Supplemental Information

Circuit models of low dimensional shared variability in cortical networks

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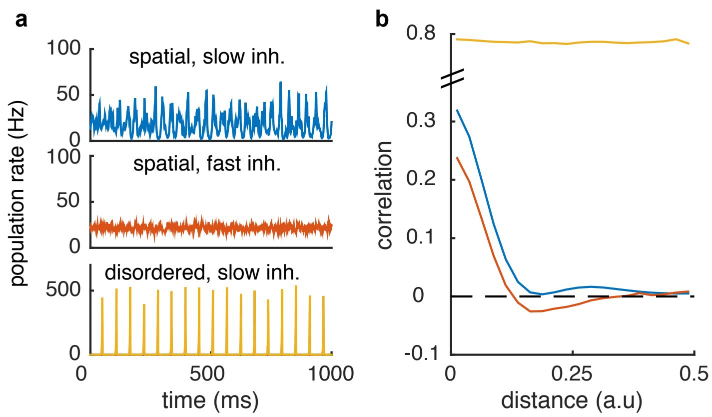
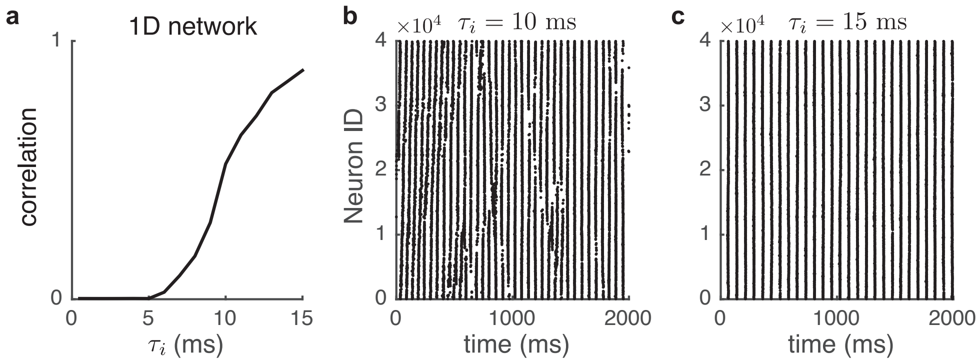
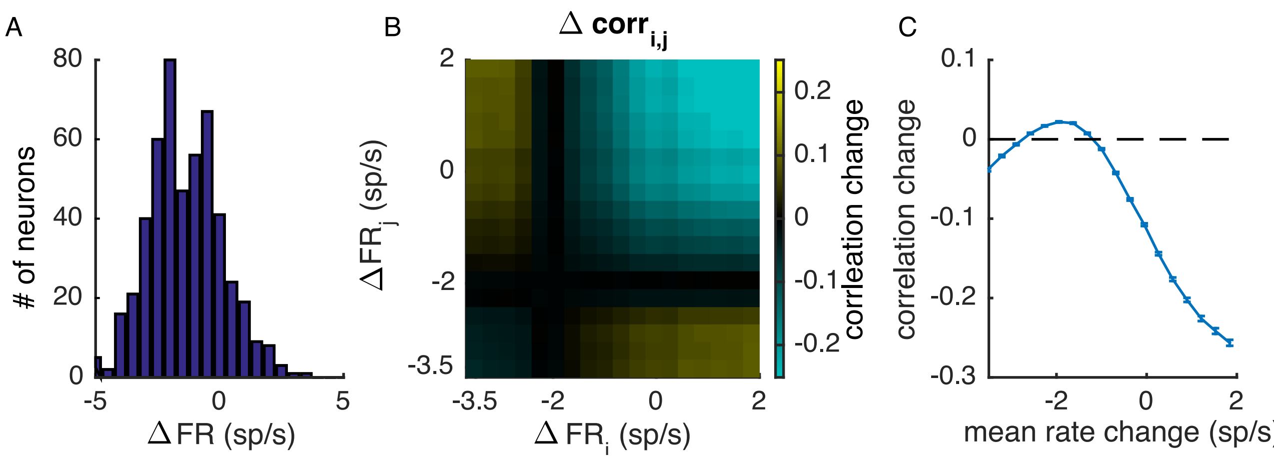


