

Modeling the dynamics of mouse iron body distribution: hepcidin is necessary but not sufficient

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Abstract

This document presents the full mathematical model as calculated by the COPASI software. The model is created automatically by the COPASI software based on the model's reaction stoichiometry and kinetic rate laws of each reaction. Included are also algebraic equations used to calculate several quantities of interest. \mathcal{N} represents the Avogadro number; Injected represents the total injected radioactive iron in particle numbers. The subscript PN indicates that the variable is the number of particles rather than concentration (e.g. Tf_{PN} for the number of particles of Tf and [Tf] for its concentration).

Full mathematical model

$$\begin{aligned}
\frac{d([FeDuo^*] \cdot V_{Duodenum})}{dt} &= - \left(\frac{VDuoNTBI \cdot V_{Duodenum} \cdot [FeDuo^*]}{(Km + [FeDuo]^* + [FeDuo]) \cdot \left(1 + \frac{[Hepcidin]}{Ki}\right)} \right) \\
&\quad + 2 \cdot (kInDuo \cdot [Fe2Tf^{**}] \cdot V_{Plasma}) \\
&\quad + (kInDuo \cdot [Fe1Tf^{**}] \cdot V_{Plasma}) \\
&\quad + (kInDuo \cdot [Fe2Tf^{**}] \cdot V_{Plasma}) \\
&\quad - (kDuoLoss \cdot [FeDuo^*] \cdot V_{Duodenum}) \\
\frac{d([FeDuo] \cdot V_{Duodenum})}{dt} &= + V_{Duodenum} \cdot (vDiet) \\
&\quad - \left(\frac{VDuoNTBI \cdot V_{Duodenum} \cdot [FeDuo]}{(Km + [FeDuo] + [FeDuo^*]) \cdot \left(1 + \frac{[Hepcidin]}{Ki}\right)} \right) \\
&\quad - (kDuoLoss \cdot [FeDuo] \cdot V_{Duodenum}) \\
&\quad + 2 \cdot (kInDuo \cdot [Fe2Tf] \cdot V_{Plasma}) \\
&\quad + (kInDuo \cdot [Fe1Tf] \cdot V_{Plasma}) \\
&\quad + (kInDuo \cdot [Fe2Tf^{**}] \cdot V_{Plasma}) \\
\frac{d([FeBM^*] \cdot V_{BoneMarrow})}{dt} &= + 2 \cdot (kInBM \cdot [Fe2Tf^{**}] \cdot V_{Plasma}) \\
&\quad + (kInBM \cdot [Fe1Tf^{**}] \cdot V_{Plasma}) \\
&\quad + (kInBM \cdot [Fe2Tf^{**}] \cdot V_{Plasma}) \\
&\quad - (kInRBC \cdot [FeBM^*] \cdot V_{BoneMarrow}) \\
&\quad - (kBMSpleen \cdot [FeBM^*] \cdot V_{BoneMarrow}) \\
\frac{d([FeBM] \cdot V_{BoneMarrow})}{dt} &= + 2 \cdot (kInBM \cdot [Fe2Tf] \cdot V_{Plasma}) \\
&\quad + (kInBM \cdot [Fe1Tf] \cdot V_{Plasma}) \\
&\quad + (kInBM \cdot [Fe2Tf^{**}] \cdot V_{Plasma}) \\
&\quad - (kInRBC \cdot [FeBM] \cdot V_{BoneMarrow}) \\
&\quad - (kBMSpleen \cdot [FeBM] \cdot V_{BoneMarrow}) \\
\frac{d([FeRBC^*] \cdot V_{RBC})}{dt} &= - (vRBCSpleen \cdot [FeRBC^*] \cdot V_{RBC}) \\
&\quad + (kInRBC \cdot [FeBM^*] \cdot V_{BoneMarrow}) \\
\frac{d([FeRBC] \cdot V_{RBC})}{dt} &= - (vRBCSpleen \cdot [FeRBC] \cdot V_{RBC}) \\
&\quad + (kInRBC \cdot [FeBM] \cdot V_{BoneMarrow})
\end{aligned}$$

