Supplementary Information for "Analysis of noise mechanisms in cell size control"

Saurabh Modi, Cesar Augusto Vargas-Garcia, Khem Raj Ghusinga, and Abhyudai Singh

S1 Noise analysis in the generalized adder

For the generalized adder strategy for growth, the newborn cell-size V_n progresses through discrete cell cycles as

$$V_{n+1} = (aV_n + \Delta_{n,\alpha_n})\beta_n. \tag{S1.1}$$

Here Δ_{n,α_n} takes the form

$$\Delta_{n,\alpha_n} = \Delta_n F(\bar{\alpha}) \left(1 + c\ S_{\bar{\alpha}} \frac{\alpha_n - \bar{\alpha}}{\bar{\alpha}}\right). \tag{S1.2}$$

By taking the expectation on both sides in (S1.1) we obtain the mean newborn cell-size

$$\langle V_{n+1} \rangle = (a \langle V_n \rangle + \langle \Delta_{n,\alpha_n} \rangle) \langle \beta \rangle. \tag{S1.3}$$

We assume the first moment is finite as for biological systems usually the mean newborn cell-sizes does not grow unboundedly. Then $\lim_{n\to\infty} \langle V_{n+1} \rangle = \lim_{n\to\infty} \langle V_n \rangle = \langle V \rangle$. Thus the mean newborn cell-size is

$$\langle V \rangle = \langle \Delta \rangle \frac{F(\bar{\alpha}) \langle \beta \rangle}{1 - a \langle \beta \rangle}. \tag{S1.4}$$

Where $F(\bar{\alpha})$ is the dependence of Δ_{n,α_n} on the mean growth-rate as defined in the main text. Next we derive the expression for the second order moment of newborn cell-size. To do this we assume the second moment/variance of newborn cell-size similar to the mean is finite. We then let $\lim_{n\to\infty} \langle V_{n+1}^2 \rangle = \lim_{n\to\infty} \langle V_n^2 \rangle = \langle V^2 \rangle$. Squaring (S1.1) on both sides and taking expectation gives

$$\langle V^2 \rangle = (a^2 \langle V^2 \rangle + \langle \Delta^2 \rangle F(\bar{\alpha})^2 (1 + c^2 S_{\bar{\alpha}}^2 CV_{\alpha}^2) + 2a \lim_{n\to\infty} \langle V_n \Delta_{n,\alpha_n} \rangle) \langle \beta^2 \rangle. \tag{S1.5}$$

Where $S_{\bar{\alpha}}$ is the log sensitivity of the mean cell-size on the mean growth-rate as defined in the main text. Notice that Δ_{n,α_n} and V_n are not independent due to the memory in growth-rate between consecutive cell-cycles. Hence to derive $\lim_{n\to\infty} \langle \Delta_{n,\alpha_n} V_n \rangle$ we expand V_n in terms of previous cell-cycle newborn cell-sizes

$$\lim_{n\to\infty} \langle \Delta_{n,\alpha_n} V_n \rangle = \lim_{n\to\infty} \left\langle \Delta_{n,\alpha_n} \beta_n (aV_{n-1} + \Delta_{n-1,\alpha_{n-1}}) \right\rangle \tag{S1.6}$$

$$= \lim_{n\to\infty} \frac{\left\langle \Delta_{n,\alpha_n} (aV_{n-1} + \Delta_{n-1,\alpha_{n-1}}) \right\rangle}{2} \tag{S1.7}$$

$$= \lim_{n\to\infty} \frac{a^n}{2^n} \langle \Delta_{n,\alpha_n} V_0 \rangle + \lim_{n\to\infty} \frac{1}{2} \sum_{r=1}^{n} \left\langle \Delta_{n,\alpha_n} \Delta_{n-r,\alpha_{n-r}} \right\rangle \left(\frac{a}{2}\right)^{r-1}. \tag{S1.8}$$

Recall that $a \in [0, 1]$, hence dropping the first term we can write

$$\lim_{n\to\infty} \langle \Delta_{n,\alpha_n} V_n \rangle = \lim_{n\to\infty} \frac{1}{2} \sum_{r=1}^{n} \left\langle \Delta_{n,\alpha_n} \Delta_{n-r,\alpha_{n-r}} \right\rangle \left(\frac{a}{2}\right)^{r-1}. \tag{S1.9}$$

