Supplemental material: bursting on a two states stochastic model for gene transcription in *Drosophila* embryos

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We present the demonstration of the negative binomial limit for the probability distribution of ϕ_n

$$\phi_n = \frac{N^n}{n!}\frac{(A\epsilon)_n}{(\epsilon)_n}\mathrm{M}(A\epsilon+n,\epsilon+n,-N). \tag{1}$$

such that

$$\phi_n \to \tilde{\phi}_n = \frac{(A\epsilon)_n}{n!}\left(\frac{\delta}{1+\delta}\right)^n\left(\frac{1}{1+\delta}\right)^{A\epsilon}. \tag{2}$$

We demonstrate this by recalling that

$$M(a,b,N(z-1)) = \frac{1}{\Gamma(a)}\int_0^\infty \mathrm{e}^{-t}t^{a-1}\mathrm{M}(0,b,N(z-1)t)dt,$$

introducing the rescaling

$$N \propto b\delta, \tag{3}$$

and recalling the fact that, as $b\to\infty$,

$$\mathrm{M}(0,b,bz)\to \mathrm{e}^z.$$

By convergence under the integration we have

$$\begin{aligned}\mathrm{M}(a,b,N(z-1)) &\sim \frac{1}{\Gamma(a)}\int_0^\infty \mathrm{e}^{-t}t^{a-1}\mathrm{e}^{\delta(z-1)t}dt\\ &\sim \frac{1}{\Gamma(a)}\int_0^\infty \mathrm{e}^{-(1+\delta(1-z))t}t^{a-1}dt.\end{aligned}$$

Making the variable change $u=(1+\delta(1-z))t$ gives

$$\frac{1}{\Gamma(a)}\int_0^\infty \mathrm{e}^{-(1+\delta(1-z))t}t^{a-1}dt = (1+\delta(1-z))^{-a},$$

so that the generating function $\phi(z)$ for the probabilities ϕ_n can be written.

$$\phi(z)\sim\left(\frac{1}{1+\delta(1-z)}\right)^a. \tag{4}$$

where $a = A\epsilon$ and the negative binomial distribution governs mRNA numbers in transcriptional bursts.

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