Electronic Supplementary Material

Gawryszewski FM (2017) Colour vision models: a practical guide, some simulations, and *colourvision* R package.

1. Methods for tetrachromatic colour vision models

1.1. Colour hexagon model for tetrachromats

The tetrachromat version of Chittka (1992) colour hexagon was derived by Thery & Casas (2002). The photoreceptor outputs are calculated in the same way as the trichromatic version (main text eqn. 1-4). Then, these values are projected into a tridimensional colour space, a hexagonal trapezohedron, by the following formulae:

$$x = \frac{\sqrt{2}\sqrt{3}}{3}(E_3 - E_4) \qquad \text{(Eq. S1)}$$

$$y = E_1 - \frac{1}{3}(E_2 + E_3 + E_4) \qquad \text{(Eq. S2)}$$

$$z = \frac{2\sqrt{2}}{3}\left[\frac{1}{2}(E_3 + E_4) - E_2\right] \qquad \text{(Eq. S1)}$$

1.2. Endler and Mielke (2005) model

Model calculation follow the same steps as in the trichromatic version, but with four photoreceptor outputs. Then, f_i is transformed so that photoreceptor outputs $u + s + m + l = 1$:

$$u = \frac{f_1}{f_1 + f_2 + f_3 + f_4} \qquad \text{(Eq. S4)}$$

$$s = \frac{f_2}{f_1 + f_2 + f_3 + f_4} \qquad \text{(Eq. S5)}$$

$$m = \frac{f_3}{f_1 + f_2 + f_3 + f_4} \qquad \text{(Eq. S6)}$$

$$l = \frac{f_4}{f_1 + f_2 + f_3 + f_4} \quad \text{(Eq. S7)}$$

Photoreceptor outputs are then used to project a tridimensional colour space (tetrahedron) by the following formulae:

$$x = \sqrt{\frac{3}{2}}\left(\frac{1 - 2\mathrm{s} - \mathrm{m} - \mathrm{u}}{2}\right) \quad \text{(Eq. S8)}$$

$$y = \frac{-1 + 3\mathrm{m} + \mathrm{u}}{2\sqrt{2}} \quad \text{(Eq. S9)}$$

$$z = \mathrm{u} - \frac{1}{4} \quad \text{(Eq. S10)}$$

1.3. Receptor noise limited models: linear and log-linear versions

For linear-RNL and log-RNL models, photoreceptor outputs and photoreceptor noise of each photoreceptor type are used to find colour locus in the chromaticity diagram using the following formulae (Renoult et al. 2017):

$$A = \sqrt{\frac{(e_3e_4)^2 + (e_2e_4)^2 + (e_2e_3)^2}{(e_2e_3e_4)^2 + (e_1e_3e_4)^2 + (e_1e_2e_4)^2 + (e_1e_2e_3)^2}} \quad \text{(Eq. S11)}$$

$$a = \frac{(e_2e_3)^2}{(e_3e_4)^2 + (e_2e_3)^2 + (e_2e_4)^2} \quad \text{(Eq. S12)}$$

$$b = \frac{(e_2e_4)^2}{(e_3e_4)^2 + (e_2e_3)^2 + (e_2e_4)^2} \quad \text{(Eq. S13)}$$

$$c = \frac{(e_3e_4)^2}{(e_3e_4)^2 + (e_2e_3)^2 + (e_2e_4)^2} \quad \text{(Eq. S14)}$$

$$x = \sqrt{\frac{1}{e_3^2 + e_4^2}}(f_4 - f_3) \qquad \text{(Eq. S15)}$$

$$y = \sqrt{\frac{e_3^2 + e_4^2}{(e_3 e_4)^2 + (e_2 e_3)^2 + (e_2 e_4)^2}}\left[f_2 - \left(f_4 \frac{e_3^2}{e_3^2 + e_4^2} + f_3 \frac{e_4^2}{e_3^2 + e_4^2}\right)\right] \qquad \text{(Eq. S16)}$$

$$z = A[f_1 - (af_4 + bf_3 + cf_2)] \qquad \text{(Eq. S17)}$$

1.4. Distance between colour loci

Chromaticity distance between pair of reflectance spectra (a and b) are found by calculating the Euclidian distance between their colour loci in the colour space:

$$\Delta S = \sqrt{(x_a - x_b)^2 + (y_a - y_b)^2 + (z_a - z_b)^2} \qquad \text{(Eq. S18)}$$

By definition background reflectance lays at the centre of the background ($x = 0, y = 0$). Therefore, the distance of the observed object against the background is given by:

$$\Delta S = \sqrt{x^2 + y^2 + z^2} \qquad \text{(Eq. S19)}$$

In the original noise receptor model (Vorobyev and Osorio 1998) ΔS between pair of reflectance spectra (a and b) is calculated directly, without finding colour locus in the colour space:

$$\Delta S = \sqrt{\frac{(e_1 e_2)^2(\Delta f_4 - \Delta f_3)^2 + (e_1 e_3)^2(\Delta f_4 - \Delta f_2)^2 + (e_1 e_4)^2(\Delta f_3 - \Delta f_2)^2 + (e_2 e_3)^2(\Delta f_4 - \Delta f_1)^2 + (e_2 e_4)^2(\Delta f_3 - \Delta f_1)^2 + (e_3 e_4)^2(\Delta f_2 - \Delta f_1)^2}{(e_1 e_2 e_3)^2 + (e_1 e_3 e_4)^2 + (e_1 e_3 e_4)^2 + (e_2 e_3 e_4)^2}}$$

(Eq. S20)

Where Δf_i is the difference between photoreceptor *i* output for the reflectance spectrum a and b ($\Delta f_i = f_{a_i} - f_{b_i}$). Using equation (S24) will give the same value as calculating ΔS using equations

(S12-18) and then equation (S19).