Again note that Δ_{n,α_n} and $\Delta_{n-r,\alpha_{n-r}}$ are dependent due to the memory in growth-rate. Here we use the assumption that α_n follows an autoregressive process given in eqn. (5) of the main text. To obtain $\left\langle \Delta_{n,\alpha_n} \Delta_{n-r,\alpha_{n-r}} \right\rangle$ we expand these in terms of growth-rate as

$$\begin{aligned} \left\langle \Delta_{n,\alpha_n} \Delta_{n-r,\alpha_{n-r}} \right\rangle &= \langle \Delta \rangle^2 \bar{\alpha}^2 \left\langle (1 + cS_{\bar{\alpha}} \frac{\alpha_n - \bar{\alpha}}{\bar{\alpha}})(1 + cS_{\bar{\alpha}} \frac{\alpha_{n-r} - \bar{\alpha}}{\bar{\alpha}}) \right\rangle && \text{(S1.10)} \\ &= \langle \Delta \rangle^2 \bar{\alpha}^2 (1 + c^2 S_{\bar{\alpha}}^2 \frac{\langle (\alpha_n - \bar{\alpha})(\alpha_{n-r} - \bar{\alpha}) \rangle}{\bar{\alpha}^2})) && \text{(S1.11)} \\ &= \langle \Delta \rangle^2 \bar{\alpha}^2 (1 + c^2 S_{\bar{\alpha}}^2 \rho_\alpha^r CV_\alpha^2). && \text{(S1.12)} \end{aligned}$$

Substituting (S1.12) in (S1.9) we can write

$$\lim_{n\to\infty} \langle \Delta_{n,\alpha_n} V_n \rangle = \frac{\langle \Delta \rangle^2 \bar{\alpha}^2}{2} \left(\frac{1}{1 - a/2} + c^2 S_{\bar{\alpha}}^2 CV_\alpha^2 \frac{\rho_\alpha}{1 - a\rho_\alpha/2} \right). \tag{S1.13}$$

For the second order moment of V substituting the above expression in (S1.5) we obtain,

$$\left\langle V^2 \right\rangle = \frac{F(\bar{\alpha})^2 \left\langle \beta^2 \right\rangle}{1 - a^2 \left\langle \beta^2 \right\rangle} \left(\left\langle \Delta^2 \right\rangle + a\bar{\alpha} \left\langle \Delta \right\rangle^2 \left(\frac{2}{2 - a} + c^2 S_{\bar{\alpha}}^2 \frac{2CV_\alpha^2 \rho_\alpha}{2 - a\rho_\alpha} \right) \right). \tag{S1.14}$$

Thus dividing the second order moment above by the square of the mean in (S1.4) and using $CV_{\hat{\alpha}}^2$ as defined in the main text we obtain the variation in newborn cell-size as,

$$\begin{aligned} CV_V^2 = &\frac{(CV_\beta^2 + 1)(2 - a)^2}{4 - a^2 - a^2 CV_\beta^2} \Bigg(CV_{\hat{\alpha}}^2 + 1)(1 + CV_\Delta^2) \\ &+ 2a \left(\frac{1}{2 - a} + \frac{CV_{\hat{\alpha}}^2 \rho_\alpha}{2 - a\rho_\alpha} \right) \Bigg) - 1. \end{aligned} \tag{S1.15}$$

S2 Noise analysis in mixer model

In this section, we derive the mean and variation in newborn cell-size for two cases of the mixer model: a timer followed by generalized adder and generalized adder followed by timer.

