THEORETICAL MODEL

A theoretical model was established to denote our hypothesis that base evolutionary forces have led to co-adaptation between defense and life span, affecting the balance of cost vs. benefit trade-offs between defense and life span. Based on the assumption that the evolution of defense is driven mainly by development and that life span is in itself optimal in different environments, we established a model for the optimal level of defense for different life span (L) scenarios (early vs. late flowering plants), taking into account costs of allocating resources away from growth and reproduction to defense. The cost of defense is proportional to the amount of resources needed for production and maintenance of that defense (McKey 1974). The energy (σ) available for a plant is limited by natural resources and has to be divided between defense (d) and growth (g).

$$d + g = 1 = \sigma$$

$$g = 1 - d$$

Plants face a trade-off between size at reproduction and chances of survival upon infection. The more a plant grows until it produces seeds, the higher the number of seeds it can produce, and hence its fitness (W) (Mitchell-Olds 1996). On the other hand, survival is crucial for completing the reproductive cycle.

$$W = \text{final size } s * \text{survival rate } r$$

The amount of energy invested into defense over growth was determined by the risk of dying before reproduction, which was defined as the risk of becoming infected (parasite prevalence τ per unit of time, $\tau * L$) multiplied with the risk of dying when infected (virulence υ, which is a function of d, $0 \leq \upsilon \leq 1$). This meant a survival rate of $0 \leq \mathbf{d} \leq \sigma$.

$$s\ (r\ (\sigma, g), L) = \sigma * g * L = \sigma r\ (\sigma - d)\ L$$

$$\tau * L * d + 1 - \tau * L = 1 - \tau * L * (1 - d)$$

How did fitness then change with defense and life span?

$$W\ (d) = \sigma\ (\sigma - d) * L * [1 - \tau * \upsilon * L\ (1 - d)] \quad |\ \sigma = 1$$

$$W\ (d) = (1 - d) * L * [1 - \tau * \upsilon * L\ (1 - d)] = (1 - d)^2 * L^2 * \tau * \upsilon + (1 - d) * L$$

$$W\ (d) = (1 - d) * L - (1 - d)^2 * L^2\ \tau * \upsilon$$

Maximizing for defense and growth:

$$W\ (g) = g * L - g2 * L^2 * \tau * \upsilon$$

$$\frac{\partial\, W\ (g)}{\partial\, g} = L - 2g * L^2 * \tau * \upsilon \doteq 0$$

$$g = \frac{L}{2\, L^2 * \tau * \upsilon} = \frac{1}{2\, ft * \tau * \upsilon} = 1 - d$$

The optimal level of defense to produce the highest fitness in each life span scenario would then be:

$$d = 1 - \frac{1}{2\, L * \tau * \upsilon}$$

Based on this model, we expected a positive correlation between life span and strength of defense, meaning that in annual plants, where lifespan scales with flowering time, late flowering plants would have stronger defense than early flowering plants.