2. Simulations with tetrachromatic vision

Model simulation parameters were the same as in the trichromatic simulation, except that instead of honeybee photoreceptors, I used the average photoreceptor sensitivity curves of birds (only birds with UV λ_{max} cones; data from Hart & Vorobyev 2005 available in Endler & Mielke 2005; Figure S1). I estimated receptor noise using eqn. 21, with a ratio of 1:2:2:4 photoreceptor types in the retina (from shortest to longest lambda-max; *Leiothrix lutea*) and a noise-to-signal ratio of 0.1 (data available in Vorobyev & Osorio 1998; Vorobyev *et al.* 1998). Results are presented in Figures S4-S10.

3. Simulations using Gaussian reflectance curves

Model simulation parameters were the same as in the original simulations, except that instead of a logistic function, I used a Gaussian function to generate stimulus reflectance spectra:

$$R(\lambda) = ae^{-\frac{(\lambda-\lambda_p)^2}{2b^2}} \qquad \text{(Eq. S17)}$$

Where R is the reflectance value at wavelength λ, a gives the curve maximum reflectance value (%), b controls the width of the curve, and λ_p is the wavelength (nm) of maximum reflectance. I used a maximum value of $a = 50\%$ reflectance, and a width of $b = 0.04$. I generated curves with wavelength of maximum reflectance varying from 300 to 700 nm with 5 nm intervals, in a total of 81 reflectance spectra (Figure S2 and S3). Results are presented in Figures S11-S14.

4. References

Endler, J.A. & Mielke, P. (2005). Comparing entire colour patterns as birds see them. *Biological Journal Of The Linnean Society*, **86**, 405–431.

Hart, N.S. & Vorobyev, M. (2005). Modelling oil droplet absorption spectra and spectral sensitivities of bird cone photoreceptors. *Journal of Comparative Physiology A: Sensory, Neural, and Behavioral Physiology*, **191**, 381–392.

Renoult, J.P., Kelber, A. & Schaefer, H.M. (2017). Colour spaces in ecology and evolutionary biology. *Biological Reviews Of The Cambridge Philosophical Society*, **92**, 292–315.

Thery, M. & Casas, J. (2002). Predator and prey views of spider camouflage. *Nature*, **415**, 133–133.

Vorobyev, M. & Osorio, D. (1998). Receptor noise as a determinant of colour thresholds. *Proceedings of the Royal Society B: Biological Sciences*, **265**, 351–358.

Vorobyev, M., Osorio, D., Bennett, A.T.D., Marshall, N.J. & Cuthill, I.C. (1998). Tetrachromacy, oil droplets and bird plumage colours. *Journal of Comparative Physiology A: Sensory, Neural, and Behavioral Physiology*, **183**, 621–633.

5. Figures

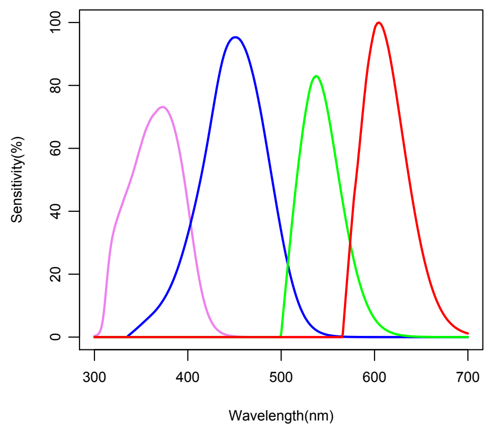

Figure S1. Average photoreceptor sensitivity curves of birds (only birds with UV λ_{max} cones; λ_{max} of cones with oil droplets; data from Hart & Vorobyev 2005 available in Endler and Mielke 2005; used for tetrachromatic model simulations.

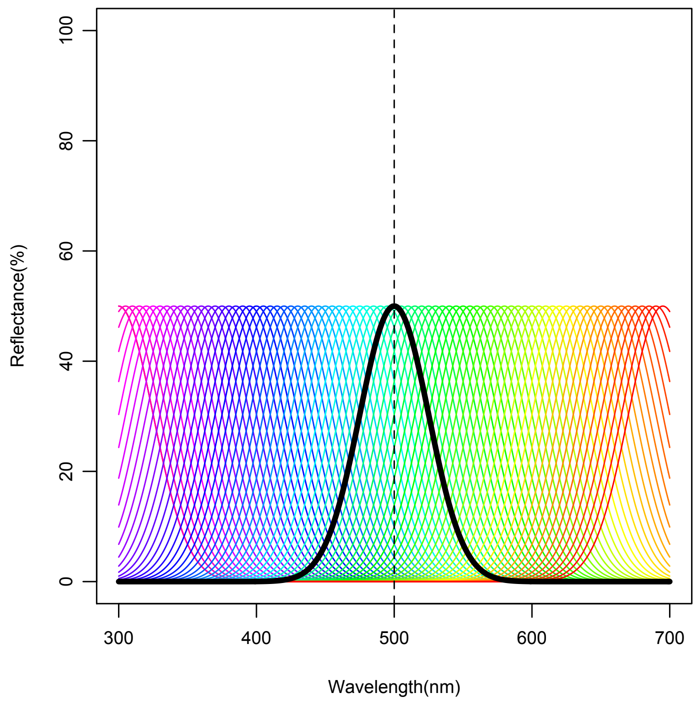

Figure S2. Reflectance spectra generated by a Gaussian function with wavelength of maximum reflectance varying from 300 to 700nm at 5nm intervals. Spectrum colours are arbitrary. In black is shown a reflectance curve with wavelength of maximum reflectance at 500nm.