S2-a Timer followed by generalized adder

The newborn cell-size V_{n+1} in the $(n + 1)^{th}$ cell cycle newborn cell-size is given by

$$V_{n+1} = (a(1 + f_n)V_n + \Delta_{n,\alpha_n})\, \beta_n. \tag{S2.1}$$

Taking expectation on both sides we get the mean newborn cell-size,

$$\langle V_{n+1} \rangle = (a(1 + \langle f \rangle)\langle V_n \rangle + \langle \Delta_{n,\alpha_n} \rangle)\, \beta_n. \tag{S2.2}$$

We assume the first moment is finite and use the expression for $\langle \Delta_{n,\alpha_n} \rangle$ from (S1.2) to obtain the mean newborn cell-size

$$\langle V \rangle = \frac{\langle \beta \rangle F(\bar{\alpha}) \langle \Delta \rangle}{1 - a \langle \beta \rangle (1 + \langle f \rangle)}. \tag{S2.3}$$

To obtain the second order moment we square (S2.1) on both sides and take expectation

$$\left\langle V^2\right\rangle = (a^2(1+2\langle f\rangle+\langle f^2\rangle)\left\langle V^2\right\rangle + \langle\Delta^2\rangle F(\bar\alpha)^2(1+c^2S^2_{\bar\alpha}CV^2_\alpha) + 2a(1+\langle f\rangle)\lim_{n\to\infty}\langle V_n\Delta_{n,\alpha_n}\rangle\left\langle\beta^2\right\rangle. \tag{S2.4}$$

Here Δ_{n,α_n} and V_n are dependent due to the memory in growth-rate between consecutive cell-cycles. Hence to derive $\lim_{n\to\infty}\langle\Delta_{n,\alpha_n}V_n\rangle$ we expand V_n in terms of previous cell-cycle newborn cell-sizes

$$\lim_{n\to\infty}\langle\Delta_{n,\alpha_n}V_n\rangle = \lim_{n\to\infty}\left\langle(1+f_n)\Delta_{n,\alpha_n}\beta_n(aV_{n-1}+\Delta_{n-1,\alpha_{n-1}})\right\rangle \tag{S2.5}$$

$$= \lim_{n\to\infty}\left(\frac{a(1+\langle f\rangle)}{2}\right)^n\langle\Delta_{n,\alpha_n}V_0\rangle + \lim_{n\to\infty}\frac{1}{2}\sum_{r=1}^{n}\left\langle\Delta_{n,\alpha_n}\Delta_{n-r,\alpha_{n-r}}\right\rangle\left(\frac{a(1+\langle f\rangle)}{2}\right)^{r-1}. \tag{S2.6}$$

From the definition of $a\in[0,1]$ and f_n with $\langle f\rangle<1$, $a(1+\langle f\rangle)\in[0,2)$, hence

$$\lim_{n\to\infty}\langle\Delta_{n,\alpha_n}V_n\rangle = \lim_{n\to\infty}\frac{1}{2}\sum_{r=1}^{n}\left\langle\Delta_{n,\alpha_n}\Delta_{n-r,\alpha_{n-r}}\right\rangle\left(\frac{a(1+\langle f\rangle)}{2}\right)^{r-1}. \tag{S2.7}$$

The expression for $\left\langle\Delta_{n,\alpha_n}\Delta_{n-r,\alpha_{n-r}}\right\rangle$ is same as that derived in the generalized adder case

$$\left\langle\Delta_{n,\alpha_n}\Delta_{n-r,\alpha_{n-r}}\right\rangle = \langle\Delta\rangle^2\bar\alpha^2(1+c^2S^2_{\bar\alpha}\rho^r_\alpha CV^2_\alpha). \tag{S2.8}$$

Substituting (S2.8) in (S2.7) results in

$$\lim_{n\to\infty}\langle\Delta_{n,\alpha_n}V_n\rangle = \frac{\langle\Delta\rangle^2\bar\alpha^2}{2}\left(\frac{1}{1-a(1+\langle f\rangle)/2}+\frac{\rho_\alpha c^2S^2_{\bar\alpha}CV^2_\alpha}{1-a(1+\langle f\rangle)\rho_\alpha/2}\right). \tag{S2.9}$$