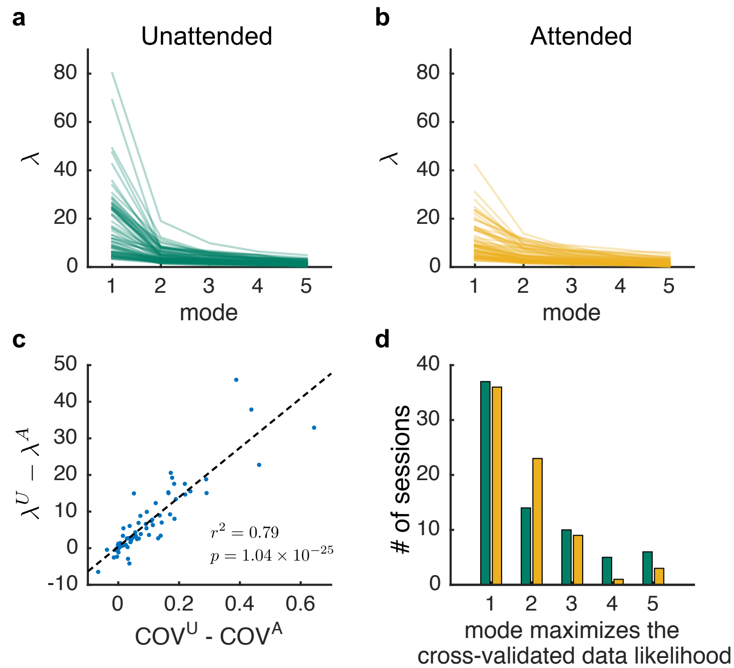
Supplemental Figure S1: **a**, Examples of mean population rates from a spatially ordered model with slow inhibition (blue) (Fig. 2b, green), a spatially ordered model with fast inhibition (red) (Fig. 2b, purple), and a disordered model with slow inhibition (Fig. 2a, green). **b**, Pair-wise correlation as a function of distance between neuron pairs for the three models.



Supplemental Figure S2: **a**, One-dimensional spatial model shows a rapid increase in mean pairwise correlation with increasing time scale of inhibitory synaptic current (compare with Fig. 2e). **b**, Example rasters of network activity when $\tau_i = 10$ ms. **c**, Same as **b** with $\tau_i = 15$ ms. Parameters of the one dimensional model are the same as those in the two-dimensional spatial model in Fig. 2b, except that neurons are ordered on interval [0, 1].



Supplemental Figure S3: Relationship between correlation change and firing rate change of the pair by attention. (A) Histogram of firing rate change by attention. (B) Correlation change of neuron pair i and j as a function for firing rate change of neuron i (x-axis) and firing rate change of neuron j (y-axis). (C) Correlation change as a function of the mean firing rate change of the pair. Parameters of the network are the same as Fig. 3 with $\mu_I = 0.2$ pA for unattended state and $\mu_I = 0.35$ pA for attended state. A total of 500 excitatory neurons are sampled from MT layer and there is a total of 2025 spike counts per neuron to compute the correlation for each attentional state. Δ=Attended-Unattended.



Supplemental Figure S4: Factor analysis of the multi-electrode recordings from V4[1]. **a**, The first five largest eigenvalues of the shared component of the spike count covariance matrix. Each line is for data from each recording session (72 in total, trial number and unit number of each session see Table S1). **b**, Same as **a** for the attended state. **c**, The difference between the largest eigenvalues and the difference between the mean covariances from the unattended and attended states are correlated. **d**, Histogram of the modes that maximize the cross-validated data likelihood across sessions. More details see Experimental methods and Statistical methods.