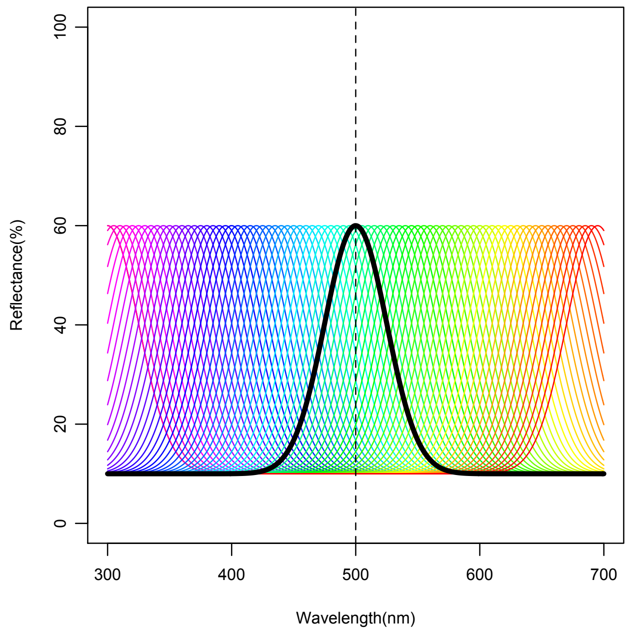

Figure S3. Ten percent point added to the reflectance spectra generated by a Gaussian function (Figure S2). Spectrum colours are arbitrary. In black is shown a reflectance curve with wavelength of maximum reflectance at 500nm.

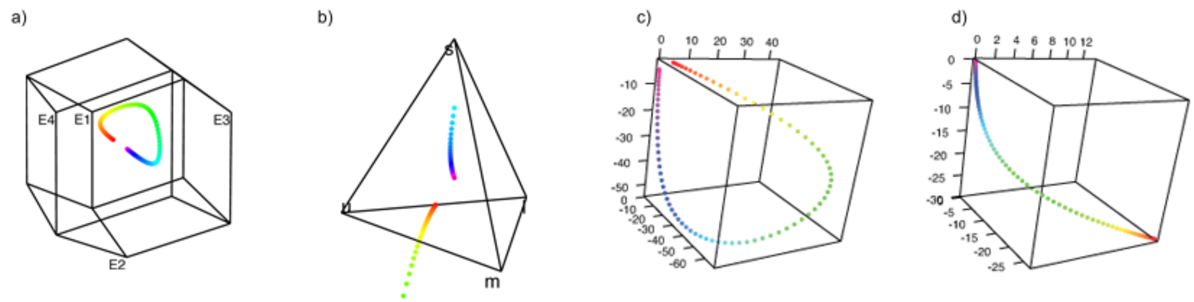

Figure S4. Chromaticity diagrams of the basic setup of colour vision model simulations: a) Chittka (1992), b) Endler & Mielke (2005) model, and b) linear and c) log-linear Receptor Noise Limited models (Linear-RNL and Log-RNL; Vorobyev & Osorio 1998; Vorobyev et al. 1998). Colours correspond to reflectance spectra from Figure 1d (main text).