Further from (S2.4) , the second order moment of newborn cell-size can be expressed as

$$\left\langle V^2\right\rangle = \langle\beta^2\rangle\Bigg(\langle\Delta\rangle^2\bar\alpha^2a(1+\langle f\rangle)\left(\frac{1}{1-a(1+\langle f\rangle)/2}+\frac{\rho_\alpha c^2S^2_{\bar\alpha}CV^2_\alpha}{1-a(1+\langle f\rangle)\rho_\alpha/2}\right) + \langle\Delta^2\rangle F(\bar\alpha)^2(1+c^2S^2_{\bar\alpha}CV^2_\alpha)\Bigg)\frac{1}{1-a^2\langle\beta^2\rangle((1+\langle f\rangle)^2+\langle f\rangle^2CV^2_f)}. \tag{S2.10}$$

Thus from the above and (S2.3) with $CV^2_{\hat\alpha}$ as defined in the main text we obtain the variation in newborn cell-size as

$$CV^2_V = \frac{(CV^2_\beta+1)(2-a(1+\langle f\rangle))^2}{4-a^2(CV^2_\beta+1)((1+\langle f\rangle)^2+\langle f\rangle^2CV^2_f)}\Bigg(2a(1+\langle f\rangle)\times \left(\frac{1}{2-a(1+\langle f\rangle)}+\frac{\rho_\alpha CV^2_{\hat\alpha}}{2-a\rho_\alpha(1+\langle f\rangle)}\right)+(1+CV^2_\Delta)(1+CV^2_{\hat\alpha})\Bigg)-1. \tag{S2.11}$$

S2-b Generalized adder followed by timer

For this case the newborn cell-size in the n^{th} cell-cycle is given by

$$V_{n+1} = (aV_n+\Delta_{n,\alpha_n})(1+f_n)\beta_n. \tag{S2.12}$$

Using the same method described in the previous section we obtain the mean newborn cell-size,

$$\langle V\rangle = \frac{\langle\Delta\rangle F(\bar\alpha)(1+\langle f\rangle)}{2-a(1+\langle f\rangle)}. \tag{S2.13}$$

The second moment of newborn cell-size can be derived similar to the previous section

$$\left\langle V^2\right\rangle = (1+2\langle f\rangle+\langle f^2\rangle)\left\langle \beta^2\right\rangle (a^2\left\langle V^2\right\rangle + \langle\Delta^2\rangle F(\bar{\alpha})^2(1+c^2S_{\bar{\alpha}}^2CV_{\alpha}^2)+2a\lim_{n\to\infty}\langle V_n\Delta_{n,\alpha_n}\rangle). \quad \text{(S2.14)}$$

Here the term $\lim_{n\to\infty}\langle V_n\Delta_{n,\alpha_n}\rangle$ can be expressed as

$$\lim_{n\to\infty}\langle\Delta_{n,\alpha_n}V_n\rangle = \frac{1+\langle f\rangle}{2}\lim_{n\to\infty}\sum_{r=1}^{n}\left\langle\Delta_{n,\alpha_n}\Delta_{n-r,\alpha_{n-r}}\right\rangle\left(\frac{a(1+\langle f\rangle)}{2}\right)^{r-1} \quad \text{(S2.15)}$$

$$= \frac{1+\langle f\rangle}{2}\lim_{n\to\infty}\sum_{r=1}^{n}\langle\Delta\rangle^2\bar{\alpha}^2(1+c^2S_{\bar{\alpha}}^2\rho_{\alpha}^rCV_{\alpha}^2)\left(\frac{a(1+\langle f\rangle)}{2}\right)^{r-1} \quad \text{(S2.16)}$$

$$= \frac{(1+\langle f\rangle)\langle\Delta\rangle^2\bar{\alpha}^2}{2}\left(\frac{1}{1-a(1+\langle f\rangle)/2}+\frac{\rho_{\alpha}c^2S_{\bar{\alpha}}^2CV_{\alpha}^2}{1-a(1+\langle f\rangle)\rho_{\alpha}/2}\right). \quad \text{(S2.17)}$$