Session #	Unit #	Trial # (Att.)	Trial # (Unatt.)	Session #	Unit #	Trial # (Att.)	Trial # (Unatt.)
1	50	702	361	37	21	364	744
2	80	361	702	38	38	744	364
3	27	408	349	39	22	306	338
4	49	349	408	40	44	338	306
5	26	795	1038	41	46	516	480
6	35	1038	795	42	62	480	516
7	56	280	624	43	43	620	509
8	73	624	280	44	55	509	620
9	26	1109	992	45	43	601	543
10	47	992	1109	46	52	543	601
11	30	673	824	47	40	484	658
12	47	824	673	48	62	658	484
13	33	597	622	49	37	452	320
14	79	622	597	50	63	320	452
15	24	242	619	51	37	548	657
16	51	619	242	52	65	657	548
17	14	261	583	53	39	369	282
18	39	583	261	54	57	282	369
19	26	399	666	55	41	413	501
20	46	666	399	56	53	501	413
21	25	696	800	57	35	601	486
22	46	800	696	58	49	486	601
23	30	751	861	59	36	392	414
24	56	861	751	60	61	414	392
25	25	670	724	61	35	483	421
26	50	724	670	62	50	421	483
27	28	470	414	63	32	592	433
28	52	414	470	64	55	433	592
29	25	1728	1843	65	35	692	439
30	50	1843	1728	66	53	439	692
31	32	515	643	67	36	655	437
32	67	643	515	68	49	437	655
33	16	412	566	69	36	390	352
34	42	566	412	70	47	352	390
35	19	752	730	71	32	507	385
36	42	730	752	72	71	385	507

Table S1: Units number and trial number of each recording session for the Factor analysis of the multi-electrode recordings from V4[1] (Fig. 4a, Fig. S4).

Supplementary Methods

Hidden variable model

First, we consider the attentional effect on noise correlations in one cortical area. Let R be the response variable of the neurons in that area and H be the external source of variability which projects to R with strength β; $R = X + \beta H$. Here we take the variance of H, $\mathrm{Var}^\alpha(H)$, to be dependent on the attentional state $\alpha \in \{U, A\}$, and for now X is an attention independent source of fluctuating input. Denote $P_H = \frac{\beta^2 \mathrm{Var}^U(H)}{\mathrm{Var}^U(R)} = \frac{\beta^2 \mathrm{Var}^U(H)}{\mathrm{Var}(X) + \beta^2 \mathrm{Var}^U(H)}$ as the influence of H on R ($0 < P_H < 1$). Then the constraint on H is:

$$\frac{\underset{U-A}{\Delta}\mathrm{Var}(H)}{\mathrm{Var}^U(H)} = \frac{1}{P_H}\frac{\underset{U-A}{\Delta}\mathrm{Var}(R)}{\mathrm{Var}^U(R)}. \tag{1}$$

Here $\underset{U-A}{\Delta}\mathrm{Var}(H) = \mathrm{Var}^U(H) - \mathrm{Var}^A(H)$ (same for $\underset{U-A}{\Delta}\mathrm{Var}(R)$). The population data provide values for $\underset{U-A}{\Delta}\mathrm{Var}(R)/\mathrm{Var}^U(R)$. In Fig. 1c (main text) the change in $\mathrm{Var}(H)$, $\underset{U-A}{\Delta}\mathrm{Var}(H)/\mathrm{Var}^U(H)$, is plotted as a function of the influence of H on R, P_H, with the V4 data (Fig. 1b) determining $\underset{U-A}{\Delta}\mathrm{Var}(R)/\mathrm{Var}^U(R) = 0.3$.

Next, we consider the correlation between two cortical areas. Let R be the neural response from MT and X be the neural response from V1. Suppose all the variability in R is from X and the transfer function of X to R can be linearly approximated as $\delta R = \alpha \delta X$, then

$$\begin{aligned}\mathrm{Var}(R) &= \alpha^2 \mathrm{Var}(X),\\ \mathrm{cov}(R, X) &= \alpha \mathrm{Var}(X).\end{aligned}$$

which gives

$$\mathrm{Var}(R) = \mathrm{cov}(R, X)^2/\mathrm{Var}(X). \tag{2}$$

Hence any decrease in $\mathrm{Var}(R)$ by attention predicts a decrease in $\mathrm{cov}(R, X)$, which is in contradiction with the electrophysiological recordings[2] (Fig. 1b,d).