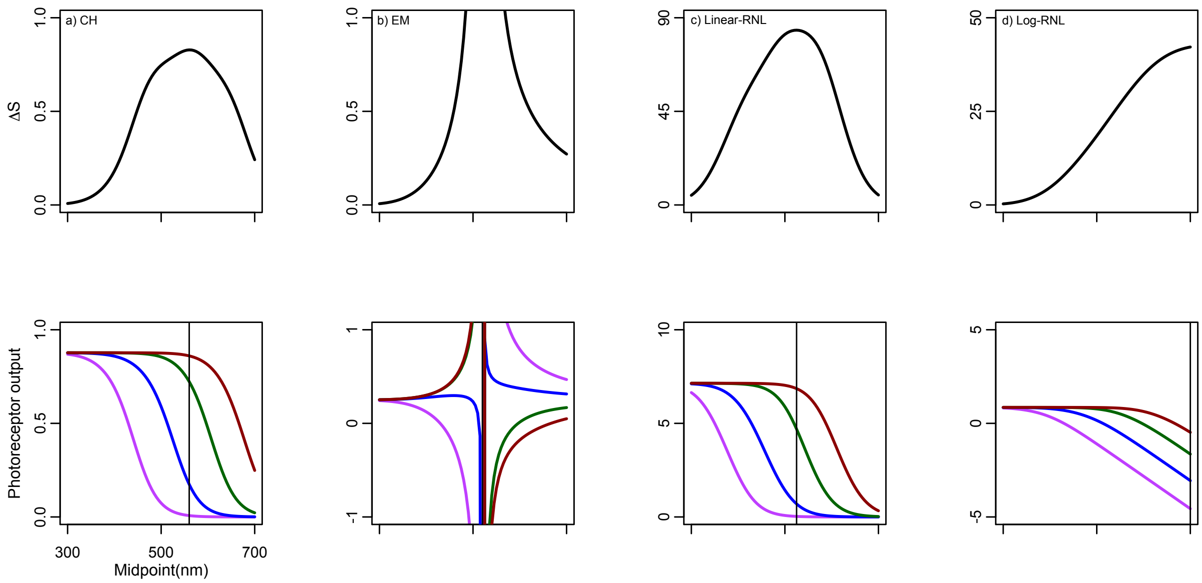

Figure S5. ΔS and photoreceptor outputs of the basic setup of colour vision model simulations: a) Chittka (1992); b) Endler & Mielke (2005), and b) linear and c) log-linear Receptor Noise Limited models (Linear-RNL and Log-RNL; Vorobyev & Osorio 1998; Vorobyev *et al.* 1998). Variation in ΔS-values as a function of reflectance spectra with midpoints from 300 to 700nm (top row). Photoreceptor output values as a function of the same reflectance spectra (bottom row). Violet, blue, green, and red colours represent UV, short, middle and long λ_{max} photoreceptor types. Vertical lines represent midpoint of maximum ΔS-values.

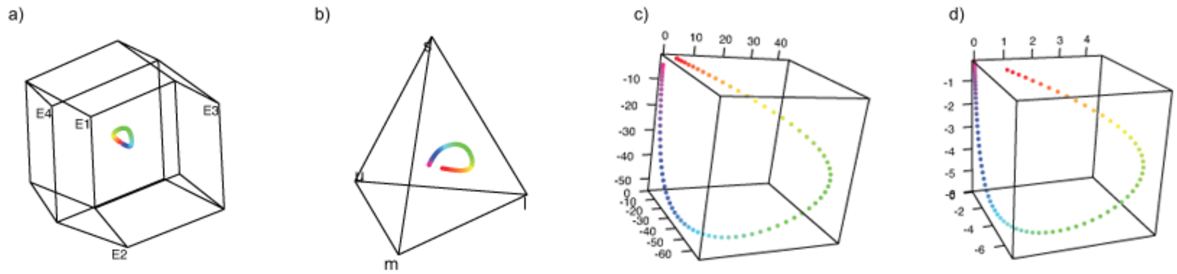

Figure S6. Chromaticity diagrams with 10 percent point added to reflectance values: a) Chittka (1992), b) Endler & Mielke (2005) model, and b) linear and c) log-linear Receptor Noise Limited models (Linear-RNL and Log-RNL; Vorobyev & Osorio 1998; Vorobyev et al. 1998). Colours correspond to reflectance spectra from Figure 2a (main text).

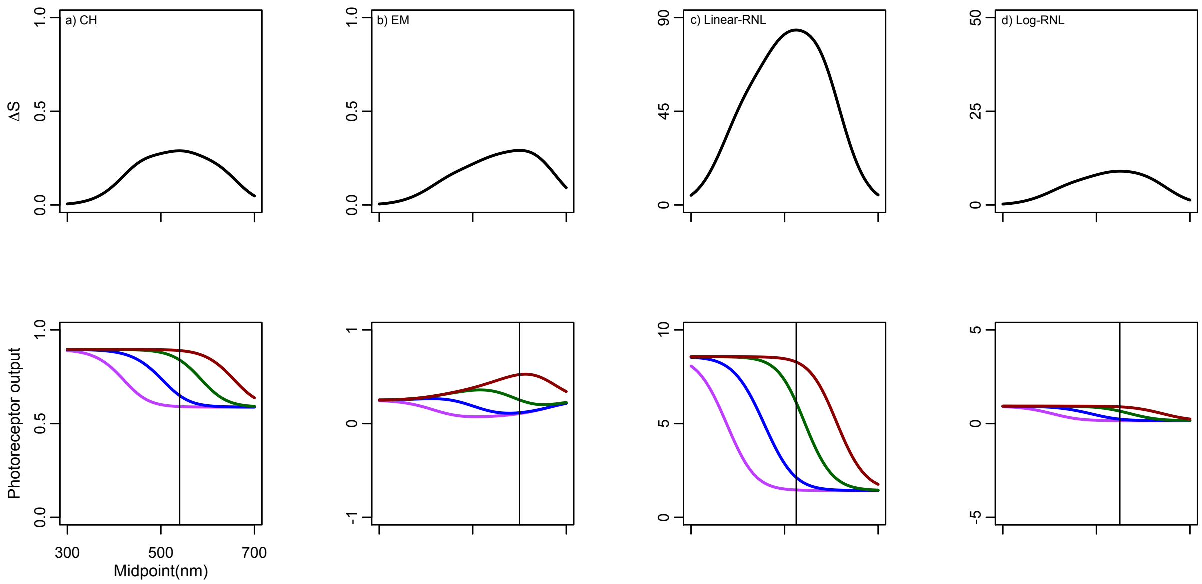

Figure S7. ΔS and photoreceptor outputs of the second setup of colour vision model simulations - 10 percent point added to stimulus reflectance spectra: a) Chittka (1992); b) Endler & Mielke (2005), and b) linear and c) log-linear Receptor Noise Limited models (Linear-RNL and Log-RNL; Vorobyev & Osorio 1998; Vorobyev *et al.* 1998). Variation in ΔS-values as a function of reflectance spectra with midpoints from 300 to 700nm (top row). Photoreceptor output values as a function of the same reflectance spectra (bottom row). Violet, blue, green, and red colours represent UV, short, middle and long λ_{max} photoreceptor types. Vertical lines represent midpoint of maximum ΔS-values. For comparison, scales are the same as in Figure S5.