Substituting this in (S2.14) we can write the second moment as

$$\left\langle V^2\right\rangle = \langle\beta^2\rangle\Bigg(\langle\Delta\rangle^2\bar{\alpha}^2a(1+\langle f\rangle)\left(\frac{1}{1-a(1+\langle f\rangle)/2}+\frac{\rho_{\alpha}c^2S_{\bar{\alpha}}^2CV_{\alpha}^2}{1-a(1+\langle f\rangle)\rho_{\alpha}/2}\right)$$
$$+\langle\Delta^2\rangle F(\bar{\alpha})^2(1+c^2S_{\bar{\alpha}}^2CV_{\alpha}^2)\Bigg)\frac{(1+\langle f\rangle)^2+\langle f\rangle^2CV_f^2}{1-a^2\left\langle\beta^2\right\rangle((1+\langle f\rangle)^2+\langle f\rangle^2CV_f^2)}. \quad \text{(S2.18)}$$

Thus from the above expression and (S2.13), with $CV_{\hat{\alpha}}^2$ as defined in the main text we obtain the variation in newborn cell-size

$$CV_V^2 = \left(\frac{\langle f\rangle^2CV_f^2}{(1+\langle f\rangle)^2}+1\right)\frac{(CV_{\beta}^2+1)(2-a(1+\langle f\rangle))^2}{4-a^2(CV_{\beta}^2+1)((1+\langle f\rangle)^2+\langle f\rangle^2CV_f^2)}\Bigg(2a(1+\langle f\rangle)\times$$
$$\left(\frac{1}{2-a(1+\langle f\rangle)}+\frac{\rho_{\alpha}CV_{\hat{\alpha}}^2}{2-a\rho_{\alpha}(1+\langle f\rangle)}\right)+(1+CV_{\Delta}^2)(1+CV_{\hat{\alpha}}^2)\Bigg)-1. \quad \text{(S2.19)}$$

Comparing this expression with (S2.11) we can write

$$\underbrace{CV_V^2}_{\text{timer after adder}} = \underbrace{(CV_V^2+1)}_{\text{adder after timer}}\times\left(\frac{\langle f\rangle^2CV_f^2}{(1+\langle f\rangle)^2}+1\right)-1. \quad \text{(S2.20)}$$

From the above expression we can see that the variation in newborn cell-size is higher for the adder followed by timer case compared to the opposite.

S3 Power-law exponent of cell-size in mixer model

Introducing a timer phase fold-change with noise in the generalized adder can cause higher order moments to become infinite. The power-law exponent is a parameter that determines the threshold of moments above which all moments are infinite. If the power-law exponent is m then all the moments above $m-1$ are unbounded. To obtain the power-law exponent for newborn cell-size, we raise both sides in (S2.1) to $m-1$, and take expectation on both sides

$$\langle V_{n+1}^{m-1}\rangle = \langle(aV_n+\Delta_{n,\alpha_n})^{m-1}(1+f_n)^{m-1}\rangle\langle\beta_n^{m-1}\rangle \quad \text{(S3.1)}$$

$$= \langle(aV_n(1+f_n))^{m-1}\rangle\langle\beta_n^{m-1}\rangle \quad \text{(S3.2)}$$
$$+\sum_{k=1}^{m-1}\binom{m-1}{k}\langle\Delta_{n,\alpha_n}^k\rangle\langle(aV_n)^{m-1-k}(1+f_n)^{m-1}\rangle\langle\beta_n^{m-1}\rangle.$$

Now using the assumption that all higher order moments of f_n, Δ_n and α_n are finite. This shows that all the higher order moments of Δ_{n,α_n} is finite. Also by definition of power-law exponent, all moments of V_n lower than $m-1$ are finite. Hence all the terms except the first are finite and are irrelevant for the boundedness of moments of newborn cell size. Now let $\lim_{n\to\infty}\langle V_n^{m-1}\rangle = \lim_{n\to\infty}\langle V_{n-1}^{m-1}\rangle = \langle V^{m-1}\rangle$. Expanding V_n in the first term in (S3.2) and taking the limit $n\to\infty$ gives

$$\langle V^{m-1}\rangle = \lim_{n\to\infty}\left\langle (1+f_n)^{m-1}\right\rangle^n \langle V^{m-1}\rangle\langle\beta_n^{m-1}\rangle^n a^{n(m-1)} + \text{ Lower order moments.} \tag{S3.3}$$

From which the condition for the power-law exponent m becomes

$$1 = \left\langle (1+f_n)^{m-1}\right\rangle \left\langle \beta_n^{m-1}\right\rangle a^{(m-1)}. \tag{S3.4}$$