Assume a hidden source of variability, H, that projects to both R and X with strengths β and κ, respectively. Specifically, $R = \alpha X + \beta H$ and $X = X_0 + \kappa H$, where $\mathrm{cov}(X_0, H) = 0$. Suppose $\mathrm{Var}(X) = 1$ and β and κ are attention independent, then

$$\begin{aligned}\mathrm{Var}(R) &= \alpha^2 + \left(\beta^2 + 2\beta\kappa\alpha\right)\mathrm{Var}(H),\\ \mathrm{cov}(R, X) &= \alpha + \beta\kappa \mathrm{Var}(H).\end{aligned}$$

In order to have an attention-mediated simultaneous reduction in Var(R) and an increase in cov(R, X) we need α increases and Var(H) decreases with attention. The relative change in cov(R, X) by attention is

$$\frac{\underset{A-U}{\Delta}\text{cov}(R,X)}{\text{cov}^U(E,X)} = \frac{\underset{A-U}{\Delta}\alpha - \beta\kappa \underset{U-A}{\Delta}\text{Var}(H)}{\alpha_U + \beta\kappa\text{Var}^U(H)}. \tag{3}$$

An attention-mediated increase of the correlation between V1 ans MT implies $\underset{A-U}{\Delta}\text{cov}(R, X) > 0$, which gives

$$\underset{A-U}{\Delta}\alpha > \beta\kappa \underset{U-A}{\Delta}\text{Var}(H). \tag{4}$$

The reduction in Var(R) by attention is

$$\begin{aligned}
\underset{U-A}{\Delta}\text{Var}(R) &= -\left(\alpha_A^2 - \alpha_U^2\right) + \beta^2 \underset{U-A}{\Delta}\text{Var}(H) + 2\beta\kappa\left(\alpha_U\text{Var}^U(H) - \alpha_A\text{Var}^A(H)\right) \\
&= -\left(2\alpha_U + \underset{A-U}{\Delta}\alpha\right)\underset{A-U}{\Delta}\alpha + \beta^2\underset{U-A}{\Delta}\text{Var}(H) + 2\beta\kappa\alpha_U\underset{U-A}{\Delta}\text{Var}(H) - 2\beta\kappa\text{Var}^A(H \\
&= \beta^2\underset{U-A}{\Delta}\text{Var}(H) - 2\alpha_U\underset{A-U}{\Delta}\text{cov}(R,X) - \left(\underset{A-U}{\Delta}\alpha\right)^2 - 2\beta\kappa\text{Var}^A(H)\underset{A-U}{\Delta}\alpha
\end{aligned}$$

Hence the relative reduction in Var(R) is

$$\frac{\underset{U-A}{\Delta}\text{Var}(R)}{\text{Var}^U(R)} = \frac{\underset{U-A}{\Delta}\text{Var}(H)}{\text{Var}^U(H)}P_H - 2\alpha_U\frac{\underset{A-U}{\Delta}\text{cov}(R,X)}{\text{cov}^U(R,X)}\frac{\text{cov}^U(R,X)}{\text{Var}^U(R)} - \frac{\left(\underset{A-U}{\Delta}\alpha\right)^2 + 2\beta\kappa\text{Var}^A(H)}{\text{Var}^U(R)} \tag{5}$$

The second term from the RHS of Eq. (5) is

$$\begin{aligned}
2\alpha_U\frac{\text{cov}^U(R,X)}{\text{Var}^U(R)}\frac{\underset{A-U}{\Delta}\text{cov}(E,X)}{\text{cov}^U(E,X)} &= \frac{2\alpha_U^2 + 2\beta\kappa\alpha_U\text{Var}^U(H)}{\alpha_U^2 + (\beta^2 + 2\beta\kappa\alpha_U)\,\text{Var}^U(H)}\frac{\underset{A-U}{\Delta}\text{cov}(R,X)}{\text{cov}^U(R,X)} \\
&> (1 - P_H)\frac{\underset{A-U}{\Delta}\text{cov}(R,X)}{\text{cov}^U(R,X)}
\end{aligned}$$

