

# Supplementary material

to "The mechanics of predator-prey interactions: first principles of physics predict predator-prey size ratios"

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## Supplementary methods

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## Supplementary references

# Supplementary methods

## 1 Drag Coefficient

For the calculation of motion, drag coefficient ( $C_d$ ) is calculated using an empirical rule (Turton & Levenspiel, 1986) that offers a good approximation for very low as well as very high Reynolds numbers (see Figure S1).

$$C_d = \left[ 0.352 + \left( 0.124 + \frac{24}{Re} \right)^{0.5} \right]^2 \quad (\text{eqn S.1})$$

where  $Re$  is the Reynolds number, which is calculated as the following:

$$Re = \frac{\rho_m v L_b}{\mu} \quad (\text{eqn S.2})$$

where  $v$  is speed,  $L_b$  is body length,  $\rho_m$  is medium density, and  $\mu$  is medium dynamic viscosity.

## 2 Framework for the calculation of speed and work

Although animal motion is diverse, it is possible to define a common pattern. Animal motion can be represented as an oscillatory movement (Bejan & Marden, 2006), a pattern observed in swimming, running or flying animals. Thus, following this idea, we define a general framework for species motion.

Considering one oscillation, motion can be decomposed into a vertical and a horizontal component (Figure 2, main text). Both are essential. The horizontal component represents the translational motion (i.e., the distance traveled between two points). However, this horizontal motion is impossible without a vertical motion that either lifts the body or the surrounding medium, allowing the

horizontal movement (Bejan & Marden, 2006). The muscular output creates a force that is split between these two components.

### Vertical component

Vertical motion sequence during one oscillation includes three phases. The first phase is the active phase: a vertical muscular force ( $F_{Mv}$ ) is applied during the stroke period  $t_{force}$ . The body is lifted by this muscular force and Archimedes' force (due to medium density), but gravity and drag work in the opposite direction. The overall vertical force ( $F_v$ ) writes as

$$F_v = F_{Mv} + g\rho_m V_b - gM_b - \frac{1}{2}vS_b\rho_m C_d \quad (\text{eqn S.3})$$

where  $v$  is instantaneous speed,  $S_b$  is the cross-section surface of the body, and  $C_d$  is the drag coefficient. According to Newton's second law, acceleration is equal to force divided by mass. It is also known that acceleration is the first derivative of speed with respect to time. Thus, instantaneous speed can be derived from equation S.3:

$$\frac{dv}{dt} = \frac{F_{Mv}}{M_b} + \frac{g\rho_m V_b}{M_b} - g - \frac{1}{2} \frac{vS_b\rho_m C_d}{M_b} \quad (\text{eqn S.4})$$

During the second phase, the muscular force stops (at time  $t_{force}$ ), and the body pursues its lift by inertia until it stops (at time  $t_2$ ). It is the inertial ascending phase.

$$\frac{dv}{dt} = \frac{g\rho_m V_b}{M_b} - g - \frac{1}{2} \frac{vS_b\rho_m C_d}{M_b} \quad (\text{eqn S.5})$$

Third, the body falls (or sinks) passively back to its original vertical position (from time  $t_2$  to  $t_3$ ).

During this inertial descending phase, motion is favored by gravity, but Archimedes' force and drag work in opposition.

$$\frac{dv}{dt} = -\frac{g\rho_m V_b}{M_b} + g - \frac{1}{2} \frac{v S_b \rho_m C_d}{M_b} \quad (\text{eqn S.6})$$

At the end of one oscillation the body will end up at its original vertical position so that the animal stays at the same altitude or depth. The vertical component of the force thus sets the duration of the oscillation.

### **Horizontal component**

The horizontal component includes two phases. The first phase is the active phase during which an horizontal muscular force ( $F_{Mh}$ ) allows a displacement of the body. This force is applied during the stroke period  $t_{force}$  (same as vertical component). For the horizontal component, since we do not pay attention to vertical forces, only drag matters.

$$\frac{dv}{dt} = \frac{F_{Mh}}{M_b} - \frac{1}{2} \frac{v S_b \rho_m C_d}{M_b} \quad (\text{eqn S.7})$$

During the second phase (inertial phase), the force stops (at time  $t_{force}$ ), and the body pursues its motion by inertia until it stops (from time  $t_{force}$  to  $t_3$  at the maximum).

$$\frac{dv}{dt} = -\frac{1}{2} \frac{v S_b \rho_m C_d}{M_b} \quad (\text{eqn S.8})$$

The total time ( $t_3$ ) for both vertical and horizontal components is the same, as well as the duration of the active phase ( $t_{force}$ ). It explains why the allocation of muscular force between the two components has a strong impact on the result because a total allocation to the vertical component

is useless, since the individual stays at the same place (horizontally), while a total allocation to the horizontal component is inefficient, since the individual cannot displace itself or the medium to move forward.