To obtain a closed form expression for the term $\left\langle (1+f_n)^{m-1}\right\rangle$, we write it as

$$\left\langle (1+f_n)^{m-1}\right\rangle = \left\langle \exp\left((m-1)\log\left(1+f_n\right)\right)\right\rangle. \tag{S3.5}$$

Then we use the Taylor series approximation of $\log(1+f_n)$ with respect to f_n about $\langle f\rangle$. Also using the fact that $CV_f^2 \ll 1$ and truncating upto the first order term gives

$$\left\langle (1+f_n)^{m-1}\right\rangle \approx \left\langle \exp\left((m-1)\log(1+\langle f\rangle) + \frac{m-1}{1+\langle f\rangle}(f_n - \langle f\rangle)\right)\right\rangle \tag{S3.6}$$

For low noise limit $CV_f^2 \ll 1$, we assume f_n's are approximately gaussian random variables. Thus, we can write

$$\left\langle \exp\left((m-1)\log(1+\langle f\rangle) + \frac{m-1}{1+\langle f\rangle}(f_n - \langle f\rangle)\right)\right\rangle = \tag{S3.7}$$

$$(1+\langle f\rangle)^{m-1}\exp\left(\frac{\langle (f_n-\langle f\rangle)^2\rangle(m-1)^2}{2(1+\langle f\rangle)^2}\right) \tag{S3.8}$$

$$\implies \left\langle (1+f_n)^{m-1}\right\rangle \approx (1+\langle f\rangle)^{m-1}\exp\left(\frac{\langle f\rangle^2 CV_f^2 (m-1)^2}{2(1+\langle f\rangle)^2}\right) \tag{S3.9}$$

To find $\langle\beta_n^{m-1}\rangle$ assuming $\beta_n \sim Beta(r,p)$

$$\left\langle \beta_n^{m-1}\right\rangle = \frac{\Gamma(m+r-1)\Gamma(r+p)}{\Gamma(m+r+p-1)\Gamma(r)} \tag{S3.10}$$

where $r = \left(\frac{1}{CV_\beta^2}\left(\frac{1}{\langle\beta\rangle}-1\right)-1\right)\langle\beta\rangle$ and $p = \left(\frac{1}{\langle\beta\rangle}-1\right)r$. Hence to obtain the dependence of m on the partitioning errors and timer-phase noise in Fig. 3 in the main text we numerically solve the equation

$$\begin{aligned} 0 = &\log\left(\frac{\Gamma(m+r-1)\Gamma(r+p)}{\Gamma(m+r+p-1)\Gamma(r)}\right) + (m-1)\log(1+\langle f\rangle) \\ &+ \frac{\langle f\rangle^2 CV_f^2(m-1)^2}{2(1+\langle f\rangle)^2} + (m-1)\log a. \end{aligned} \tag{S3.11}$$

In some limit $\langle \beta_n^{m-1} \rangle$ can be simplified further. In the case where partitioning is very precise with partitioning noise $CV_\beta^2 \ll 1$ and $(m-1)(m-2) \ll 1/CV_\beta^2$ we find that $\langle \beta_n^{m-1} \rangle$ can be simplified by Taylor expansion about $\beta_n = \langle \beta \rangle$. Truncating upto the second order term we obtain

$$\begin{aligned}\left\langle \beta_n^{m-1} \right\rangle \approx \Big\langle \langle\beta\rangle^{m-1} + (m-1)\langle\beta\rangle^{m-2}(\beta_n - \langle\beta\rangle) \qquad \text{(S3.12)}\\ + \langle\beta\rangle^{m-3}\frac{(m-1)(m-2)}{2}(\beta_n - \langle\beta\rangle)^2 \Big\rangle \\ = \langle\beta\rangle^{m-1} + \langle\beta\rangle^{m-1}\frac{(m-1)(m-2)}{2}CV_\beta^2. \qquad \text{(S3.13)}\end{aligned}$$

$$\begin{aligned}\log(\left\langle \beta_n^{m-1} \right\rangle) = (m-1)\log\langle\beta\rangle + \log\left(1 + \frac{(m-1)(m-2)}{2}CV_\beta^2\right) \qquad \text{(S3.14)}\\ \approx (m-1)\log\langle\beta\rangle + \frac{(m-1)(m-2)}{2}CV_\beta^2 \qquad \text{(S3.15)}\end{aligned}$$

