

Parameter	Definition	Units
<i>Biomass and substrate components</i>		
$X_H$	Concentration of heterotrophic bacteria	$\text{gCOD m}^{-3}$
$X_A$	Concentration of ammonium-oxidizing bacteria (AOB)	$\text{gCOD m}^{-3}$
$X_N$	Concentration of nitrite-oxidizing bacteria (NOB)	$\text{gCOD m}^{-3}$
$X_I$	Concentration of inert solids	$\text{gTS m}^{-3}$
$S_O$	Concentration of dissolved oxygen	$\text{gO}_2 \text{ m}^{-3}$
$S_N$	Concentration of nitrite	$\text{gN m}^{-3}$
$S_A$	Concentration of ammonium	$\text{gN m}^{-3}$
$S_C$	Concentration of soluble, readily biodegradable organics	$\text{gCOD m}^{-3}$
<i>Kinetic and stoichiometric coefficients</i>		
$\mu_{H\max}$	Maximum growth rate of heterotrophs	$\text{d}^{-1}$
$b_H$	Aerobic endogenous respiration rate constant for heterotrophs	$\text{d}^{-1}$
$K_{OH}$	Affinity constant for oxygen of heterotrophs	$\text{gO}_2 \text{ m}^{-3}$
$K_C$	Affinity constant for organics of heterotrophs	$\text{gCOD m}^{-3}$
$Y_H$	Yield coefficient for heterotrophs growing aerobically	$\text{gCOD}_{XH} \text{ g}^{-1}\text{COD}_{SC}$
$\mu_{A\max}$	Maximum growth rate of AOB	$\text{d}^{-1}$
$b_A$	Aerobic endogenous respiration rate constant for AOB	$\text{d}^{-1}$
$K_{OA}$	Affinity constant for oxygen of AOB	$\text{gO}_2 \text{ m}^{-3}$
$K_A$	Affinity constant for ammonium of AOB	$\text{gN m}^{-3}$
$Y_A$	Yield coefficient for AOB	$\text{gCOD}_{XA} \text{ g}^{-1}\text{NH}_4\text{-N}$
$\mu_{N\max}$	Maximum growth rate of NOB	$\text{d}^{-1}$
$b_N$	Aerobic endogenous respiration rate constant for NOB	$\text{d}^{-1}$
$K_{ON}$	Affinity constant for oxygen of NOB	$\text{gO}_2 \text{ m}^{-3}$
$K_N$	Affinity constant for $\text{NO}_2^-$ of NOB	$\text{gN m}^{-3}$
$Y_N$	Yield coefficient for NOB	$\text{gCOD}_{XN} \text{ g}^{-1}\text{NO}_2\text{-N}$
$f_{XI}$	Fraction live biomass being convert to inert material during endogenous respiration	
$i_{NX}$	Nitrogen content in biomass	$\text{gNH}_4\text{-N g}^{-1}\text{COD}$
<i>Physical parameters</i>		
$D_{W\_O}$	Diffusion coefficient of oxygen in water	$\text{m}^2 \text{ s}^{-1}$
$D_{W\_N}$	Diffusion coefficient of nitrite in water	$\text{m}^2 \text{ s}^{-1}$
$D_{W\_A}$	Diffusion coefficient of ammonium in water	$\text{m}^2 \text{ s}^{-1}$
$D_{W\_C}$	Diffusion coefficient of organic carbon in water	$\text{m}^2 \text{ s}^{-1}$
$D_e$	Effective diffusion coefficient, i.e. diffusion coefficient in biofilm	$\text{m}^2 \text{ s}^{-1}$
$f_{VS}$	Fraction of the total solids that is live, active bacteria	
$f_{XH}$	Fraction of the live bacteria that is aerobic heterotrophs	
$f_{XA}$	Fraction of the live bacteria that is AOB	
$f_{XN}$	Fraction of the live bacteria that is NOB	
$S_{O,\text{bulk}}$	Concentration dissolved oxygen in bulk liquid	$\text{gO}_2 \text{ m}^{-3}$
$S_{N,\text{bulk}}$	Concentration nitrite in bulk liquid	$\text{gN m}^{-3}$
$S_{A,\text{bulk}}$	Concentration ammonium in bulk liquid	$\text{gN m}^{-3}$

$S_{C,bulk}$	Concentration organic carbon in bulk liquid	$gCOD\ m^{-3}$
$L$	Biofilm thickness	m
$\Delta x$	Thickness of layer in biofilm	m
$X_{TS}$	Biofilm density	$gTS\ m^{-3}$
$\delta_{BL}$	Bulk liquid-biofilm diffusion boundary layer thickness	M