$$\frac{d([FeSpleen^*] \cdot V_{Spleen})}{dt} = + (vRBCSpleen \cdot [FeRBC^*] \cdot V_{RBC})$$

$$- \left(\frac{VSpleenNTBI \cdot V_{Spleen} \cdot [FeSpleen^*]}{(Km + [FeSpleen^*] + [FeSpleen]) \cdot \left(1 + \frac{[Hepcidin]}{Ki}\right)} \right)$$

$$+ (kBMSpleen \cdot [FeBM^*] \cdot V_{BoneMarrow})$$

$$\frac{d([FeSpleen] \cdot V_{Spleen})}{dt} = + (vRBCSpleen \cdot [FeRBC] \cdot V_{RBC})$$

$$- \left(\frac{VSpleenNTBI \cdot V_{Spleen} \cdot [FeSpleen]}{(Km + [FeSpleen] + [FeSpleen^*]) \cdot \left(1 + \frac{[Hepcidin]}{Ki}\right)} \right)$$

$$+ (kBMSpleen \cdot [FeBM] \cdot V_{BoneMarrow})$$

$$\frac{d([FeLiver^*] \cdot V_{Liver})}{dt} = + (kInLiver \cdot [Fe1Tf^{**}] \cdot V_{Plasma})$$

$$- \left(\frac{VLiverNTBI \cdot V_{Liver} \cdot [FeLiver^*]}{(Km + [FeLiver^*] + [FeLiver]) \cdot \left(1 + \frac{[Hepcidin]}{Ki}\right)} \right)$$

$$+ (kInLiver \cdot [Fe2Tf^{**}] \cdot V_{Plasma})$$

$$+ 2 \cdot (kInLiver \cdot [Fe2Tf^{**}] \cdot V_{Plasma})$$

$$\frac{d([FeLiver] \cdot V_{Liver})}{dt} = - \left(\frac{VLiverNTBI \cdot V_{Liver} \cdot [FeLiver]}{(Km + [FeLiver] + [FeLiver^*]) \cdot \left(1 + \frac{[Hepcidin]}{Ki}\right)} \right)$$

$$+ 2 \cdot (kInLiver \cdot [Fe2Tf] \cdot V_{Plasma})$$

$$+ (kInLiver \cdot [Fe1Tf] \cdot V_{Plasma})$$

$$+ (kInLiver \cdot [Fe2Tf^{**}] \cdot V_{Plasma})$$

$$\frac{d([FeRest^*] \cdot V_{RestOfBody})}{dt} = - \left(\frac{VRestNTBI \cdot V_{RestOfBody} \cdot [FeRest^*]}{(Km + [FeRest^*] + [FeRest]) \cdot \left(1 + \frac{[Hepcidin]}{Ki}\right)} \right)$$

$$+ (kInRest \cdot [Fe1Tf^{**}] \cdot V_{Plasma})$$

$$+ (kInRest \cdot [Fe2Tf^{**}] \cdot V_{Plasma})$$

$$+ 2 \cdot (kInRest \cdot [Fe2Tf^{**}] \cdot V_{Plasma})$$

$$- V_{RestOfBody} \cdot (kRestLoss \cdot [FeRest^*])$$

$$\frac{d([FeRest] \cdot V_{RestOfBody})}{dt} = + 2 \cdot (kInRest \cdot [Fe2Tf] \cdot V_{Plasma})$$

$$- V_{RestOfBody} \cdot (kRestLoss \cdot [FeRest])$$

$$- \left(\frac{VRestNTBI \cdot V_{RestOfBody} \cdot [FeRest]}{(Km + [FeRest] + [FeRest^*]) \cdot \left(1 + \frac{[Hepcidin]}{Ki}\right)} \right)$$

