Supporting Information

S3 Appendix. Smoothing and extension.

There is one problem that can arise from the fact that the indicator functional is discontinuous [1]. The effect is particularly clear when the multilevel approach is applied to compute a univariate posterior with a very steep region in the CDF. The result is the multilevel estimator around this steep region becomes unstable. This effect can be negated through usage of a Lipschitz continuous approximation to the indicator functional. For this work we have replaced $\mathbb{1}_{A_{\mathbf{s}}}$ with a smooth functional $g_{\mathbf{s}}$ defined by,

$$g_{\mathbf{s}}(\theta) = \prod_{i=1}^{\kappa} f\left(\frac{s_i - \theta_i}{\delta_i}\right), \quad f(y) = \begin{cases} 1 & y \leq -1 \\ \frac{5}{8}y^3 - \frac{9}{8}y + \frac{1}{2} & -1 < y < 1. \\ 0 & y \geq 1 \end{cases}$$

where δ_i is the width of the region the indicator is being smoothed over in the ith dimension.

The multilevel estimator does not enforce the condition of a true CDF, that it must be a monotonically increasing function. If $P(S, \mu)$ is extension of the estimator to $\mathbb{R}^{\kappa}$, then a monotonically increasing extension is

$$\bar{F}(\mathbf{s}) = \frac{1}{2}\left(\sup_{\theta \in A_{\mathbf{s}}} P(S, \mu)(\theta) + \inf_{\theta \notin A_{\mathbf{s}}} P(S, \mu)(\theta)\right).$$

We note, without proof, that this construction cannot increase the RMSE error in terms of the L^{∞}-norm.

References

1. Giles MB, Nagapetyan T, Ritter K. Multilevel Monte Carlo approximation of cumulative distribution function and probability densities. SIAM/ASA Journal on Uncertainty Quantification. 2015;3:267–295. doi:10.1137/140960086.