### Force allocation and work

Vertical ( $F_{Mv}$ ) and horizontal ( $F_{Mh}$ ) muscular forces are applied simultaneously during stroke ( $t_{force}$ ). Thus, an animal can use up to its maximal muscular output for motion

$$0 < (F_{Mv} + F_{Mh}) \leq F_{Max} \quad (\text{eqn S.9})$$

Due to the recursive relationship between speed and drag, equations S.4 to S.8 have to be solved numerically. Their integration through time gives the distance covered during one phase.

Then, knowing the forces ( $F_{Mv} + F_{Mh}$ ) applied during a period of time ( $t_{force}$ ) and the distance covered during that period of time in both vertical ( $x_v$ ) and horizontal plan ( $x_h$ ), a work can be calculated, which is the energetic cost for motion.

$$\text{Work} = \int_{t_0}^{t_{force}} F_{Mv} x_v \, dx + \int_{t_0}^{t_{force}} F_{Mh} x_h \, dx \quad (\text{eqn S.10})$$

This work can be divided by the time of a whole oscillation (from  $t_0 = 0$  to  $t_3$ ), thus having a cost *per time* ( $\text{Cost}_{pt}$ ).

$$\text{Cost}_{pt} = \frac{\text{Work}}{t_3} \quad (\text{eqn S.11})$$

### 3 Encounter rate

In order to be consistent throughout the whole study, prey is assumed to fill 1% of the total volume of the medium (White, Ernest, Kerkhoff, & Enquist, 2007). Therefore, small prey is more abundant than large prey. Then, encounter rate ( $E_r$ ) is calculated following Rothschild & Osborn (1988):

$$E_r = \frac{\pi\omega_{Prey}D_{pred}^2(v_{Prey}^2 + 3v_{Pred}^2)}{3v_{Pred}} \quad \text{when } v_{Pred} > v_{Prey} \quad (\text{eqn S.12})$$

$$E_r = \frac{\pi\omega_{Prey}D_{pred}^2(v_{Pred}^2 + 3v_{Prey}^2)}{3v_{Prey}} \quad \text{when } v_{Pred} < v_{Prey} \quad (\text{eqn S.13})$$

Where  $\omega_{Prey}$  is prey abundance,  $D_{pred}$  is the predator detection distance (see main text),  $v_{Pred}$  and  $v_{Prey}$  are the species-specific speeds for the predator and prey respectively (see main text).

### 4 Handling time

Handling time is calculated as the sum of ingestion and digestion times. Ingestion time ( $I_t$ ) can be described as the time needed for ingestion of a bite ( $B_t$ ) times the number of bites needed to consume a whole prey.

$$I_t = B_t \frac{M_{prey}}{B_s} \quad (\text{eqn S.14})$$

where  $B_s$  is bite size, and  $M_{prey}$  is prey mass.

Bite diameter ( $B_d$ ) depends on predator body mass (Wilson & Kerley, 2003)

$$B_d = B_0 \left( \frac{M_{pred}}{M_{0b}} \right)^{0.32} \quad (\text{eqn S.15})$$

Where  $M_{pred}$  is predator mass,  $B_0$  is bite size at reference size (set at 0.26 mm), and  $M_{0b}$  is reference size (set at 2.9 kg). Calibration has been done using published data (Wilson & Kerley, 2003). Assuming that a bite is generally spherical, the bite diameter can easily be transformed in a corresponding mass. Therefore, bite size ( $B_s$ ) is

$$B_s = \rho_b \frac{4}{3} \pi \left( \frac{B_d}{2} \right)^3 \quad (\text{eqn S.16})$$

where  $\rho_b$  is body density.

Bite time ( $B_t$ ) is the time needed to ingest a bite of size  $B_s$  (Laca, Ungar, & Demment, 1994)

$$B_t = 0.1 B_s^2 \quad (\text{eqn S.17})$$

If the prey is smaller than the size of one bite (i.e.,  $M_{prey} < B_s$ ), bite time is assumed to be equal to the time for one bite (not a fraction of this time).

