Supporting Information

S1 Appendix. Derivation of average acceptance rate.

Theorem 1 *Let* $\{\mathbf{X}_t\} : \Omega \to \mathbb{N}^\kappa$ *be a DSCT Markov process, let* $\mathbf{X}_d$ *be observational data of a single realization,* $\{\mathbf{X}_t(\bar{\omega})\}$ *for some fixed* $\bar{\omega} \in \Omega$*, at discrete times* $t_1, t_2, \ldots, t_{N_t}$*, that is,* $\mathbf{X}_d \in \left[\mathbf{X}_{t_1}(\bar{\omega}), \mathbf{X}_{t_2}(\bar{\omega}), \ldots, \mathbf{X}_{t_{N_t}}(\bar{\omega})\right]$ *and let* A_ϵ *denote the acceptance rate of ABC rejection with acceptance threshold* ϵ *and the metric* ρ *given by*

$$\rho(\{\mathbf{X}_t(\omega)\}, \mathbf{X}_d) = \sum_{i=0}^{N_t} |\mathbf{X}_{t_i}(\omega) - \mathbf{X}_{t_i}(\bar{\omega}))|_2^2 \,. \tag{1}$$

Then, for fixed N_t*,* $A_\epsilon = O(\epsilon^\kappa)$*.*

Proof: The average acceptance rate is the probability of a simulation being accepted over all of parameter space. That is,

$$A_\epsilon = \int_\Omega \int_{\mathbb{R}^M} p\left(\{\mathbf{X}_t(\omega)\}, \mathbf{X}_d\right) < \epsilon \,|\, \theta)\, p\left(\theta\right) \mathrm{d}\theta \mathrm{d}\omega. \tag{2}$$

We define a κN_t-dimensional ball of radius ϵ centered on $\mathbf{X}_d$ as

$$B(\epsilon, \mathbf{X}_d) = \{\omega \in \Omega : \rho(\{\mathbf{X}_t(\omega)\}, \mathbf{X}_d) < \epsilon\} \,.$$

Since $\int_\Omega p\left(\rho(\{\mathbf{X}_t(\omega)\}, \mathbf{X}_d) < \epsilon\right) \mathrm{d}\omega = \int_{B(\epsilon,\mathbf{X}_d)} p\left(\{\mathbf{X}_t(\omega)\}\right) \mathrm{d}\omega$, we simplify Equation (2) to give

$$A_\epsilon = \int_{B(\epsilon,\mathbf{X}_d)} \int_{\mathbb{R}^M} p\left(\{\mathbf{X}_t(\omega)\} \,|\, \theta\right) p\left(\theta\right) \mathrm{d}\theta \mathrm{d}\omega = \int_{B(\epsilon,\mathbf{X}_d)} p\left(\{\mathbf{X}_t(\omega)\}\right) \mathrm{d}\omega. \tag{3}$$

Since $[\mathbb{N}^\kappa]^{N_t}$ is countable, then $p\left(\{\mathbf{X}_t(\omega)\}\right)$ is bounded over $B(\epsilon, \mathbf{X}_d)$. Let $c = \sup_{\omega \in B(\epsilon,\mathbf{X}_d)} |p\left(\{\mathbf{X}_t(\omega)\}\right)|$. It follows that

$$A_\epsilon \leq cV, \tag{4}$$

where V is the volume of the ball $B(\epsilon, \mathbf{X}_d)$. Under the definition of ρ, we have $V = O(\epsilon^{\kappa N_t})$. Therefore, for fixed N_t, $A_\epsilon = O(\epsilon^\kappa)$. □