$$+ (kInRest \cdot [Fe1Tf] \cdot V_{Plasma})$$

$$+ (kInRest \cdot [Fe2Tf^{**}] \cdot V_{Plasma})$$

$$\begin{aligned}
\frac{d([FeOutside^*] \cdot V_{RestOfBody})}{dt} &= + (kDuoLoss \cdot [FeDuo^*] \cdot V_{Duodenum}) \\
&\quad + V_{RestOfBody} \cdot (kRestLoss \cdot [FeRest^*]) \\
\frac{d([Hepcidin] \cdot V_{Plasma})}{dt} &= + V_{Plasma} \cdot vHepcidinSynthesis \\
&\quad - V_{Plasma} \cdot (kHepcidinDecay \cdot [Hepcidin]) \\
\frac{d([NTBI^*] \cdot V_{Plasma})}{dt} &= + \left(\frac{VDuoNTBI \cdot V_{Duodenum} \cdot [FeDuo^*]}{(Km + [FeDuo^*] + [FeDuo]) \cdot \left(1 + \frac{[Hepcidin]}{Ki}\right)} \right) \\
&\quad + \left(\frac{VRestNTBI \cdot V_{RestOfBody} \cdot [FeRest^*]}{(Km + [FeRest^*] + [FeRest]) \cdot \left(1 + \frac{[Hepcidin]}{Ki}\right)} \right) \\
&\quad - V_{Plasma} \cdot (kFe1Tf_Fe2Tf \cdot [Fe1Tf**] \cdot [NTBI^*]) \\
&\quad + \left(\frac{VSpleenNTBI \cdot V_{Spleen} \cdot [FeSpleen^*]}{(Km + [FeSpleen^*] + [FeSpleen]) \cdot \left(1 + \frac{[Hepcidin]}{Ki}\right)} \right) \\
&\quad - V_{Plasma} \cdot (kFe1Tf_Fe2Tf \cdot [Fe1Tf] \cdot [NTBI^*]) \\
&\quad + \left(\frac{VLiverNTBI \cdot V_{Liver} \cdot [FeLiver^*]}{(Km + [FeLiver^*] + [FeLiver]) \cdot \left(1 + \frac{[Hepcidin]}{Ki}\right)} \right) \\
&\quad - V_{Plasma} \cdot (kNTBI_Fe1Tf \cdot [NTBI^*] \cdot [Tf]) \\
\frac{d([NTBI] \cdot V_{Plasma})}{dt} &= + \left(\frac{VDuoNTBI \cdot V_{Duodenum} \cdot [FeDuo]}{(Km + [FeDuo] + [FeDuo^*]) \cdot \left(1 + \frac{[Hepcidin]}{Ki}\right)} \right) \\
&\quad + \left(\frac{VSpleenNTBI \cdot V_{Spleen} \cdot [FeSpleen]}{(Km + [FeSpleen] + [FeSpleen^*]) \cdot \left(1 + \frac{[Hepcidin]}{Ki}\right)} \right) \\
&\quad + \left(\frac{VLiverNTBI \cdot V_{Liver} \cdot [FeLiver]}{(Km + [FeLiver] + [FeLiver^*]) \cdot \left(1 + \frac{[Hepcidin]}{Ki}\right)} \right) \\
&\quad - V_{Plasma} \cdot (kNTBI_Fe1Tf \cdot [NTBI] \cdot [Tf]) \\
&\quad + \left(\frac{VRestNTBI \cdot V_{RestOfBody} \cdot [FeRest]}{(Km + [FeRest] + [FeRest^*]) \cdot \left(1 + \frac{[Hepcidin]}{Ki}\right)} \right) \\
&\quad - V_{Plasma} \cdot (kFe1Tf_Fe2Tf \cdot [Fe1Tf] \cdot [NTBI]) \\
&\quad - V_{Plasma} \cdot (kFe1Tf_Fe2Tf \cdot [Fe1Tf**] \cdot [NTBI]) \\
\frac{d([Fe2Tf**] \cdot V_{Plasma})}{dt} &= - (kInBM \cdot [Fe2Tf**] \cdot V_{Plasma}) \\
&\quad - (kInDuo \cdot [Fe2Tf**] \cdot V_{Plasma}) \\
&\quad + V_{Plasma} \cdot (kFe1Tf_Fe2Tf \cdot [Fe1Tf**] \cdot [NTBI^*]) \\
&\quad - (kInLiver \cdot [Fe2Tf**] \cdot V_{Plasma}) \\
&\quad - (kInRest \cdot [Fe2Tf**] \cdot V_{Plasma})
\end{aligned}$$