Digestion time ( $D_t$ ) depends on predator and prey body sizes (Hendriks, 1999)

$$D_t = D_{t0} \frac{M_{prey}}{M_{pred}} M_{pred}^{0.25} \quad (\text{eqn S.18})$$

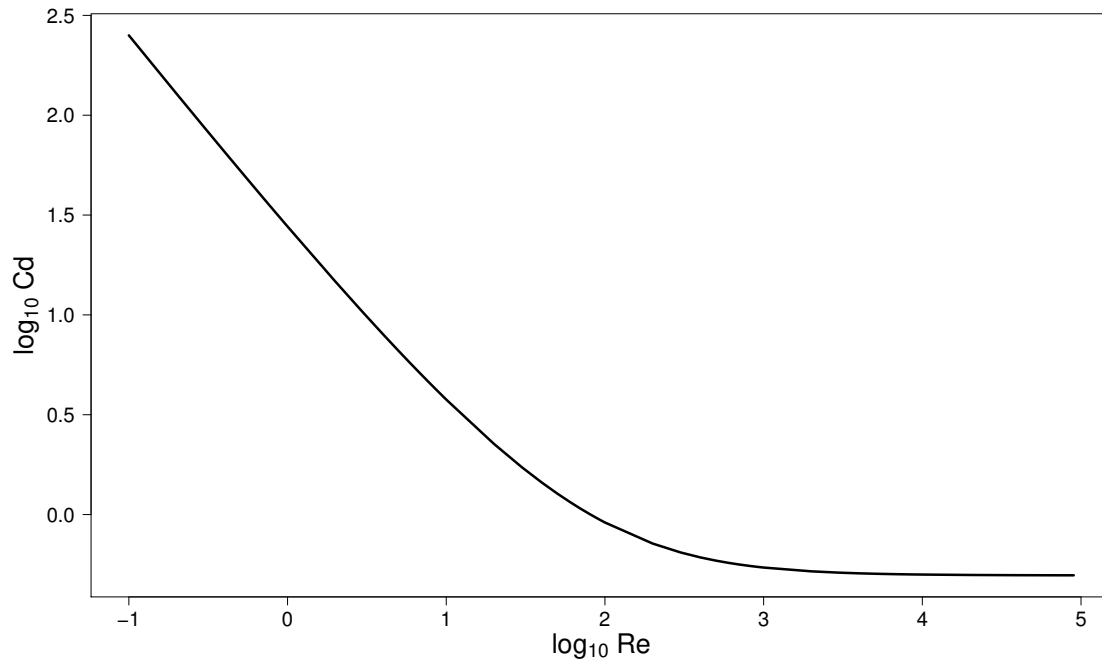
where  $D_{t0}$  is digestion time for 1 kg organism (set as  $2.3 * 10^4 \text{ s.kg}^{-1}$ ). Therefore, handling time ( $t_h$ ) is