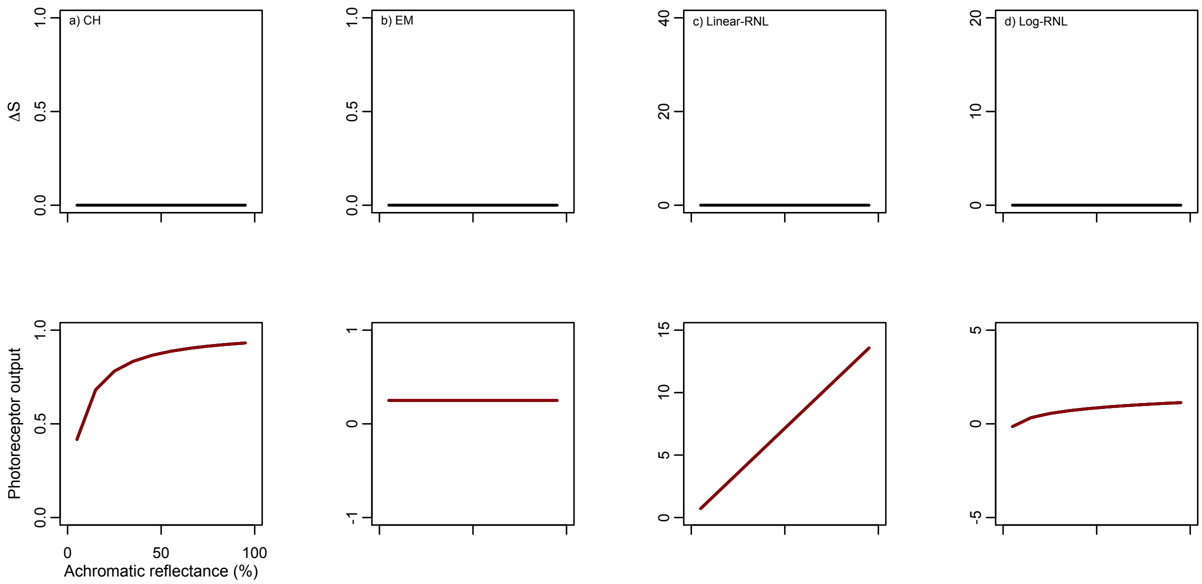

Figure S8. ΔS and photoreceptor outputs of the third setup of colour vision model simulations – achromatic stimulus against achromatic background: c) Chittka (1992), b) Endler & Mielke (2005), and b) linear and c) log-linear Receptor Noise Limited models (Linear-RNL and Log-RNL; Vorobyev & Osorio 1998; Vorobyev *et al.* 1998). Variation in ΔS-values as a function of spectra with achromatic reflectance from 5% to 95% (top row). Photoreceptor output values as a function of the same reflectance spectra (bottom row). Photoreceptors are colour coded by their λ_{max} photoreceptor, however they do not appear because are all superimposed. With the exception of c) Linear-RNL, scales are the same as in Figure S5.

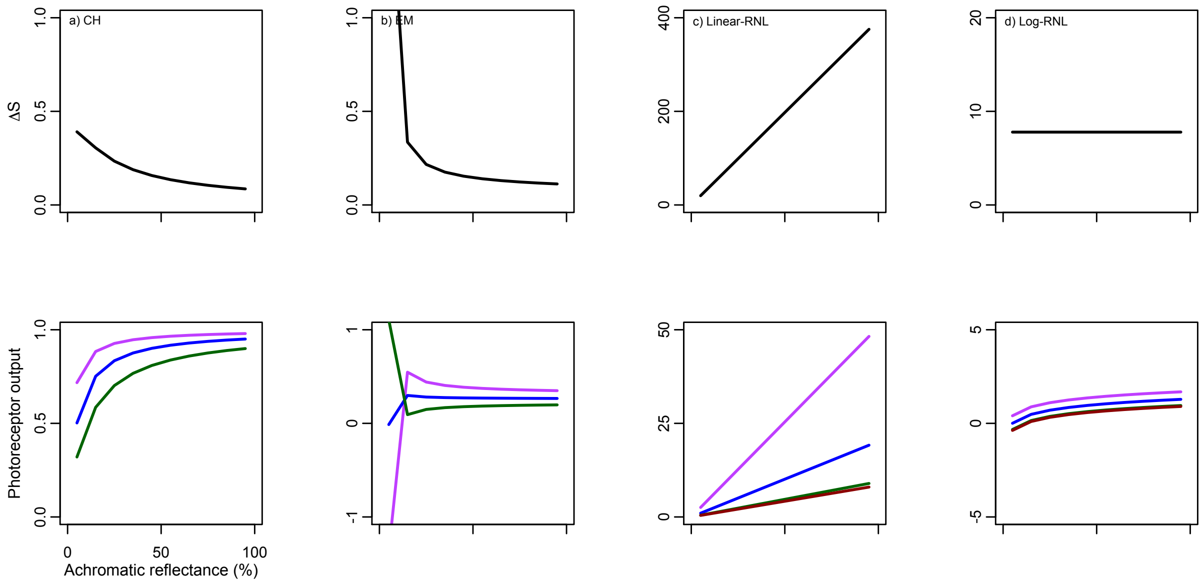

Figure S9. ΔS and photoreceptor outputs of the fourth setup of colour vision model simulations – achromatic stimulus against chromatic background: a) Chittka (1992), b) Endler & Mielke (2005), and c) linear and d) log-linear Receptor Noise Limited models (Linear-RNL and Log-RNL; Vorobyev & Osorio 1998; Vorobyev *et al.* 1998). Variation in ΔS-values as a function of spectra with achromatic reflectance from 5% to 95% (top row). Photoreceptor output values as a function of the same reflectance spectra (bottom row). Violet, blue, green and red colours represent UV, short, middle and long λ_{max} photoreceptor types. With the exception of c) Linear-RNL, scales are the same as in Figure S5.