Substituting (S3.13) and (S3.9) in (S3.4) we get

$$m = \frac{-\log\langle\beta\rangle - \log a + CV_\beta^2/2 - \log(1+\langle f\rangle)}{\frac{\langle f\rangle^2 CV_f^2}{2(1+\langle f\rangle)^2} + CV_\beta^2/2} + 1. \qquad \text{(S3.16)}$$

S4 *C. crescentus* parameters and model predicted exponent

To obtain the parameters of partitioning at cell-division we use the single cell data of cell-size at birth and before division published in [1]. The single-cell data of sizes was obtained for *C. crescentus* in the balanced growth conditions at the temperature 31 C. The sizes were measured within 2% precision. The data set consists of 11906 pairs of values of cell sizes before and after division for consecutive generations. The sample of partitioning variables β_n is obtained by taking the ratio of lengths before and after division for all generations. We obtain the confidence intervals for $\langle\beta\rangle$ and CV_β^2 by bootstrapping. Bootstrapping is performed by randomly sampling partitioning variables with replacement from the original data set 10000 times. Then we calculate $\langle\beta\rangle$ and CV_β^2 for each sample and find the 95% confidence interval for these parameters across samples.

Parameter	95% Confidence interval
$\langle\beta\rangle$	0.5772 ± 0.0002
CV_β^2	0.0085 ± 0.0004
$\langle f\rangle$	0.376 ± 0.003
CV_f^2	0.1325 ± 0.0041
m	29 ± 0.9
m_V	15.6 ± 1.1
m_Δ	13.5 ± 1.9
m_f	19.6 ± 2.3

Table I: Estimated parameters

The parameters $\langle f\rangle$ and CV_f^2 in the timer phase were calculated from the distribution of $\alpha_n t_n$ in Fig. 3(B) in [2]. Here t_n is the time in the timer-phase. We take the exponential of $\alpha_n t_n$ to obtain the distribution of fold-change $1+f_n = \exp(\alpha_n t_n)$. This distribution was then bootstrapped to obtain the confidence intervals of mean fold-change $1+\langle f \rangle$ and CV_f^2 [2]. We then derive m by using (S3.11). To derive the confidence intervals in m we incorporate uncertainty in timer-phase and partitioning parameters. This is done by simultaneously bootstrapping the parameters β, CV_β^2, $\langle f \rangle$ and CV_f^2 for 10000 realizations and calculating m for each set of parameters. The parameter values are shown in Table I.

S5 Power law fitting

S5-a Estimation of power law exponent from raw data

In addition to the contribution of the timer phase noise processes, the boundedness of moments of Δ_n and f_n have an impact on the exponent of newborn cell-size. Hence we need to estimate the power law exponents m_V, m_f and, m_Δ from the raw data in [2]. To obtain these we use the maximum likelihood estimate method given in [3] shown for a general variable x below. We first fit the distribution of sample data assuming that it is power law,

$$P(x) = \frac{(k-1)}{x_{min}} \left(\frac{x_{min}}{x}\right)^k. \tag{S5.1}$$

Here, $P(x)$ is the probability density function of the power-law fit, k is the power law exponent and x_{min} is the cut-off above which the power law distribution holds. We conveniently fix a cut-off threshold and find the maximum likelihood estimate of power-law exponent

$$k = 1 + N \left(\sum_{i=1}^{N} \log \frac{x_i}{x_{min}} \right)^{-1}. \tag{S5.2}$$

Here N is the total number of samples above the cut-off x_{min} in the data. x_i are the sample values above the cut-off.

We apply the above method to data for newborn cell-size, added cell-size, and timer-phase fold-change and obtain power-law exponents shown in Fig. S1 and Table I. The single cell data for newborn cell-size of the adherent cells is obtained from Fig. 3E of [2]. Further for the single cell data of added cell-size in adder phase we extract the data from Fig. 3E of [2]. For the single cell fold-change data we obtain the data from Fig. 3D of [2]. All these figures show single cell scatter plots of the respective parameters. The confidence intervals on the experimentally derived power-law exponents can then be obtained by bootstrapping the raw data and using the maximum likelihood estimate for each bootstrapped sample.