$$\frac{d([Fe2Tf^{**}] \cdot V_{Plasma})}{dt} = + V_{Plasma} \cdot (k_{Fe1Tf_Fe2Tf} \cdot [Fe1Tf] \cdot [NTBI^*])$$

$$+ V_{Plasma} \cdot (k_{Fe1Tf_Fe2Tf} \cdot [Fe1Tf^{**}] \cdot [NTBI])$$

$$- (k_{InDuo} \cdot [Fe2Tf^{**}] \cdot V_{Plasma})$$

$$- (k_{InLiver} \cdot [Fe2Tf^{**}] \cdot V_{Plasma})$$

$$- (k_{InBM} \cdot [Fe2Tf^{**}] \cdot V_{Plasma})$$

$$- (k_{InRest} \cdot [Fe2Tf^{**}] \cdot V_{Plasma})$$

$$\frac{d([Fe2Tf] \cdot V_{Plasma})}{dt} = - (k_{InBM} \cdot [Fe2Tf] \cdot V_{Plasma})$$

$$- (k_{InLiver} \cdot [Fe2Tf] \cdot V_{Plasma})$$

$$- (k_{InRest} \cdot [Fe2Tf] \cdot V_{Plasma})$$

$$- (k_{InDuo} \cdot [Fe2Tf] \cdot V_{Plasma})$$

$$+ V_{Plasma} \cdot (k_{Fe1Tf_Fe2Tf} \cdot [Fe1Tf] \cdot [NTBI])$$

$$\frac{d([Fe1Tf^{**}] \cdot V_{Plasma})}{dt} = - V_{Plasma} \cdot (k_{Fe1Tf_Fe2Tf} \cdot [Fe1Tf^{**}] \cdot [NTBI^*])$$

$$- (k_{InLiver} \cdot [Fe1Tf^{**}] \cdot V_{Plasma})$$

$$- (k_{InBM} \cdot [Fe1Tf^{**}] \cdot V_{Plasma})$$

$$- (k_{InRest} \cdot [Fe1Tf^{**}] \cdot V_{Plasma})$$

$$- (k_{InDuo} \cdot [Fe1Tf^{**}] \cdot V_{Plasma})$$

$$- V_{Plasma} \cdot (k_{Fe1Tf_Fe2Tf} \cdot [Fe1Tf^{**}] \cdot [NTBI])$$

$$+ V_{Plasma} \cdot (k_{NTBI_Fe1Tf} \cdot [NTBI^*] \cdot [Tf])$$

$$\frac{d([Fe1Tf] \cdot V_{Plasma})}{dt} = + V_{Plasma} \cdot (k_{NTBI_Fe1Tf} \cdot [NTBI] \cdot [Tf])$$