With inequality (4), the third term from the RHS of Eq. (5) is

$$\begin{aligned}
\frac{\left(\underset{A-U}{\Delta}\alpha\right)^2 + 2\beta\kappa\mathrm{Var}^A(H)\underset{A-U}{\Delta}\alpha}{\mathrm{Var}^U(R)} &= \frac{\underset{A-U}{\Delta}\alpha\left(\underset{A-U}{\Delta}\alpha + 2\beta\kappa\mathrm{Var}^A(H)\right)}{\beta^2\mathrm{Var}^U(H)}P_H \\
&> \frac{\left(\beta\kappa\underset{U-A}{\Delta}\mathrm{Var}(H)\right)\left(\beta\kappa\underset{U-A}{\Delta}\mathrm{Var}(H) + 2\beta\kappa\mathrm{Var}^A(H)\right)}{\beta^2\mathrm{Var}^U(H)}P_H \\
&= \frac{\beta^2\kappa^2\underset{U-A}{\Delta}\mathrm{Var}(H)\left(\mathrm{Var}^U(H) + \mathrm{Var}^A(H)\right)}{\beta^2\mathrm{Var}^U(H)}P_H \\
&> \kappa^2\mathrm{Var}^U(H)\frac{\underset{U-A}{\Delta}\mathrm{Var}(H)}{\mathrm{Var}^U(H)}P_H
\end{aligned}$$

Hence,

$$\frac{\underset{U-A}{\Delta}\mathrm{Var}(R)}{\mathrm{Var}^U(R)} < \left(1-\kappa^2\mathrm{Var}^U(H)\right)\frac{\underset{U-A}{\Delta}\mathrm{Var}(H)}{\mathrm{Var}^U(H)}P_H - \frac{\underset{A-U}{\Delta}\mathrm{cov}(R,X)}{\mathrm{cov}^U(R,X)}(1-P_H)$$

which gives the constraint on H as

$$\frac{\underset{U-A}{\Delta}\mathrm{Var}(H)}{\mathrm{Var}^U(H)}P_H > \frac{\underset{U-A}{\Delta}\mathrm{Var}(R)}{\mathrm{Var}^U(R)}\frac{1}{1-\kappa^2\mathrm{Var}^U(H)} + \frac{\underset{A-U}{\Delta}\mathrm{cov}(R,X)}{\mathrm{cov}^U(R,X)}\frac{1-P_H}{1-\kappa^2\mathrm{Var}^U(H)}. \tag{6}$$

where $0 < 1-\kappa^2\mathrm{Var}^U(H) = \frac{\mathrm{Var}^U(X_0)}{\mathrm{Var}(X)} < 1$, since $\mathrm{Var}(X) = \mathrm{Var}(X_0) + \kappa^2\mathrm{Var}(H) = 1$. Therefore, the lower bound on $\frac{\underset{U-A}{\Delta}\mathrm{Var}(H)}{\mathrm{Var}^U(H)}P_H$ increases with the relative increase in $\mathrm{cov}(R,X)$, $\frac{\underset{A-U}{\Delta}\mathrm{cov}(R,X)}{\mathrm{cov}^U(R,X)}$, and the projection strength of H on X, κ.

Fig. 1c (bottom) is plotted using Eq. (1) and Fig. 1e (bottom) is plotted using Eq. 6 with $\frac{\underset{A-U}{\Delta}\mathrm{cov}(R,X)}{\mathrm{cov}^U(R,X)} = 1$ and $\kappa^2\mathrm{Var}(H)$ ranges from 0 to 0.5 for different curves.

Supplementary Movie Captions

Movie S1: Spiking activities of a spatially ordered network with fast inhibition (Fig. 2aii). Each black dot indicates that the neuron at spatial position (x; y) fired within one millisecond of the time stamp shown on top.

Movie S2: Same as Movie S1 for a spatially ordered network with slow inhibition (Fig. 2aiv).

Movie S3: Spiking activities of the MT excitatory population from the three layered spiking network model in the unattended state (Fig. 3a, 4b).

Movie S4: Same as Movie S3 for a network with broader inhibition in MT (Fig. 4d).

References

1. Cohen, M. and Maunsell, J. Attention improves performance primarily by reducing interneuronal correlations. *Nature neuroscience* **12**(12), 1594–1600 (2009).

2. Ruff, D. A. and Cohen, M. R. Attention increases spike count correlations between visual cortical areas. *Journal of Neuroscience* **36**(28), 7523–7534 (2016).