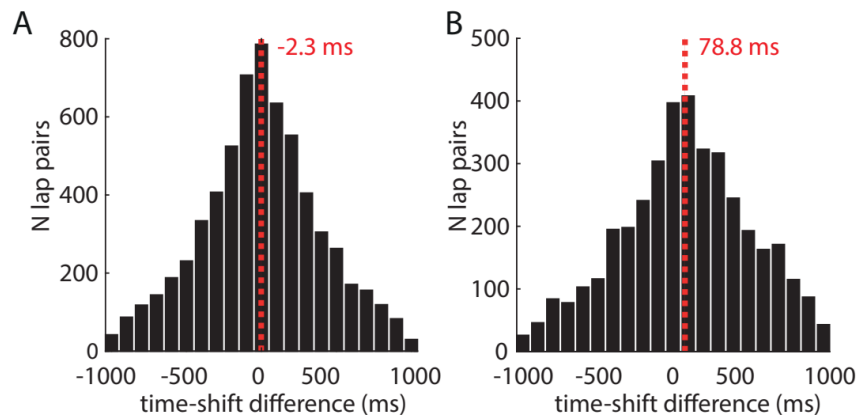
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992 **Figure 5. Distinct grid modules provide a nearly simultaneous readout of gradually increasing**

993 **time shifts. (A)** Histogram showing the distributions of time shift differences between all grid cells

994 from the same modules. **(B)** Histogram showing the distributions of time shift differences between all

995 grid cells from different modules. Dashed red lines show average time shift differences.

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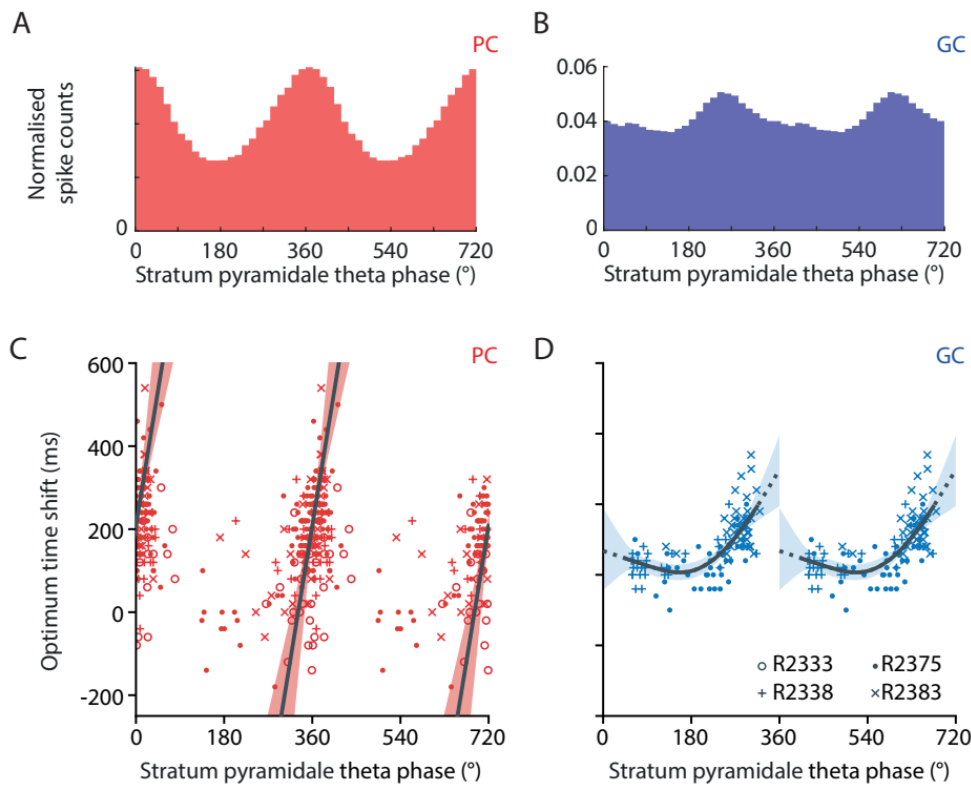
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1009 **Figure 6. Time shifts in place and grid cells are correlated with their preferred CA1 pyramidal**
1010 **theta phase.** Histogram showing normalised spike counts of place cells (A) and grid cells (B) with
1011 respect to stratum pyramidal theta phase. Scatter plot showing each place cell (C) and grid cell (D)
1012 recording's preferred theta phase against its optimum time shift. PC; place cell; GC: grid cell Grid cells
1013 showed a nonlinear relationship saturating at lower preferred theta phases. Trend lines (black) reflect a
1014 circular-linear regression for place cells (C) and a cubic-spline regression for grid cells (D; four basis
1015 functions spaced equally over 0–360°). Shaded regions show the 2.5–97.5th percentile confidence
1016 intervals (bootstrap; 1000 samples). Only cells showing significant theta-phase coupling are included.

1017