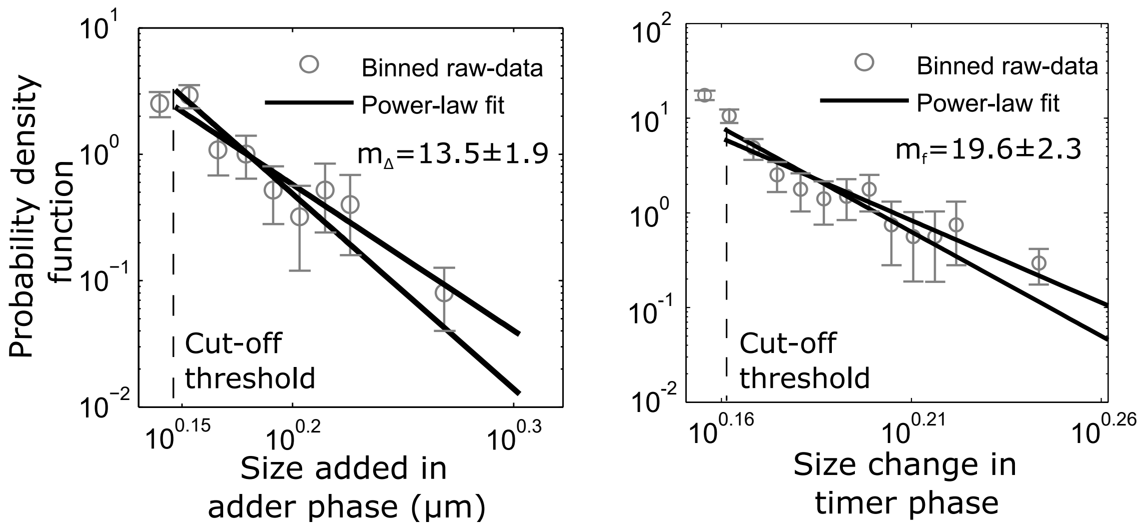

Figure S1: Figures show the derived power-law exponents of added cell-size in adder phase and size fold-change in timer-phase. *Left* The power-law exponent of added cell size in the adder phase explains the exponent of newborn cell size. The cut-off threshold used is 1.4 μm with 185 data points. The raw data is binned starting at 1.1853 μm using bin widths of 0.0433 μm with the last bin 0.3028 μm wide. The dots shows the ratio of the normalized frequency of samples in each bin to the width of the bin. The normalization is done with respect to total number of points above the threshold. The error bars show the 95% confidence interval of the height of each bin with the median as the dots. *Right* The power-law exponent of size fold-change is shown . The cut-off threshold used is 1.448 with 264 data points. The raw data is binned using bin-width 0.0214 with the last bin 0.15 wide. The dots shows the normalized ratio of number of samples in each bin to the width of the bin. The normalization is done with respect to the total number of samples above the threshold. The error bars show the 95% confidence interval of the height of each bin with the median as the dots.

References

[1] Srividya Iyer-Biswas, Charles S Wright, Jonathan T Henry, Klevin Lo, Stanislav Burov, Yihan Lin, Gavin E Crooks, Sean Crosson, Aaron R Dinner, and Norbert F Scherer. Scaling laws governing stochastic growth and division of single bacterial cells. *Proceedings of the National Academy of Sciences*, 111:15912–15917, 2014.

[2] Shiladitya Banerjee, Klevin Lo, Thomas Kuntz, Matthew K. Daddysman, Aaron R. Dinner, and Norbert F. Scherer. Crossover in the dynamics of cell wall growth controls bacterial division times. *bioRxiv 047589*, 2016. http://biorxiv.org/content/early/2016/04/07/047589.

[3] Aaron Clauset, Cosma Rohilla Shalizi, and M.E.J. Newman. Power-law distributions in emperical data. *SIAM Review*, 51:661–703, 2009.