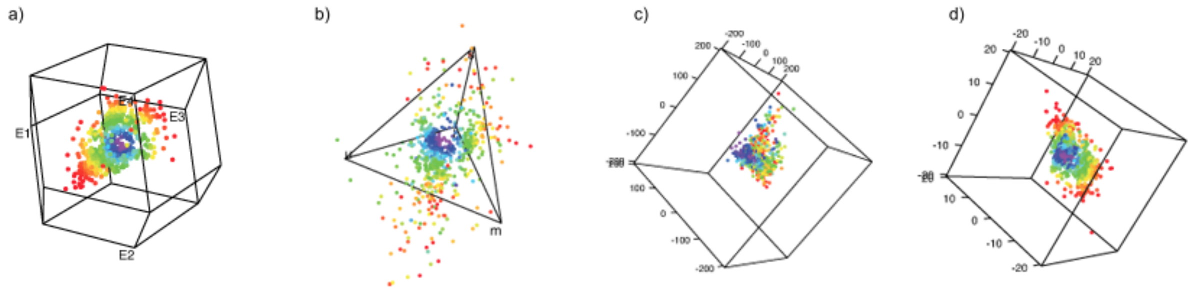

Figure S10. Flower reflectance spectra (N=858) projected into chromaticity diagrams: a) Chittka (1992), b) Endler & Mielke (2005), and c) linear and d) log-linear Receptor Noise Limited models (Linear-RNL and Log-RNL; Vorobyev & Osorio 1998; Vorobyev *et al.* 1998). To facilitate model comparison, point colours correspond to chromaticity distances in the CH chromaticity diagram.

