Supplementary information for the article "How evolution draws trade-offs"

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SI figures

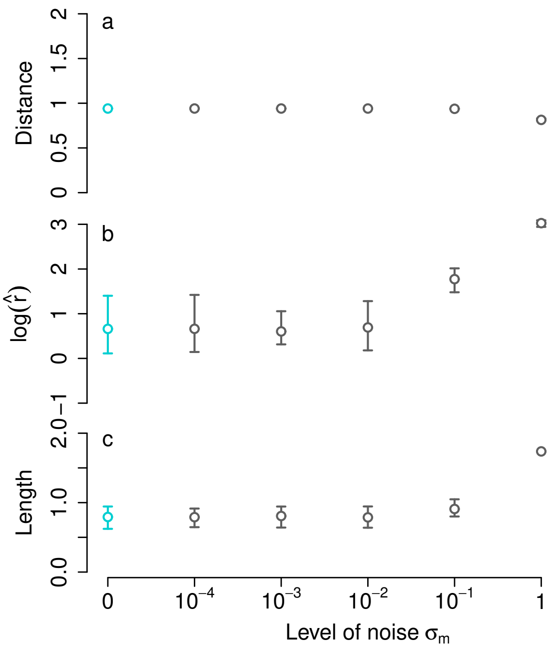

Figure S1 – The trade-off shape changes in response to the level of noise in hormones and receptors gene expressions. The noise is modeled by multiplying each genetically encoded gene expression by 10^{ϵ}, where $\epsilon = \mathcal{N}(0, \sigma_{\mathbf{n}})$. Dots and error bars represent the mean and between-quantile difference $(q(0.9) - q(0.1))$ of the distribution of shape parameters calculated in 50 replicate populations (see main text for details)

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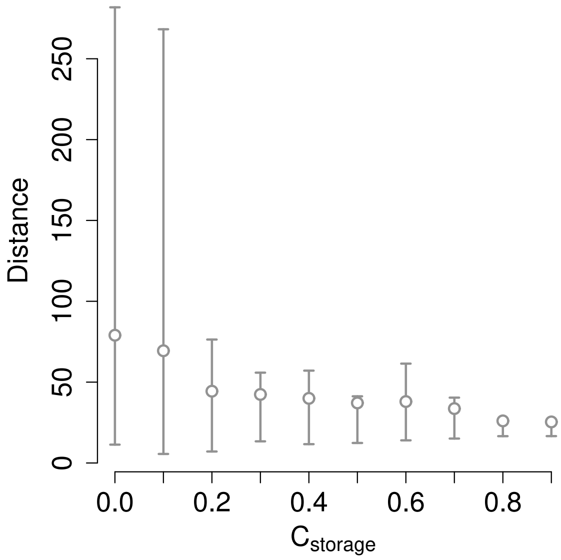

Figure S2 – The mean population difference in conformation between hormone and receptor decreases as the cost associated with storage increases. Dots and bars represent respectively to the mean and the difference between quantiles (0.1 and 0.9) of the distribution of conformation differences.