$$t_h = I_t + D_t \quad (\text{eqn S.19})$$

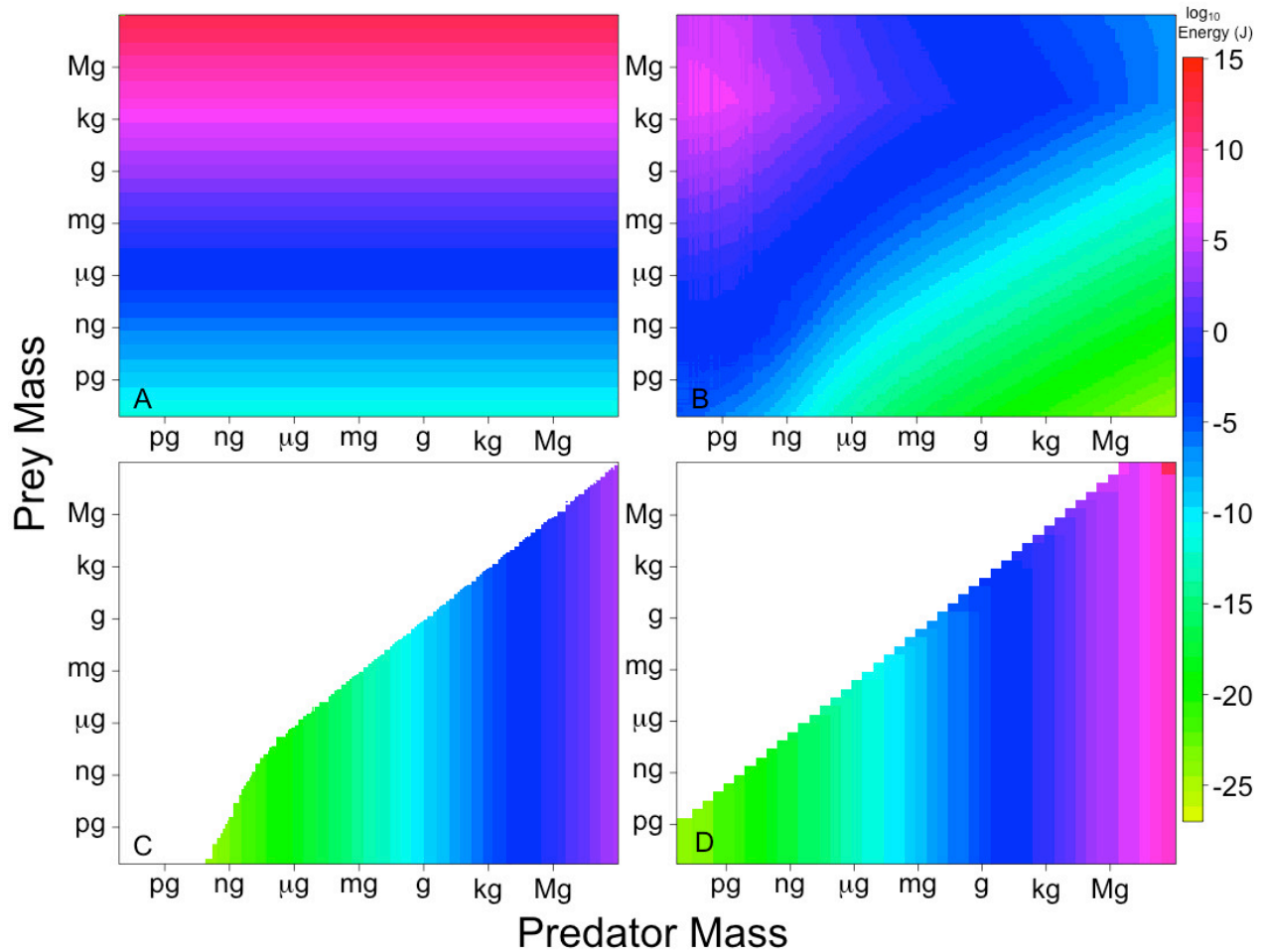
Supplementary table S 1: Biological parameters

Symbol	Parameter	Value	Unit
$M_b$	body mass		kg
$V_b$	body volume		m <sup>3</sup>
$S_b$	cross-section surface of the body		m <sup>2</sup>
$E$	energy from the prey		J
$R_{dm}$	ash-free dry mass to wet mass ratio	0.16	dimensionless
$R_{ed}$	energy to ash-free dry mass ratio	$23 * 10^6$	J.kg <sup>-1</sup>
$C_{met}$	metabolic rate		J.s <sup>-1</sup>
$D_{detec}$	detection distance		m
$d_0$	detection distance at reference size	0.225	m
$M_{0d}$	reference mass for detection distance	0.0376	kg
$B_d$	bite diameter		m
$B_s$	bite size		kg
$B_t$	bite time		s
$E_r$	encounter rate		s <sup>-1</sup>
$P_{suc}$	capture probability		dimensionless
$I_t$	ingestion time		s
$D_t$	digestion time		s
$D_{t0}$	reference digestion time	$2.3 * 10^4$	s.kg <sup>-1</sup>
$F_{max}$	maximal muscular output		N
$F_{Mv}$	vertical muscular force		N
$F_{Mh}$	horizontal muscular force		N
$t_{force}$	stroke period duration		s
$v$	instantaneous speed		m.s <sup>-1</sup>
$x_t$	horizontal translational distance		m
$Cost_{pt}$	cost (work) <i>per time</i>		J.s <sup>-1</sup>
$C_s$	searching cost		J
$C_c$	capture cost		J
$C_h$	handling cost		J