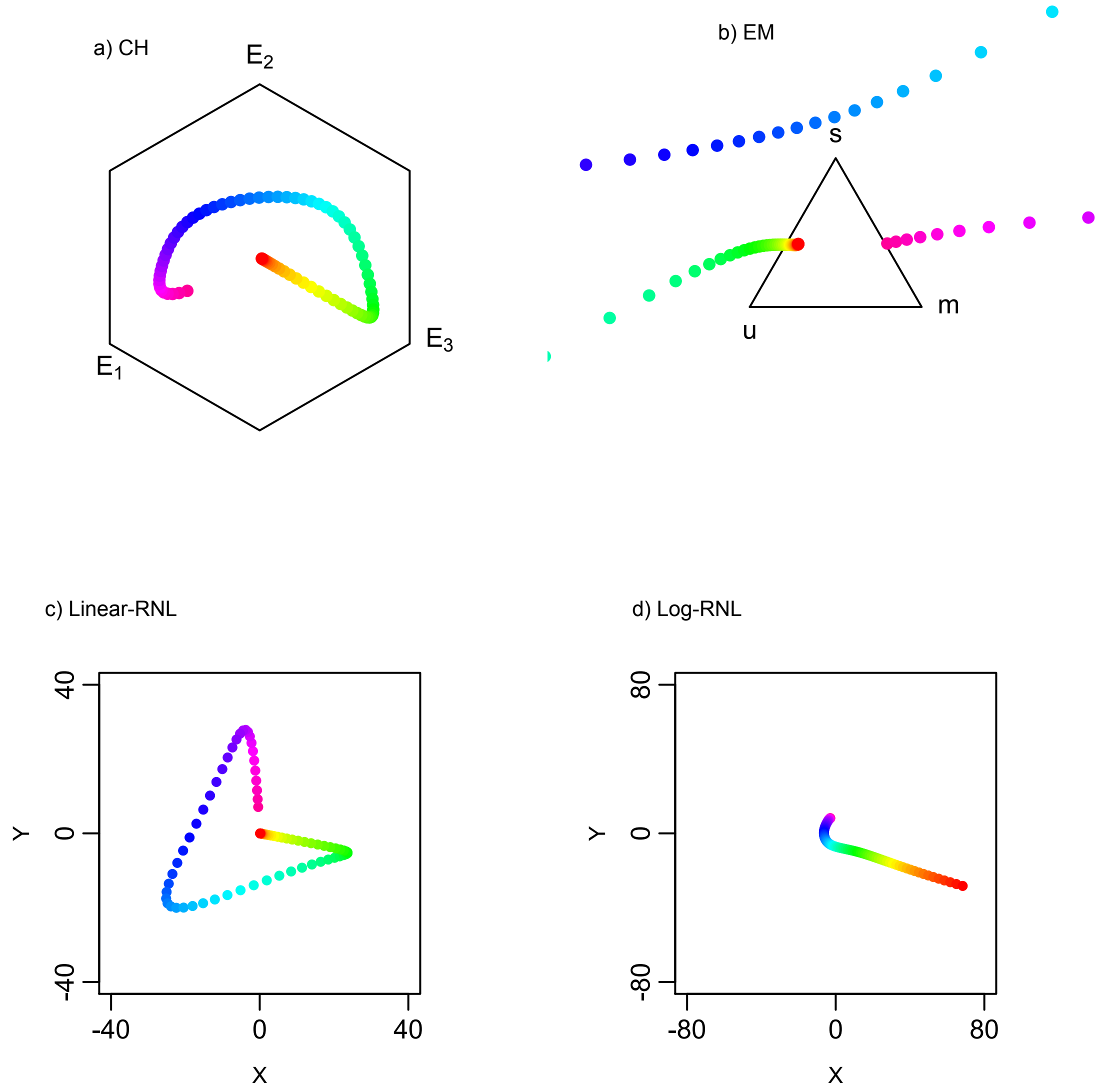

Figure S11. Chromaticity diagrams of the basic setup of colour vision model simulations with Gaussian reflectance spectra: Chittka (1992) colour hexagon (CH), Endler & Mielke (2005) colour triangle (EM), and linear and log-linear Receptor Noise Limited models (Linear-RNL and Log-RNL; Vorobyev & Osorio 1998; Vorobyev et al. 1998). Colours correspond to reflectance spectra from Figure S2.

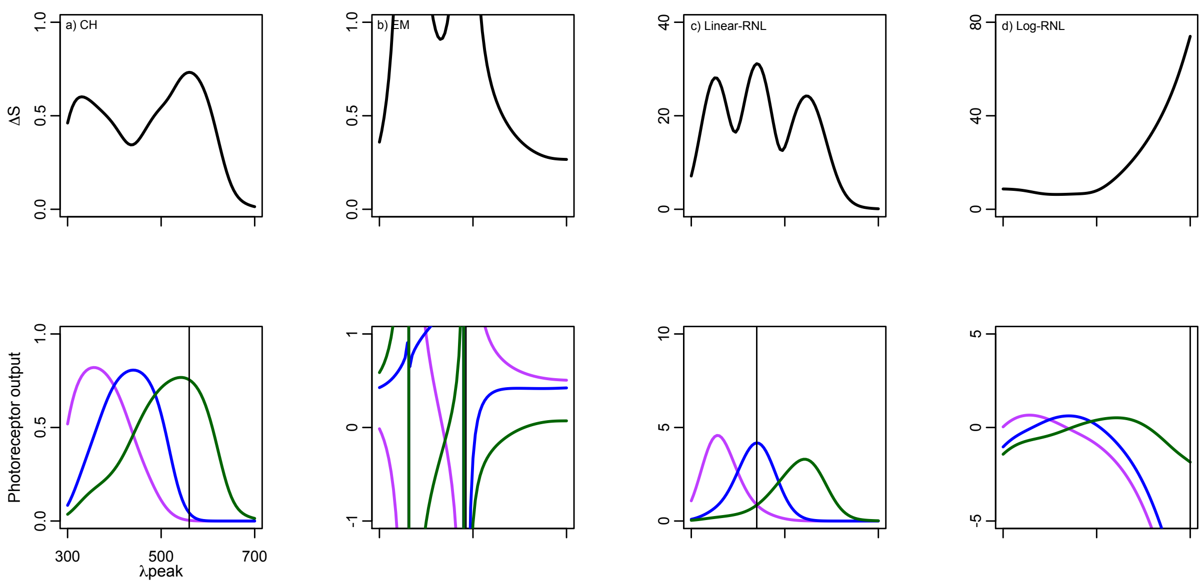

Figure S12. ΔS and photoreceptor outputs of the basic setup of colour vision model simulations with Gaussian reflectance curves (Figure S2): Chittka (1992) colour hexagon (CH), Endler & Mielke (2005) colour triangle (EM), and linear and log-linear Receptor Noise Limited models (Linear-RNL and Log-RNL; Vorobyev & Osorio 1998; Vorobyev *et al.* 1998). Variation in ΔS-values as a function of wavelengtn of maximum reflectance (λ_{peak}) from 300 to 700nm (top row). Photoreceptor output values as a function of the same reflectance spectra (bottom row). Violet, blue and green colours represent short, middle and long λ_{max} photoreceptor types. Vertical lines represent midpoint of maximum ΔS-values.