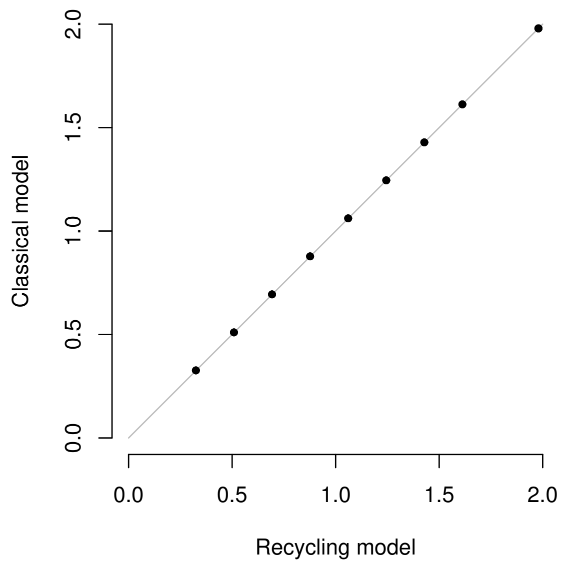

Figure S3 – Comparison of fitnesses obtained in our standard model with those obtained in a model where receptors only are recycled – instead of both the hormone and the receptor being recycled. In this latter model, eq. [1] becomes $\frac{d[H_k]}{dt} = \alpha_k - \sum_{i=1}^{n_r} \sum_{j=1}^{n_s} \left(k_{\mathsf{on}_{ik}} \times [R_{ij}] \times [H_k] \right) - k_d \times [S_D] \times [H_k]$, such that the term corresponding to the hormone released after complex dissociation is canceled. Dots correspond to different genotypes, which were randomly generated by simulating three mutations from the initial conditions defined in the main text. The line represents the situation where both models yield identical fitnesses.

SI text 1 – Solution for the energy allocation dynamics

Equations (10) and (11) in the main text yield three distinct phases for the energy allocation dynamics, as illustrated by figure 1.

Phase 1. As stated in the main text, phase 1 starts at t_0 (the meal) and goes on as long as $[E] > E_{\mathrm{om}}$, $E_{\mathrm{om}} = 0.08$ being a concentration threshold above which energy is stored. At t_0, $[E] = Eo$ and decreases until $[E] = E_{\mathrm{om}}$ (t_1). From equation (10), we obtain

$$[E](t) = \frac{b \times E_{\mathrm{om}}}{a+b} \times (1 - \mathrm{e}^{-(a+b)\times t}) + Eo \times \mathrm{e}^{-(a+b)\times t}. \tag{S1}$$

We find t_1 from the equality $[E](t_1) = E_{\mathrm{om}}$:

$$t_1 = -\ln\left(\frac{E_{\mathrm{om}} \times a}{Eo \times (a+b) - b \times E_{\mathrm{om}}}\right) \times \frac{1}{a+b} \tag{S2}$$

Finally, substituting $[E](t)$ in equation (11) we obtain :

$$[Es](t_1) = \frac{b \times (1 - C_{\mathrm{storage}})}{a+b} \times \left(-a \times E_{\mathrm{om}} \times t_1 + (Eo - \frac{b \times E_{\mathrm{om}}}{a+b}) \times (1 - e^{-(a+b)\times t_1})\right) \tag{S3}$$

Phase 2. Here the resource is released from the storage structure until $[E_{\mathrm{s}}](t) = 0$ (t_2). At this point, we have :

In order to obtain t_2, we assume that $[E]$ is constant during this phase, so $\frac{d[E]}{dt} = 0$ and $b([E] - E_{\mathrm{om}}) = -a[E]$. Therefore,

$$\frac{d[E_{\mathrm{s}}]}{dt} = -a[E]. \tag{S4}$$

From equation (S4) we obtain

$$[E_{\mathrm{s}}](t) = [E_{\mathrm{s}}](t_1) + a \times t_1 \times \frac{bE_{\mathrm{om}}}{a+b} - a \times E_{\mathrm{om}} \times t, \tag{S5}$$

which equals 0 at t_2. We thus find t_2 :

$$t_2 = (E_{\mathrm{s}}(t_1) + a \times \frac{b \times E_{\mathrm{om}}}{(a+b)} \times t_1) \times \frac{(a+b)}{a \times b \times E_{\mathrm{om}}}. \tag{S6}$$

Finally we calculate :

$$[E](t_2) = \frac{b \times E_{\mathrm{om}}}{a+b} \times (1 - \mathrm{e}^{-(a+b)\times t_2}) + Eo \times \mathrm{e}^{-(a+b)\times t_2} \tag{S7}$$

Phase 3. Phase 3 begins when $[E_{\mathrm{s}}]$ reaches 0, such that $[E]$ decreases until it reaches the critically low value $E_{\mathrm{min}} = 0.01$ (t_3). During this phase, we find from equation (10) that

$$[E](t) = \frac{b \times E_{\mathrm{om}}}{a+b} \times \frac{e^{-a\times t}}{e^{-a\times t_2}}, \tag{S8}$$

and we find t_3 by substituting $[E](t_3)$ by E_{min} in equation (S8) :

$$t_3 = \frac{\ln(E_{\mathrm{min}} \times \frac{(a+b)}{b\times E_{\mathrm{om}}}) - a \times t_2}{-a} \tag{S9}$$