$$- V_{Plasma} \cdot (k_{Fe1Tf_Fe2Tf} \cdot [Fe1Tf] \cdot [NTBI])$$

$$- (k_{InLiver} \cdot [Fe1Tf] \cdot V_{Plasma})$$

$$- (k_{InBM} \cdot [Fe1Tf] \cdot V_{Plasma})$$

$$- (k_{InRest} \cdot [Fe1Tf] \cdot V_{Plasma})$$

$$- (k_{InDuo} \cdot [Fe1Tf] \cdot V_{Plasma})$$

$$- V_{Plasma} \cdot (k_{Fe1Tf_Fe2Tf} \cdot [Fe1Tf] \cdot [NTBI^*])$$

$$\begin{aligned}
\frac{d([Tf] \cdot V_{\text{Plasma}})}{dt} = & + (\text{kInBM} \cdot [\text{Fe2Tf}^{**}] \cdot V_{\text{Plasma}}) \\
& + (\text{kInDuo} \cdot [\text{Fe2Tf}^{**}] \cdot V_{\text{Plasma}}) \\
& + (\text{kInLiver} \cdot [\text{Fe1Tf}^{**}] \cdot V_{\text{Plasma}}) \\
& + (\text{kInBM} \cdot [\text{Fe1Tf}^{**}] \cdot V_{\text{Plasma}}) \\
& + (\text{kInRest} \cdot [\text{Fe1Tf}^{**}] \cdot V_{\text{Plasma}}) \\
& + (\text{kInDuo} \cdot [\text{Fe1Tf}^{**}] \cdot V_{\text{Plasma}}) \\
& + (\text{kInBM} \cdot [\text{Fe2Tf}] \cdot V_{\text{Plasma}}) \\
& - V_{\text{Plasma}} \cdot (\text{kNTBI_Fe1Tf} \cdot [\text{NTBI}] \cdot [\text{Tf}]) \\
& + (\text{kInLiver} \cdot [\text{Fe2Tf}] \cdot V_{\text{Plasma}}) \\
& + (\text{kInRest} \cdot [\text{Fe2Tf}] \cdot V_{\text{Plasma}}) \\
& + (\text{kInDuo} \cdot [\text{Fe2Tf}] \cdot V_{\text{Plasma}}) \\
& + (\text{kInLiver} \cdot [\text{Fe1Tf}] \cdot V_{\text{Plasma}}) \\
& + (\text{kInBM} \cdot [\text{Fe1Tf}] \cdot V_{\text{Plasma}}) \\
& + (\text{kInRest} \cdot [\text{Fe1Tf}] \cdot V_{\text{Plasma}}) \\
& + (\text{kInDuo} \cdot [\text{Fe1Tf}] \cdot V_{\text{Plasma}}) \\
& + (\text{kInLiver} \cdot [\text{Fe2Tf}^{**}] \cdot V_{\text{Plasma}}) \\
& + (\text{kInBM} \cdot [\text{Fe2Tf}^{**}] \cdot V_{\text{Plasma}}) \\
& + (\text{kInRest} \cdot [\text{Fe2Tf}^{**}] \cdot V_{\text{Plasma}}) \\
& - V_{\text{Plasma}} \cdot (\text{kNTBI_Fe1Tf} \cdot [\text{NTBI}^*] \cdot [\text{Tf}]) \\
& + (\text{kInLiver} \cdot [\text{Fe2Tf}^{**}] \cdot V_{\text{Plasma}}) \\
& + (\text{kInRest} \cdot [\text{Fe2Tf}^{**}] \cdot V_{\text{Plasma}})
\end{aligned}$$