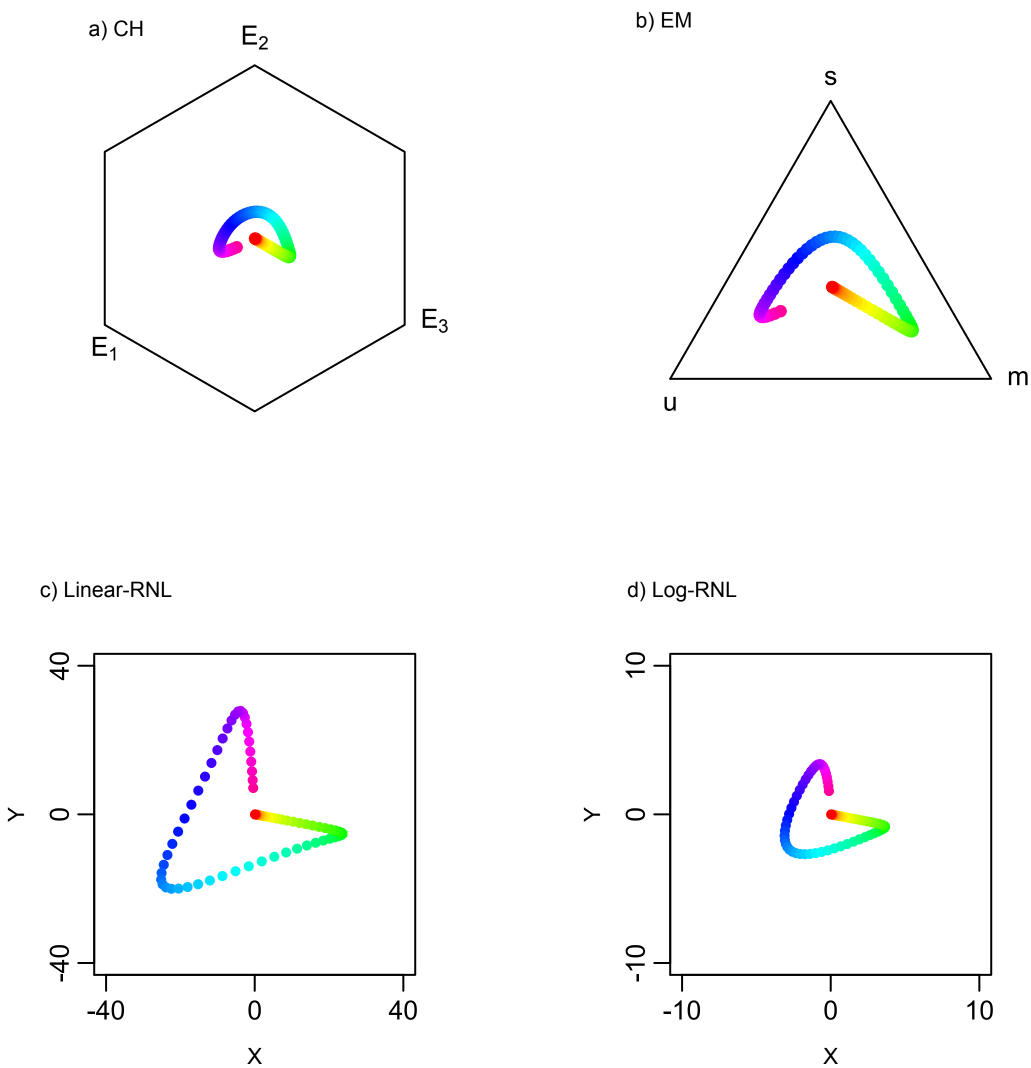

Figure S13. Chromaticity diagrams of the second simulation – 10 percent point added to reflectance values (Figure S3): Chittka (1992) colour hexagon (CH), Endler & Mielke (2005) colour triangle (EM), and linear and log-linear Receptor Noise Limited models (Linear-RNL and Log-RNL; Vorobyev & Osorio 1998; Vorobyev *et al.* 1998). Colours correspond to reflectance spectra from Figure S3.

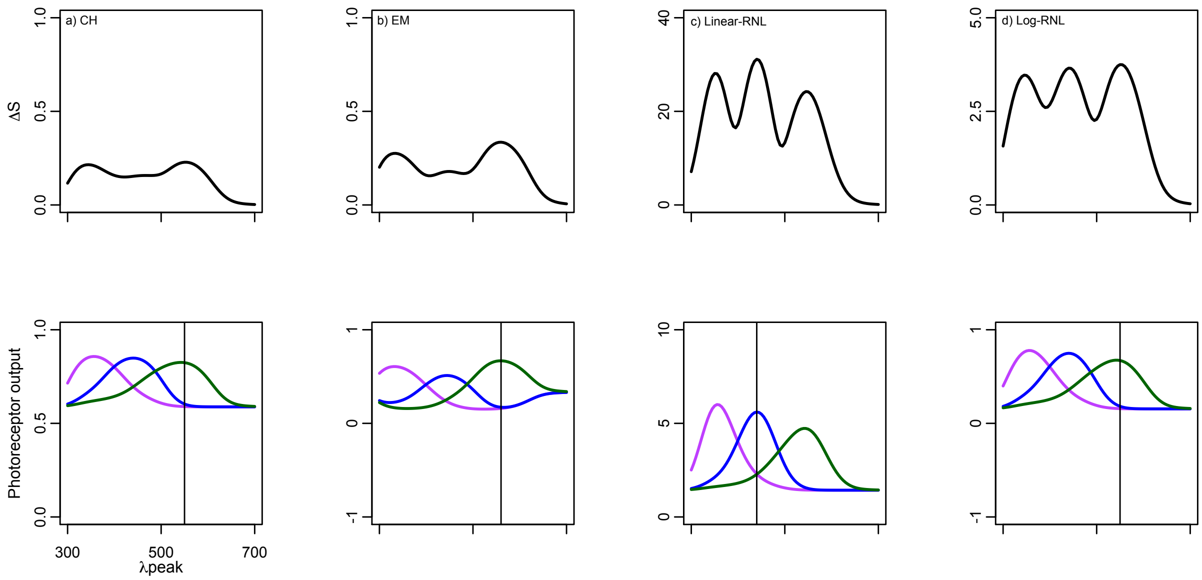

Figure 6. ΔS and photoreceptor outputs of the second setup of colour vision model simulations - 10 percent point added to stimulus reflectance spectra (Figure S3): Chittka (1992) colour hexagon (CH), Endler & Mielke (2005) colour triangle (EM), and linear and log-linear Receptor Noise Limited models (Linear-RNL and Log-RNL; Vorobyev & Osorio 1998; Vorobyev *et al.* 1998). Variation in ΔS-values as a function of reflectance spectra with wavelength of maximum reflectance (λ_{peak}) from 300 to 700nm (top row). Photoreceptor output values as a function of the same reflectance spectra (bottom row). Violet, blue and green colours represent short, middle and long λ_{max} photoreceptor types. Vertical lines represent midpoint of maximum ΔS-values. For comparison, scales are the same as in Figure 4.