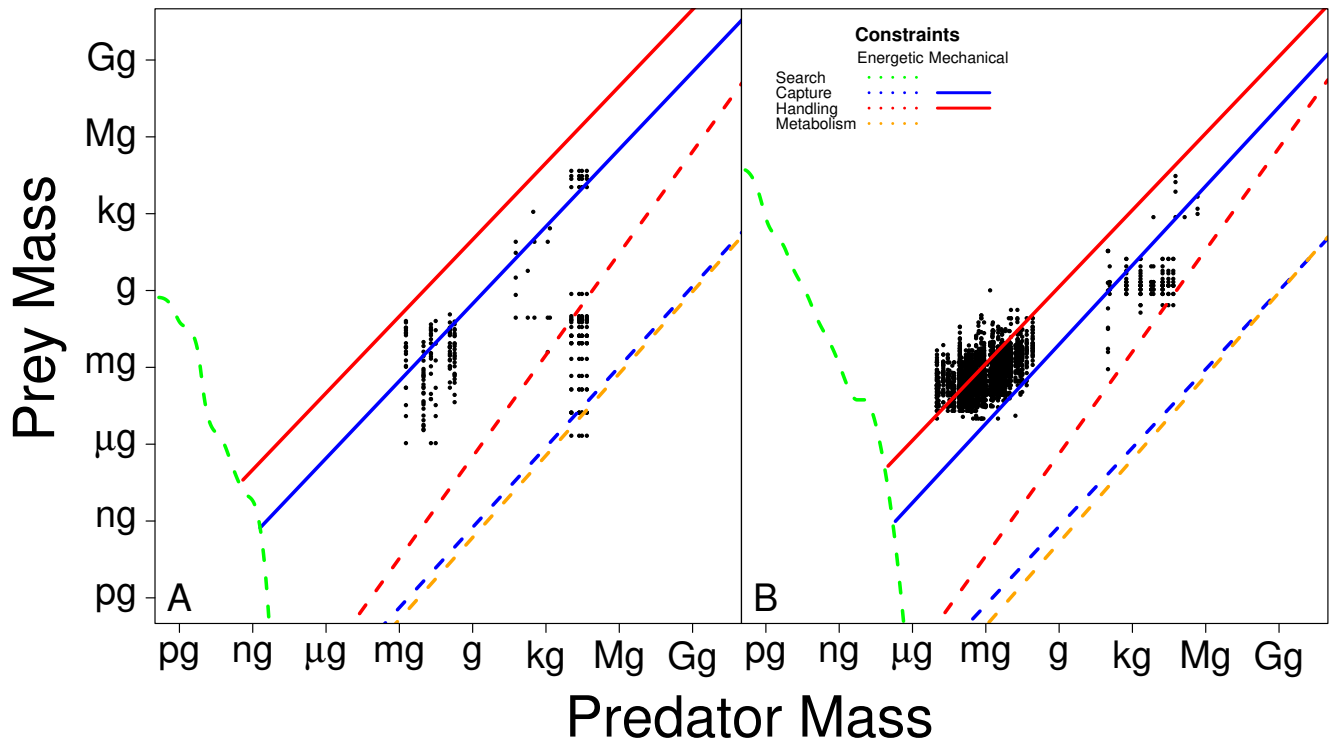




Supplementary figure S 1: Drag coefficient as a function of Reynolds number. Drag coefficient decreases when Reynolds number increases as inertia becomes more important compared to viscosity. Thus, small animals, moving at low Reynolds number, face stronger drag compared to their size than larger animals, moving at higher Reynolds number.



Supplementary figure S 2: Energetic gain and costs for each predator and prey interaction in pelagic systems. Energy given by the prey (A) increases with prey size. Searching cost (B) increases with prey size because predators need more energy to contact larger prey since prey abundance decreases with increasing prey size. This constraint is stronger for small predators because they move slowly, thus they spend more time to contact a large prey than larger predators would spend. Capture cost (C) increases mostly with predator size because a larger predator needs more energy to move. However, predators are unable to capture prey larger than an upper size threshold. This limit is stronger for small predators. Handling cost (D) increases mostly with predator size (for similar reasons than capture cost). Predators are unable to carry prey larger than an upper threshold. Handling upper limit occurs at larger prey size than capture upper limit, which means that a predator overcoming this capture limit would be able to carry larger prey.



Supplementary figure S 3: Sit-and-wait predators in aquatic (A) and terrestrial systems (B). These predators do not usually follow the main assumptions of the model. They do not move during searching time since they wait for a prey to come close to them. These predators usually live on the bottom, a complex landscape where they can hide, and where they can relax handling cost. Last, some might build traps (e.g., spider webs) that increase capture efficiency. Many sit-and-wait predators can consume prey larger than those predicted by the model. A full study of those predators is out of the scope of the present study.

## Supplementary references

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