$$\begin{aligned}
\text{Total_Fe}^* &= \text{FeDuo}_{\text{PN}}^* + \text{FeLiver}_{\text{PN}}^* + \text{FeSpleen}_{\text{PN}}^* + \text{FeRBC}_{\text{PN}}^* + \text{FeRest}_{\text{PN}}^* + \text{Fe2Tf}_{\text{PN}}^* + \text{NTBI}_{\text{PN}}^* \\
&\quad + 2 \cdot \text{Fe2Tf}_{\text{PN}}^{**} + \text{Fe1Tf}_{\text{PN}}^* + \text{FeBM}_{\text{PN}}^* \\
\text{Total_Fe} &= \text{FeDuo}_{\text{PN}} + \text{FeLiver}_{\text{PN}} + \text{FeSpleen}_{\text{PN}} + \text{FeRBC}_{\text{PN}} + \text{FeRest}_{\text{PN}} + \text{Fe2Tf}_{\text{PN}}^* + \text{NTBI}_{\text{PN}} \\
&\quad + 2 \cdot \text{Fe2Tf}_{\text{PN}} + \text{Fe1Tf}_{\text{PN}} + \text{FeBM}_{\text{PN}} \\
\text{Total_Fe (conc.)} &= \frac{\text{FeDuo}_{\text{PN}} + \text{FeLiver}_{\text{PN}} + \text{FeSpleen}_{\text{PN}} + \text{FeRBC}_{\text{PN}} + \text{FeRest}_{\text{PN}} + 2 \cdot \text{Fe2Tf}_{\text{PN}} + \text{Fe2Tf}_{\text{PN}}^* + \text{Fe1Tf}_{\text{PN}} + \text{NTBI}_{\text{PN}} + \text{FeBM}_{\text{PN}}}{N \cdot (V_{\text{Duodenum}} + V_{\text{Liver}} + V_{\text{Plasma}} + V_{\text{RBC}} + V_{\text{RestOfBody}} + V_{\text{Spleen}})} \\
\text{Total_Fe (g)} &= \frac{\text{Total_Fe} \cdot 55.845}{N} \\
\text{FePlasma} &= 2 \cdot \text{Fe2Tf}_{\text{PN}} + \text{Fe2Tf}_{\text{PN}}^* + \text{Fe1Tf}_{\text{PN}} + \text{NTBI}_{\text{PN}} \\
\text{FePlasma (conc.)} &= \frac{\text{FePlasma}}{V_{\text{Plasma}} \cdot N} \\
\text{FePlasma}^* &= 2 \cdot \text{Fe2Tf}_{\text{PN}}^{**} + \text{Fe2Tf}_{\text{PN}}^* + \text{Fe1Tf}_{\text{PN}}^* + \text{NTBI}_{\text{PN}}^* \\
\text{FePlasma}^* (\text{conc.}) &= \frac{\text{FePlasma}^*}{V_{\text{Plasma}} \cdot N} \\
\text{Total_Tf} &= \text{Tf}_{\text{PN}} + \text{Fe1Tf}_{\text{PN}}^* + \text{Fe1Tf}_{\text{PN}} + \text{Fe2Tf}_{\text{PN}}^{**} + \text{Fe2Tf}_{\text{PN}}^* + \text{Fe2Tf}_{\text{PN}} \\
\text{Total_Tf (conc.)} &= \frac{\text{Total_Tf}}{N \cdot V_{\text{Plasma}}} \\
\text{Total_Tf mg/ml} &= \text{Total_Tf (conc.)} \cdot 80 \\
\text{TfSaturation} &= \frac{100 \cdot (2 \cdot \text{Fe2Tf}_{\text{PN}}^* + 2 \cdot \text{Fe2Tf}_{\text{PN}} + \text{Fe1Tf}_{\text{PN}}^* + \text{Fe1Tf}_{\text{PN}} + 2 \cdot \text{Fe2Tf}_{\text{PN}}^{**})}{2 \cdot (\text{Fe2Tf}_{\text{PN}}^* + \text{Fe1Tf}_{\text{PN}}^* + \text{Tf}_{\text{PN}} + \text{Fe2Tf}_{\text{PN}}^{**} + \text{Fe2Tf}_{\text{PN}} + \text{Fe1Tf}_{\text{PN}})} \\
\text{PDuo} &= \frac{100 \cdot \text{FeDuo}_{\text{PN}}^*}{\text{Injected}} \\
\text{PLiver} &= \frac{100 \cdot \text{FeLiver}_{\text{PN}}^*}{\text{Injected}} \\
\text{PSpleen} &= \frac{100 \cdot \text{FeSpleen}_{\text{PN}}^*}{\text{Injected}} \\
\text{PRBC} &= \frac{100 \cdot \text{FeRBC}_{\text{PN}}^*}{\text{Injected}} \\
\text{PRest} &= \frac{100 \cdot \text{FeRest}_{\text{PN}}^*}{\text{Injected}} \\
\text{PPlasma} &= \frac{100 \cdot (\text{NTBI}_{\text{PN}}^* + 2 \cdot \text{Fe2Tf}_{\text{PN}}^{**} + \text{Fe2Tf}_{\text{PN}}^* + \text{Fe1Tf}_{\text{PN}}^*)}{\text{Injected}} \\
\text{POutside} &= \frac{100 \cdot \text{FeOutside}_{\text{PN}}^*}{\text{Injected}} \\
\text{PBM} &= \frac{100 \cdot \text{FeBM}_{\text{PN}}^*}{\text{Injected}} \\
\text{Injected} &= 3.073 \cdot 10^{15}
\end{aligned}$$